CHAPTER 2

MICROSTRIP PATCH ANTENNAS AND MODELING TECHNIQUES
2.1 Introduction

Being conceived in the year 1953[2], the microstrip Patch Antenna has matured considerably over last 33 years with starting of serious work in 1972[2,3]. The major progresses in the Microstrip Patch Antennas are included; success in bandwidth enhancement, combating surface wave effects, development of compact antennas, active antennas and development in the analysis tools.

Microstrip Patch Antennas use radiating elements of a wide variety of shapes: square, rectangle, circle, ring, triangle, ellipse, star, L, semi-circle etc. More complex geometrical figures and combinations of simple shapes are also used for some particular applications [7,8,87]. The selection of a particular shape depends on the parameters one wishes to optimize: bandwidth, sidelobes, cross polarization, and antenna size.

Microstrip antennas are inherently narrowband structure due to their resonant nature and confinement of fields between the patch metallization and ground plane[4]. Hence narrow bandwidth is one of the principal disadvantages of microstrip patch antenna. In recent years, significant research contributions have been devoted to the band width enhancement technique of the microstrip patch antenna in general. The various techniques that have been suggested and tried to manipulate the impedance curve includes: use of thick substrate with low relative dielectric constant ($\varepsilon_r$)[4,8].
choice of suitable feeding technique[8,9,86], introduction of coupled modes[8,9,86], impedance matching and resistive loading [4,86]. There exists a large amount of open literature related to bandwidth enhancement technique and related solution that have been suggested [5,6,9-125].

In this chapter, a brief overview on the choice Substrate of microstrip patch antenna is included in the section 2.2. In section 2.3, discussion is included on feeding technique of microstrip patch antennas while an overview of wideband probe fed microstrip patch antenna is given in section 2.4. Section 2.5 gives an insight of modeling techniques that are currently available.

2.2 Choice of Substrate of Microstrip Patch Antenna

The dielectric substrate provides a stable support for the conductor strip and patches that make up connecting lines, resonator, and antennas. It ensures that the components that are implemented are properly located and firmly held in place, just as in printed circuits for electronics at lower frequencies. The substrate plays an important role in determining the electrical characteristics of the Microstrip Patch Antenna. There is no one ideal substrate; the choice rather depends on the application. For instance, conformal microstrips require flexible substrates,
while low frequency applications require high dielectric constant to keep size small[6]. Microstrip Patch Antennas use low dielectric substrates i.e. generally substrate of dielectric constant $\varepsilon_r < 2$. The use of higher value of dielectric constant ($\varepsilon_r$), affects the confinement of field and reduces the radiation efficiency of the Microstrip Patch Antenna. The bandwidth and efficiency variation with substrate height at centre frequency for rectangular microstrip patch for two different substrate are shown in Figure 2.1[5]. From the figure it is clear that the bandwidth increases almost linearly with substrate height and the bandwidth decreases with the increase of dielectric constant of the substrate.

Most commonly used substrate for microstrip patch antennas are: alumina or high dielectric ($\varepsilon_r$) substrate, composite material substrates, and honeycomb substrate. Honey-comb material is light-weight, sturdy and low $\varepsilon_r$ and is generally preferred for aerospace applications. Composite materials are obtained by adding fiberglass, quartz or ceramic in suitable proportion to the organic or synthetic materials to obtain the desired permittivity and the electrical and mechanical properties. The most commonly used combination is that of PTFE and glass and the resulting substrate have $\varepsilon_r$ between 2.17 and 2.55. Combination of PTFE and ceramic is used to produce flexible substrates with $\varepsilon_r$ near 10. These high $\varepsilon_r$ substrate or a alumina are used for miniaturization of microstrip antennas often at the cost of bandwidth.
2.3 Feeding Techniques of Microstrip Patch Antenna

