CHAPTER – 1

THEORY OF OPTICAL POLARISATION

1.1 INTRODUCTION

Light

When Light is considered as a transversely oscillating electromagnetic wave propagating from a source, only the electrical field is considered. Amplitude, frequency and phase are essential parameters. Because the electric field vibration is transverse to the direction of propagation, the orientation of the electric field vector (e-vector) is to be accounted for.

![Diagram of light propagation](image1)

Light propagation in an isotropic medium

Polarization

Polarization is the state of the e-vector orientation. Using XYZ coordinate system, Z is the direction of propagation. Since light is a transverse wave, the
polarization state can be analyzed by projecting the e-vector onto arbitrary orthogonal axes called X and Y and then evaluating these projected components. Viewed across time, the relationship of X and Y projections may be fully or partly disordered, and any ordered portion will have a phase and amplitude relation between X and Y components.

**The possible states of polarization**\(^1\)\(^2\):

**Un polarized**: A completely disordered, chaotic orientation of the e-vector over time is referred to as an un-polarized state. In this state, at any instant there is no relation of the E-vector to prior or future orientation.

**Partial polarization**:

A statistical preference for one polarization state over others is partial polarization. Light in nature is generally partially polarized.

**Linear polarization**: When the e-vector oscillates within a plane that is constant over time it is said to be linearly polarized. This plane is called the plane of polarization.

A linear polarizer transmits only the portion of incident light that is projected along its pass axis, regardless of the incident light’s degree or state of polarization. This portion can be anywhere from nearly 100% of the incident light to nearly zero. Depending upon the type of polarizer, the remainder can be reflected, refracted or absorbed. Plastic sheet polarizer reject the unwanted component by absorption, and typically transmit less than 75% even along the pass axis. Wire grid polarizer reflect and transmit orthogonal linear polarization states, and can work in strongly converging beams across a wide wavelength range, but have low extinction ratios especially at shorter wavelengths approaching the dimension of the grid spacing. Thin film polarizer’s separate the portions into reflected and transmitted beams, usually
with better than 98% efficiency, but work well only within a limited spectral and angular range. Crystal polarizer’s either reflect or refract the rejected portion, without significant absorption of either portion, and can achieve extinction ratios of 10-6:1 over a broad spectral range, but only over a small range of incident angles.

**Circular polarization:**

In this type of polarization, the phase difference between equal X and Y projections is one quarter of a wavelength. In the example of the wave propagating vertically downward, the e-vector encountering the table could point N, then E, then S, then W. This is called left circular, because it is counterclockwise when looking into the beam. It is useful to visualize a threaded rod pushing forward along its length without rotating; the threads representing the e-vector. For a stationary plane, the e-vector describes a circle over time. At any instant the e-vector describes a helix through space, but the entire helix is in constant motion forward. The case of right circular polarization could be visualized as an N-W-S-E progression of the threads.

**Elliptical polarization:**

This is more a general case than circular polarization, in which there is a phase difference between the two components. Elliptical polarization is the result of components that are either equal with non-quarter-wave phase difference, or unequal with any nonzero phase difference.
Random polarization: Due to the nature of certain lasers, random state of polarization is distinct from un-polarized state of polarization. Randomly polarized lasers usually emit a linearly or elliptically polarized beam whose orientation changes at random but finite intervals, usually to an orthogonal state.

Polarizer

Polarizer is a material that suppresses the light vibrating in different planes and allows only in one particular plane. If two polarizer’s are arranged along the optical axis and their optical axes are parallel, light passes through both of them. However, if the axes are set up at 90 degrees (crossed), the polarized light from the first fixed polarizer is absorbed by the second polarizer called analyzer. As the angle rotates from 0 to 90 degrees, the amount of light that is transmitted decreases. This effect is demonstrated in the fig.1.2. The polarizer’s are parallel at the top and crossed at the bottom.
Fig. 1.2

Polarizer with vertical transmission axis

Polarizer with horizontal transmission axis

Crossed polarizer’s
The relation between plane of incidence and plane of polarization: S and P

The plane of polarization contains the e-vector and the direction of propagation. Any optical components in the path are not considered in the definition. The plane of incidence includes the direction of propagation of the light and the surface normal to the component and is arbitrary for normal incidence. Polarization is not considered in the definition. The relationship between these two planes is of critical importance.

