INTRODUCTION

Graph theory is one of the fastest growing areas of mathematics today. It is a delightful playground for mathematicians to exploit proof techniques in discrete mathematics and to explore its application in many areas of computing, social and natural sciences.

In 1987, the frontiers of topological graph theory are being explored in numerous United Kingdom, the United States and Yugoslavia. A few of the frontier topics would include algorithms for covering-space construction, enumeration of graphs, forbidden subgraphs, characteristic properties of the relationship to graph minors, genus of graphs, computational connection, representation of higher dimensional complexes in and VLSI pay out etc.

The concept of measure of a graph is an interesting concept in the field of graph theory. It helps in determining the way to bring out the concept of multidimensional graphs. This can be done in two ways, one is the edge version and the other is the vertex version.
INTRODUCTION

Graph theory, is one of the fastest growing areas in Mathematics today. It is a delightful playground for the exploration of proof techniques in discrete mathematics and its results have application in many areas of computing, social and natural science.

In 1987, the frontiers of topological graph theory are advancing in numerous different directions, each pursued by a most of researchers including particularly active schools in France, Italy, the United Kingdom, the United States and Yugoslavia. A partial listing of Frontier topics would include algorithm for embedding problems, covering-space construction, enumeration analysis of embedding distribution, forbidden subgraphs characterizations and their relationship to graph minors, genus of groups, map theoretic connection; representation of higher dimensional manifolds by graphs and VLSI pay out etc.

The concept of measure of a set as well as a function was an interesting concept in the field of Real Analysis. In this thesis we try to bring out the concept of measure of a graph. This can be done in two ways, one is the edge version and the other is the vertex analog.
Let $G$ be a graph and $\mathcal{G}$ be a $\sigma$-field whose elements are spanning subgraphs of $G$. Then a non-negative extended real valued countably additive graph function $\mu_e : \mathcal{G} \to \mathbb{R}$ is said to be an edge measure on $G$ and the triplet $(G, \mathcal{G}, \mu_e)$ is called an edge measure space. Similarly if $\mathcal{I}$ is a $\sigma$-field of induced subgraph of $G$, then a non-negative extended real valued countably additive graph function $\mu_v : \mathcal{I} \to \mathbb{R}$ is said to be a vertex measure on $G$ and the triplet $(G, \mathcal{I}, \mu_v)$ is said to be a vertex measure space. The edge measure $\mu_e$ is said to be an edge probability measure if $\mu_e(G) = 1$ and is denoted by $P_e$. Similarly the measure $\mu_v$ is said to be vertex probability measure if $\mu_v(G) = 1$.

The concept of intersection graph was introduced in the field of graph theory in 1940's. In this thesis we try to bring out the concept of union graphs and its application to group theory and topology. Let $X$ be a non-empty set and $\mathcal{S}$ be any collection of subsets of $X$. The union graph denoted by $\mathcal{U}(\mathcal{S})$ of $\mathcal{S}$ is defined as a graph whose vertex set is $\mathcal{S}$ itself and two distinct vertices say $S_i$ and $S_j$ in $\mathcal{S}$ are said to be adjacent in $\mathcal{U}(\mathcal{S})$ if and only if $S_i \cup S_j \in \mathcal{S}$. A graph $G$ is said to be a union graph on $X$ if $\mathcal{U}(\mathcal{S}) \cong G$ for some collection $\mathcal{S}$ of subsets of $X$. If we drop the only if condition in the above definition, the resultant graph is said to be a weak union graph. Its application to group theory and topology has been discussed in this thesis.
We also try to extract the concept of union group-graph, and intersection group-graph in this thesis. Let $S_G$ be a group and $\mathcal{H}_G$ be a collection of subgroups of $S_G$. The *union group-graph* on $\mathcal{H}_G$ is a graph $\mathcal{U}(\mathcal{H}_G)$ whose vertex set is $\mathcal{H}_G$ itself and two distinct vertices $L_i$ and $L_j$ are said to be adjacent in $\mathcal{U}(\mathcal{H}_G)$ if and only if either $L_i \subseteq L_j$ or $L_j \subseteq L_i$. A graph $G$ is said to be a *union group-graph* if $\mathcal{U}(\mathcal{H}_G) \cong G$ for some collection $\mathcal{H}_G$ of subgroups of $S_G$. If $\mathcal{H}_G$ be a collection of subgroups of $S_G$ other than the identity $\{e\}$, then the *intersection group-graph* on $\mathcal{H}_G$ is a graph $\mathcal{A}(\mathcal{H}_G)$ whose vertex set is $\mathcal{H}_G$ itself and two vertices $L_i$ and $L_j$ for $i \neq j$ are adjacent in $\mathcal{A}(\mathcal{H}_G)$ if and only if $L_i \cap L_j$ contains at least one element other than $e$. A graph $G$ is said to be an *intersection group-graph* if $\mathcal{A}(\mathcal{H}_G) \cong G$ for some collection $\mathcal{H}_G$ of subgroups of $S_G$.