The excitation of the radiating element is an essential and important factor, which requires careful consideration in designing a most appropriate antenna for a particular application. A wide variety of feed mechanisms are available, not just for coupling energy to individual elements, but also for the controlled distribution of energy to linear or planar array elements. The feed element may be either co-planer with the radiating elements, or situated in a separate transmission-line layer. Now there is available ample literature on the feeding technique [6,7,8,9,87]. Therefore, a brief overview of only four most popular Microstrip Antenna Feed techniques are given. These are namely: Microstrip line, Co-axial Probe, Aperture coupling and Proximity coupling.
2.3.1 Microstrip Line Feed

Microstrip line feed is based on the principle that cutting an inset in the patch does not significantly affect the resonant frequency but that it modifies the input impedance. By properly selecting the depth of the inset, one can match the path to the transmission line without additional matching elements [87]. The feed was the first used for practical applications [88] and is the simplest way to feed a microstrip patch is to connect a microstrip line directly to the edge of the patch, with both elements located on the same substrate. A microstrip line feed is shown in figure.2.2.

![Microstrip Feed](image)

**Figure 2.2** Microstrip patch antenna with microstrip line feed.

The microstrip line feed though simple in nature but a microstrip structure with the line and patch cannot be optimized simultaneously as an antenna and a transmission line. There must be some compromise between the two so that feed line does not radiate too much at the discontinuities [89]. The spurious radiation and the accumulated reactive power below the patch (cavity effect), degrades the antenna performance and reduces its bandwidth[8].
2.3.2 Co-axial Line Feed

Co-axial line feed was among the first considered and even today one of the most popular in many application of microstrip patch antenna. In co-axial line feed, the inner conductor of the coax is extends across the dielectric substrate and is connected to the patch while the outer conductor is connected to the ground plane as shown in the figure 2.3.

![Coaxial Line Feed Diagram](image)

**Figure 2.3** Microstrip Patch antenna with coaxial line feed.

In case of coaxial line feed the intrinsic radiation from the feed is small and can be neglected for thin substrates but becomes significant with thicker substrates. Now, most of the theoretical developments consider coaxial feeds and models were developed to characterize the injection of current in to patch accurately [84,90]. However, coaxial feeds are difficult to realize in practice because drilling or punching holes through the substrate in a particular specific point is critical task, generally this operation would like to avoid. Again introducing the conductor through the holes and soldered to the patch are delicate operations that require careful handling, and mechanical control of the connection is difficult, especially for very high frequencies[86].
2.3.3 Aperture Coupling

In a conventional aperture coupling, the microstrip patch antenna consists of two substrate layers separated by a common ground plane. The radiating microstrip layer on the top of the substrate is fed through an aperture in the ground plane by a microstrip feed line lying on the bottom of the lower substrate. The important requirement is that the common ground plane should contain etched apertures accurately positioned below the microstrip patch and above the feed line [8]. Figure 2.4 shows an Aperture coupled feed microstrip patch antenna.

![Aperture coupled feed microstrip patch antenna](image)

**Figure 2.4** Aperture coupled feed microstrip patch antenna.

The aperture coupled feed technique has many attractive features [9, 91-95], one is it provides stronger coupling than a similar triplet or suspended stripline system because of higher concentration of fields above the feed line where the aperture is positioned. Further more, a relatively high-permittivity substrate can be used if required for the feed system, without compromising the radiating properties of
the lower-permittivity substrate carrying the microstrip patches. In this technique, the slot on the common ground plane is free to radiate bidirectionally. By using multilayer substrate, it can be made unidirectional radiation, but may result in strongly coupled surface wave modes which degraded in the antenna efficiency [96]. The cavity backed aperture coupled technique is used to improve the efficiency of antenna [97] as well as solves the above mentioned problem.

### 2.3.4 Proximity Coupling

In this feeding technique, the coupling of the patch and the feed line is obtained by placing the patch and the feed at different substrate levels. A thin layer of high dielectric constant substrate is used to reduce the radiation from the feed lines, whereas a thick layer of low dielectric constant substrate is used in the upper layer to increase the radiation of the patch [8,86]. The length of the feeding stub and the width-to-line ratio of the path can be used to control the match.