What is called as 'P' and 'S' polarization refers to the relation between these two planes. Returning to the example of vertical propagation, N-S linear. A mirror is placed in the beam path tilted so that the beam will reflect horizontally North or South. The beam is then P polarized with respect to the mirror. P polarization is defined as the condition where the plane of incidence includes the Plane of polarization. If the mirror is arranged to reflect the same beam horizontally east or west, the beam is considered S (senkrecht) polarized with respect to the mirror. Those more familiar with the English language than with German may find it helpful to remember these cases as ones where the e-vector "Pokes" the mirror in its vibrations, or "Slips" side to side on the Mirror. One can also imagine the orientation of a flat stone thrown onto water and use P for "Plunge" and S for "Skip".

Interaction of polarized light with matter:

Interaction with surfaces

Normal incidence surface reflection and transmission through isotropic materials are unaffected by polarization. At non-normal incidence S polarization will almost always reflect more than P Polarization, and with increasing angles it reaches 100%.
At Brewster's angle, defined as the angle whose tangent is equal to the ratio of the indices on either side of a surface, P polarized reflection goes to zero, and in the absence of absorption, Transmission reaches 100%. As the angle of incidence increases beyond $\varphi_B$, P- reflection also increases to 100%.

When light is incident upon a higher index material, 100% reflection is approached at grazing incidence. Using Snell's law for refraction the corresponding angle of internal propagation, known as the critical angle can be obtained. When light encounters the surface from the high index side, 100% reflection for P and S is attained at the critical angle and above. This phenomenon is known as total internal reflection (TIR).

While pure P or S polarization will remain P or S upon reflection or transmission, any other incident polarization state will exhibit changes. It is always preferable to choose P polarization for oblique transmission, S polarization for oblique reflection, and near normal incidence for monochromatic light.

**Historical Review**

The rotation of the orientation of linearly polarized light was first observed in 1811 in quartz by French physicist François Jean Dominique Arago. Around this same time, Jean Baptist Biot also observed the effect in liquids and gases of organic substances such as turpentine. In 1822, the English astronomer Sir John F.W.Herschel discovered that different crystal forms of quartz rotated the linear polarization in different directions. Simple polarimeters are in use since this time to measure the concentrations of simple sugars, such as glucose, in solution. In fact, one name for glucose, dextrose, refers to the fact that it causes linearly polarized light to rotate to the
right or Dexter side. Similarly, laevulose, more commonly known as fructose, causes the plane of polarization to rotate to the left. Fructose is even more strongly levorotatory than glucose which is dextrorotatory. Invert sugar, formed by adding fructose to a solution of glucose, gets its name from the fact that the conversion causes the direction of rotation to "invert" from right to left.

In 1849, Louis Pasteur\(^6\) resolved a problem concerning the nature of tartaric acid. A solution of this compound derived from living things (specifically/ wine lees) rotated the plane of polarization of light passing through it, but tartaric acid derived by chemical synthesis had no such effect, even though its reactions were identical and its elemental composition was the same. Pasteur noticed that the crystals came in two asymmetric forms that were mirror images of one another. Sorting the crystals by hand gave two forms of the compound: solutions of one form rotated polarized light clockwise, while the other form rotated light counterclockwise. An equal mix of the two had no polarizing effect on light. Pasteur deduced the molecule in question was asymmetric and could exist in two different forms that the two asymmetric crystals form, dextro and levorotatory, of tartaric acid.

In 1874, Jacobus Henricus van’t Hoff\(^7\) and Joseph Achille Le Bel independently proposed that the Phenomenon of optical activity could be explained by assuming that the chemical bonds between carbon atoms and their neighbors were directed towards the corners of a regular tetrahedron. This led to a better understanding of the three-dimensional nature of molecules.
1.2 Optical activity

**Optical rotation** or **optical activity** is the rotation of linearly polarized light as it travels through certain materials. It occurs in solutions of chiral molecules such as sucrose (sugar), solids with rotated crystal planes such as quartz, and spin-polarized gases of atoms or molecules. It is used in the sugar industry to measure syrup concentration, in optics to manipulate polarization, in chemistry to characterize substances in solution, and is being developed as a method to measure blood sugar concentration in diabetic people.