In the similar manner next, we define the union semigroup-graph and the intersection semigroup-graph. Let $S_g$ be a semigroup and $\mathcal{S}_g$ be a collection of subsemigroups of $S_g$. The *intersection semigroup-graph* $\mathcal{I}(\mathcal{S}_g)$ is a graph whose vertex set is $\mathcal{S}_g$ itself and two vertices $S_i$ and $S_j$ for $i \neq j$ are adjacent in $\mathcal{I}(\mathcal{S}_g)$ if and only if $S_i \cap S_j \neq \emptyset$. The *union semigroup-graph* is a graph $\mathcal{U}(\mathcal{S}_g)$ whose vertex set is $\mathcal{S}_g$ itself and two vertices $S_i$ and $S_j$ are adjacent if and only if $S_i \cup S_j$ is also a subsemigroup.
Three types of topologies on a spanning tree with its ends were discussed by Rienhard Diestel in a paper titled "End spaces and spanning trees" in 2005 [9]. In this thesis we try to define a topology on a graph $G$ called as \textit{graph topology}. Let $G$ be a graph and $\tau_g$ be a collection of spanning subgraphs of $G$. $\tau_g$ is called a \textit{graph topology} on $G$ if the following axioms are satisfied

(i) $\phi, G \in \tau_g$

(ii) $\tau_g$ is closed with respect to arbitrary union and finite intersection.

Then the pair $(G, \tau_g)$ is called a graph topological space.

In conformity of these studies we have obtained some new terminologies, parameters and results which form the content of this thesis. The thesis mainly divided into five chapters.

Chapter 1 consists of the basic definitions and result in Graph theory, Group theory, Real analysis, Topology and Probability. For graph theoretic results we follow [2] and [4], for analysis we depend upon [8] and [11]; for probability theory we follow [1] and [10], for modern algebra we follow [7] and topology we follow [6].

In chapter 2 we try to define the edge measure on a graph $G$. Some results relating these concepts have been defined in this chapter. More over $P_e$-in dependent spanning subgraphs and the relating results are also discussed here. For chapter 2 in addition we also follow [12], [13] and [14].
In chapter 3 we define a new operation called as \textit{induced union} on a typical class of graphs known as induced graphs. Further we have considered the concept of vertex measure on a graph $G$. $P_v$-independent induced subgraphs of $G$ and their properties were also discussed in chapter 3.

In chapter 4 we try to define some concepts, viz., union graph, union group-graph, intersection group-graph, union semigroup-graphs and intersection semigroup-graphs. Some results related to these concepts and their application to group theory and topology were also discussed here. For chapter 4 we additionally used [3] and [5].

In chapter 5 we introduced the concept of graph topological spaces and some results based on this concepts. For chapter 5 we follow [9] also.