![Proximity couple feed microstrip patch antenna](image)

**Figure 2.5** Proximity couple feed microstrip patch antenna
Using the proximity coupling, the frequency band width of a patch resonator could be significantly widened [98,99]. The special feature is that, the feed line is no longer located to an open surface and there is no need to solder different conductors, unlike co-axial feed. But a structure with two dielectric layers, however, is more complex to analyze, because the simple models developed for single layers cannot be used. The resulting structure becomes more complex to build, with two dielectric layers instead of one. Again one cannot easily connect components within the feeding circuit as it is buried inside substrate.

2.4 Wideband Probe-Fed Microstrip Patch Antenna

The microstrip antennas are having narrow impedance bandwith, typically a few percent. The impedance bandwidth of microstrip antennas is usually much smaller than the pattern band width[100]. Hence focus is given on input impedance rather than radiation pattern in the discussion of bandwidth enhancement technique. Some generic types of band width extension technique are: increasing antenna volume by incorporating parasitic elements, stacked substrates, use of foam dielectrics; creation of multiple resonances in input response by addition of external passive networks or internal structure; and incorporation of dissipative loading by adding lossy materials or resistors [8]. Pozar[101] divided these various bandwidth enhancement technique into three broad approaches: impedance matching; the use of multiple resonance; and the use of lossy materials. In this overview, the bandwidth enhancement techniques are categorised into different approaches in terms of
normally used antenna structures. These are impedance matching network; edge coupled probes; stacked coupled probes; shaped probes; capacitive coupled and slot coupled. The lossy antennas are not included here because generally lossy materials are not frequently used as it limits the radiation frequencies of the antenna.

2.4.1 Impedance Matching Networks

An impedance matching network is used to improve the impedance bandwidth of a probe feed microstrip patch antenna. In this process, without altering the antenna element, a reactive matching network is used, to compensate for the rapid frequency variations of input impedances. With this method, compared to thin substrate, a thick substrate will add some extra bandwidth. An impedance matching network is typically implemented in microstrip antenna from below the ground plane of the antenna element. The figure 2.6 shows the geometry of a microstrip patch antenna with a impedance matching network.

![Geometry of a microstrip patch antenna with a impedance matching network.](image)

**Figure 2.6** Geometry of a microstrip patch antenna with a impedance matching network.
The impedance matching network method was implemented by Pues and Capelle [14] by modeling the antenna as simple resonant circuit. Their method is unique in that it doesn't alter the radiating element itself, while a reactive matching network is used to compensate for the rapid frequency variations of the input impedance. The validity of the technique is based on the relative frequency insensitivity of the radiation pattern and gain characteristics as compared to the resonant behaviour of the input impedance. In their approach for the design of the matching network, the input impedance of a microstrip antenna should be by either a simple series resonant or a simple parallel resonant RLC circuit in the vicinity of fundamental resonant frequency. Once the RLC equivalent circuit of the antenna is obtained, a procedure similar to the design of a band pass filter[15] is used to synthesis the matching network of the microstrip patch antenna. With this approach they have achieved a increased in band width by a factor of 3.2 or 9.1 percent. Hongming An etal [16] introduced the simplified real frequency technique(SRFT) to design the loss less matching networks of microstrip antennas in order to increase bandwidth. The most significant feature of this is the numerical technique is that it does not require any analytical description of the antenna and generator, the measured or simulated impedance data are processed directly. Further more neither on a priori choice of a matching network topology nor an analytic form of the system transfer function needed. With this approach, they have managed to increase the band width for one antenna from 5.7% to 11.06% at the level of VSWR = 1.5, and for another antenna element, the band width increased from 9.4% to 16.82% for the VSWR<2. Recently, Haaj et al[30] reported an impedance bandwidth of 6.9% for a VSWR of
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1.5:1 with a parallel resonant circuit.

The advantage of using the matching network is that the radiation characteristics of the antenna element remain unchanged because the antenna element do not get altered as the matching network can be placed behind the antenna's ground plane\[8,86\]. The radiation form the matching network is also minimum. The main drawback of this methods are, the matching networks are used to excite the individual elements in an antenna array and more than one substrate layer is required to support the antenna element and the matching network for single element antennas[8,9,86].