Optical activity is a type of birefringence. Any linear polarization of light can be written as an equal combination of right-hand (RHC) and left-hand circularly (LHC) polarized light:

$$E_{\theta_0} = E_{RHC} + e^{i2\theta_0}E_{LHC}$$

where $E$ is the electric field of the light. **The relative phase between the two circular polarizations, $2\theta_0$, sets the direction of the linear polarization to $\theta_0$.** In an optically active material the two circular polarizations experience different refractive indices. The difference in the indices quantifies the strength of the optical activity,

$$\Delta n = n_{RHC} - n_{LHC}$$

This difference is a characteristic of the material (for substances in solution it is given as the specific rotation). After traveling through length $L$ of material, the two polarizations pick up a relative phase of,

$$2\Delta \theta = \frac{\Delta n L 2\pi}{\lambda}$$
where $\lambda$ is the wavelength of the light (in vacuum). Consequently, the final polarization is rotated through an angle of $\theta_0 + \Delta \theta$. Generally, the refractive index depends on the wavelength.

**The above discussion shows the importance of converting linearly polarized light into circularly polarized light using quarter wave plate.**

The variation in rotation with the wavelength of the light is called optical rotatory dispersion (ORD). ORD spectra and circular dichroism spectra are related through the Kramers–Kronig relations. Complete knowledge of one spectrum allows the calculation of the other.

In summary, the degree of rotation depends on the color of the light (the yellow sodium D line near **589nm** wavelength is commonly used for measurements), the **path length** $L$ and the properties of the material (e.g. $\Delta n$ or specific rotation and concentration).

**For a pure substance in solution, if the wavelength and path length are fixed and the specific rotation is known, the observed rotation can be used to calculate the concentration.** This usage makes the polarimeter a tool of great importance to those who trade in or use sugar syrups in bulk.

In the presence of magnetic fields, all molecules have optical activity. A magnetic field aligned in the direction of light propagating through a material will cause the rotation of the plane of linear polarization. This effect popularly referred to as the Faraday Effect is one of the first discoveries of the relationship between light and electromagnetic effects. Optical activity or rotation should not be confused with
circularly polarized light. **Circularly polarized light is often presented as a linear polarization in rotation as the light propagates. However, the polarization completely rotates through a length equal to the wavelength** (roughly one micrometer) and it can happen in vacuum.

In contrast, optical activity only occurs in a material and a complete rotation occurs in a length of millimeters to meters, depending on the material.

**1.3 Rotating polarizer:**

Laser beam passes through a rotating polarizer and then into a detector that measures intensity as shown in Fig.1.3 The polarizer also needs to be attached to a motor so that it rotates at a constant frequency. The transmitted intensity of the beam through a polarizer varies proportionally to \( \cos^2 (\theta) \) where \( \theta \) is the angle between the incident electric field wave and the direction of polarization.

To determine the intensity of a laser beam transmitted through a polarizer, a laser beam is set to pass through a polarizer while it is rotated at a constant speed. A detector is placed on the other side of the polarizer, to measure the changed intensity. By determining the frequency of the rotation, the offset of the lasers polarization and the polarizer’s direction, a \( \cos^2 \theta \) function can be tested to see if it fits the transmitted intensity that is measured. References \(^8\)-\(^10\) have used the Rotating Polarizing Technique for phase Transition studies. The method is simple and accurate.
A liquid crystal sample is placed between crossed polarizers, whose transmission axis is aligned at some angle between the fast and slow direction of the material. Because of the Birefringence nature of the sample, the incoming linearly polarized light becomes elliptically polarized. When this ray reaches the second polarizer, there is now a component that can pass through and the region appears bright. For monochromatic light (single frequency), the magnitude of the phase difference is determined by the length and the birefringence of the material. If the sample is very thin, the ordinary and extraordinary components do not get very far out of phase. Likewise, if the sample is thick, the phase difference can be large. If the phase difference equals 360 degrees, the wave returns to its original polarization state and is blocked by the second polarizer. The size of the phase shift determines the intensity of the transmitted light.

If the transmission axis of the first polarizer is parallel to either the ordinary or extraordinary directions, the light is not broken up into components and no change in the Polarization State occurs. In this case, there is no transmitted component and the
region appears dark. In a typical liquid crystal, the birefringence and lengths are not constant over the entire sample. This means that some areas appear light and others appear dark. The light and dark areas denote regions of differing director orientation, birefringence and length.

1.4 Theory of Rotating polarizer

The laser emits a beam that has an electric field of magnitude,

\[ E = E_{\text{max}} \cos(kx + \omega t) \]  \hspace{0.5cm} (1)

Where \( E_{\text{max}} \) is the amplitude of the electric field from the wave, \( k \) is the wave number, and \( \omega \) is the angular frequency, \( x \) is the distance from the source and \( t \) is the time at which the wave was emitted from the source. The intensity of the wave is given by,

\[ I = \frac{E_{\text{max}}B_{\text{max}}}{2\mu_0} \]  \hspace{0.5cm} (2)

Where \( B_{\text{max}} \) is the amplitude of the magnetic field from the wave, and \( \mu_0 \) is the permeability of free space. Since,

\[ B = \frac{E}{c} \]  \hspace{0.5cm} (3)
Where c is the speed of light. This gives,

$$I = \frac{E_{max}^2}{2c\mu_0}$$

(4)

When the beam reaches the polarizer, the only component of the electric field that is transmitted is the part that lines up with the direction of the polarizer. This gives an $E_{max}$ at the detector to be,

$$E_d = E_e \cos(\theta)$$

(5)

Where $E_d$ is the max electric field at the detector, $E_e$ is the max electric field being emitted from the laser, is the angle between the polarizer and the electric field. As the polarizer is rotated at a constant frequency, $f$, is found from,

$$\theta = 2\pi f (t + \phi),$$

(6)

Where $f$ is the frequency of the rotating polarizer and $\phi$ is the offset due to the polarizer not starting from an angle of 0 to the electric field emitted. It then becomes,

$$I = \frac{E_{max}^2 \cos(2\pi f(t + \phi))^2}{2c\mu_0}$$

(7)
If all the constants are then combined into amplitude of A, “I” is given to vary with time by,

\[ I = A\cos^2[2\pi f(t + \phi)] \]  

(8)

1.5 Optics of Liquid Crystals

The fact that light polarized along different axes travels at different velocities makes a liquid crystal a retarder. The retardation angle \( \Delta \phi \) (in radians) is simply given by the birefringence \( \Delta n \), vacuum wavelength \( \lambda_0 \), and thickness of the sample \( d \).

To obtain the Jones matrix when the director is at an angle to the horizontal axis the fact that a rotation of the coordinate system by an angle puts the director at an angle \( \theta \) with the horizontal axis must be borne in mind. So all that needs to be done is to transform MH to the proper coordinate system using rotation matrices:

\[ \Delta \phi = \frac{2\pi \Delta n d}{\lambda_0} \]

If the director is vertical, then the Jones matrix for the liquid crystal is

\[ M_V = \begin{bmatrix} e^{ix} & 0 \\ 0 & e^{iy} \end{bmatrix} \]

\[ = \begin{bmatrix} e^{-i\Delta \phi/2} & 0 \\ 0 & e^{i\Delta \phi/2} \end{bmatrix} = e^{-i\Delta \phi/2} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\Delta \phi} \end{bmatrix} \]

If the director is horizontal, then the Jones matrix is slightly different:
\[ M_H = \begin{bmatrix} e^{i\alpha} & 0 \\ 0 & e^{i\alpha} \end{bmatrix} = \begin{bmatrix} e^{i\Delta\phi/2} & 0 \\ 0 & e^{-i\Delta\phi/2} \end{bmatrix} = e^{i\Delta\phi/2} \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\Delta\phi} \end{bmatrix} \]

\[ M_\theta = R(\theta)M_H R^{-1}(\theta) = R(\theta)M_H R(-\theta), \]