2.4.2 Edge-Coupled Patches (Multi Mode Operation)

The basic idea that has been used to widen the frequency band width is that to increase the band width of resonant circuits, in particular when designing band pass filters, is to couple several resonators with very closely spaced resonances. In this approach – several radiating structures are closely coupled to each other but resonating slightly different frequencies. Only one of the elements is driven directly. The other patches are coupled through proximity effects. In an Edge coupled microstrip antenna the parasitic patches can be coupled to either to the radiating edges, the non-radiating edges or to both pair of edges. This approach has been investigated by wood [102] and then Kumar and Gupta [103-105]. Using this approach R.Garg and V.S.Reddy[106] achieved an impedance band width of 23% at VSWR=2. Figure 2.7 shows an example of such an edge coupled microstrip antenna.
Figure 2.7 Edge coupled microstrip antenna.

The edge coupled patches can be fabricated on a single layer substrate. This coplanar nature of the structure is the advantage of the Edge coupled patch antenna. The use of additional parasitic patches increases the size of the antenna element which is one of the major drawbacks. Another drawback is that as the different patches radiate with different amplitudes, the radiation patterns change significantly over the operating frequencies[8].

2.4.3 Stacked Patches

In another approach, two or more electromagnetically coupled patches are placed on top of one another or stacked [86]. This is also a multimode operation technique. In stacked patch structure, the surroundings of the two patches are slightly
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different; the resonant frequencies of the two patches are slightly offset, which increases the frequency bandwidth. The figure 2.8 shows a Stacked patch microstrip antenna.

![Stacked patch microstrip patch antenna](image)

**Figure 2.8** Stacked patch microstrip patch antenna.

Different size are also be stacked for the two patches, either to increase the bandwidth further or to realize an antenna operating at two different frequencies[107-110]. The frequency bandwidth of a microstrip patch antenna is also enhanced by the increase thickness of the double layer structure, with concomitant risk of surface wave excitation. It is possible to stack more patches, but the performance may not be much better than with only two patches [9,101]. Using this technique, Waterhouse [107] achieved a 25% impedance bandwidth for rectangular patches, Kokotoff et al [111] reported a 22% impedance bandwidth for annular ring patches and Mitchell et al [112] achieved a 33% impedance bandwidth for circular patches.

The advantages of stack patch technique are, it does not increase the
surfaces area of the element[8,86], it can be used in array configurations without creating grating lobes, its radiation patterns and phase centre also remains relatively constant over the operating frequency band and has the large number of parameters that can be used to optimization. At the same time it has the drawback that it require more than one substrate layer to support the patch and because of larger design parameter the optimization becomes more complex.

2.4.4 Capacitive Coupled Patches

The probe fed microstrip antenna on thick substrate shows inductive nature in the input impedances. To avoid this inductive nature, capacitive coupling technique is used. In this technique, a small probe fed patch is situated below the resonant patch [113,114]. The gap between them acts as a series capacitor. Figure 2.9 shows a capacitive coupling patch.

![Figure 2.9 Capacitive coupled microstrip patch antenna.](image)

For a patch where the capacitor patch is located below the resonant patch has advantages that, this approach do not increase the surface area of the element and the cross polarization levels in the H-plane are lower than the approach where the capacitor patch is located within the surface of the resonant patch. The approach of capacitor patch below the resonant patch has the disadvantages that, it require additional subtract layer to support the configuration and prone to alignment errors which create complexity in fabrication.