Where

\[ R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \]

Doing this gives the Jones with respect?matrix for a liquid crystal sample whose director is at an angle \( \alpha \) to the horizontal.

\[ M_\theta = e^{i\Delta\phi/2} \begin{bmatrix} \cos^2 \theta + \sin^2 \theta e^{-i\Delta\phi} & \sin \theta \cos \theta (e^{-i\Delta\phi} - 1) \\ \sin \theta \cos \theta (e^{-i\Delta\phi} - 1) & \sin^2 \theta + \cos^2 \theta e^{-i\Delta\phi} \end{bmatrix} \]

The goal of the experiment is to measure the birefringence of a liquid crystal as a function of temperature. If the liquid crystal sample is placed between two horizontal polarizers, one can use the Jones matrices for horizontal polarizer to derive the intensity after the second horizontal polarizer

\[ I(\theta) = I_0 \left[ 1 - \sin^2 \left( \frac{\Delta\phi}{2} \right) \sin^2 (2\theta) \right] \]

where \( I_0 \) is the intensity of the light after the first horizontal polarizer. So if values of \( I(\theta) \) are measured for many values of \( \theta \), a fit to the above function yields the value of \( \Delta\phi \). Knowing the wavelength of light and the thickness of the sample is all that is necessary to determine the birefringence from the formula given below.

\[ \Delta\phi = \frac{2\pi \Delta n d}{\lambda_0} \]
There is one interesting subtlety that arises when measuring birefringence. Determining the phase retardation $\theta$ can be done in many ways and can usually be carried out without complications. **But the intensity $I(\theta)$ is a cyclic function of the phase retardation $\Delta \phi$ with a period of $\pi$.** In order to determine the birefringence from the phase retardation, one must know which half cycle of the function is appropriate. This can be accomplished by either repeating the measurement with a sample of different thickness or using another wavelength of light with the same sample. Since the birefringence is slightly wavelength dependent, and the wavelength can only be changed by a small amount, the former method usually works the best. **This problem can be overcome in Lock-in-amplifier as the phase angle measured is independent of change in intensity.**

**Chirality:**

A chiral molecule is one that is not super imposable on its mirror image. It has the property of rotating the plane of polarization of plane-polarized monochromatic light that is passed through it. This phenomenon is called optical activity. The term **chirality** was first observed by Lord Kelvin in 1873. In the past decades chiral surfaces have received attention for their potential applications in chiral selection such as in stereo selective chemical synthesis, separation of chiral compounds, crystal growth, protein absorption, and optical activity. Usually only one origin of chirality that is a single carbon atom bearing four different substituent groups is considered. The impression given is that this arrangement is the only condition for chirality, which is not true.
The necessary and sufficient condition is:-

Non-superimposability of a molecule on its mirror image is a **necessary and sufficient condition** for Chirality; though no exception has ever been found. The non-superimposability can come about in a number of ways, and need not involve a chiral centre or even organic molecules at all.

**Molecules with restricted rotation:**

Restricted rotation about a single double bond is well known as a potential source of geometric isomerism, though it isn't the only one. Restricted rotation can also give rise to chirality.

![Diagram](image.png)

**Fig. 1.4. Definition of chirality in tilted smectic phase of bent-core molecules.**

$\hat{Z} \times \hat{n} = \left\{ \begin{array}{ll} +\hat{b} & : (+) \text{ chirality} \\ -\hat{b} & : (-) \text{ chirality} \end{array} \right.$

$Z$ is the layer normal, $b$ is the polar vector, and two possible chiral layer structures are defined by the director $n$ tilting by either along the positive (cyan-colored) or negative (magenta-colored) $X$-axis.
1.6 Application of Polarized Light Studies to Technology and Liquid Crystals:

**Polarization** has a plethora of Technological applications. The most commonly encountered example is the liquid crystal display. All radio transmitters and receivers are intrinsically polarized, special use of which is made in RADAR. In engineering, the relationship between strain and birefringence motivates the use of polarization in characterizing the distribution of stress and strain in prototypes. Electronically controlled birefringent devices are used in combination with polarizing filters such as modulators in fiber optics.