2.4.5 Slot Coupled Patches

The slot loaded microstrip patch antenna (figure 2.10) leads to increase in antenna impedance band width and a smaller size of the antenna elements. These slots force the surface currents to meander, thus, artificially increasing the antenna antenna’s electrical length without modifying its global dimensions. This effect of increase in the length of current path can be modeled as an additional series inductance. The annular slots within the surface of the patch element may also acts as a series capacitor. It is well known fact that the inductance or capacitance is key factor to widen the impedance band width of microstrip patch antenna elements[8].
In a circular resonant patch, using a annular slot around a small circular probe fed capacitor patch in the surface of the resonant patch, P.S.Hall[118] reported a 13.2% impedance band width at 10db return loss. S.K.Palit and A.Hamadi [119] achieved up to a 39% impedance band width with H-Shaped microstrip patch antenna. T.Huynh and K.F.Lee [120] reported a U-slot antenna with which can achieved an impedance band width exceed 30%. In slot loaded E-shaped microstrip patch antenna, F. Yang, X.-X. Zhang, X. Ye and Y. Rahmat-Samii[121] an impedance band width of 30.3%, Kin-Lu Wong and Wen-Hsiu Hsu [122] reported an impedance band width of 24%, while Wen-Hsiu Hsu and Kin-Lu Wong[123] reported of achieving of impedance band width exceed 25%. In a slot loaded circular microstrip antenna, J.H.Lu[124] reported an increase in impedance band width more than 2.3 times that of conventional circular microstrip patch antenna. Recently, Ricky Chair etal[125], reported of achieving an impedance band width of 28.6% with U-slot and half E-shaped microstrip patch antenna.

Figure 2.10 Slot coupled microstrip patch antenna.
The slot loaded microstrip patch antennas reduces the size of the antenna element. Microstrip patch antennas with this technique produce greater current concentration on the antennas and therefore, increase the ohmic losses and decrease the gain of the antenna. Again proper insertion of slot in accurate position on the patch and slot size optimization is a difficult task and is very much frequency sensitive.

2.5 Modeling Techniques of Microstrip Patch Antennas

2.5.1 Introduction

The most popular methods that can be used to model and analyze the probe fed microstrip patch antennas fall in to one of two broad categories: (i) approximate methods and (ii) full wave methods. The approximate method include the transmission line model[6,7,8,10], Cavity model[6,8,33,38] and segmentation model[8]. The approximate models are easy to implement for single element antenna, it gives good physical insight with very small solution time, but has the limitation of less accurate. It becomes more complex for modeling coupling between elements, with these methods. The most popular full wave methods that can be used to model probe-fed microstrip patch antennas are the method of moment(MoM), the finite element method(FEM), and the finite-difference time-domain(FDTD) method. These are the three major paradigms of full wave electromagnetic modeling.
techniques[30,126-129]. All these methods discretize the problem region and
transform the field equations into a system of linear equations. Again these methods
can be characterized into two groups, e.g. differential and integral. Differential
method such as Finite Element Method (FEM) and Finite-Difference Time-
Domain (FDTD) requires discretization of the entire problem region. Integral method
such as Method of Moments (MoM) only require discretization over the conductor
surface. The unknown electromagnetic property used in Moments Method (MoM) is
the current density, and the electric field for the FEM and FDTD (also the magnetic
field for FDTD method). The discretization process results in the electromagnetic
property of interest being approximated by a set of smaller elements, but of which the
complex amplitudes are initially unknown. The amplitudes are determined by
applying the full-wave method of choice to the agglomeration of elements. Usually,
the approximation becomes more accurate as the number of elements is increased.
The features of full wave solutions may be include as follow[5]:

i. **Accuracy**: Full wave analysis techniques generally provide the most
accurate results for input impedance, mutual coupling, radar cross-
section, etc.

ii. **Completeness**: Full-wave solutions include the effects of surface
waves, space wave radiation, and external coupling.

iii. **Versatility**: Full-wave solution can be implemented for arbitrary
microstrip elements and arrays, various types of feeding techniques,
 multilayer geometries, and for anisotropic substrates.
Computational complexity: Full-wave solutions are numerically intensive, and require careful programming in order to be computationally efficient.

There is an abundant literature on the theoretical analysis of probe-fed microstrip patch antennas. Hence, only brief reviews of the most popular methods which are used in the present study i.e. the transmission line method, the cavity method and the Method of Moments is included in this section.

2.5.2 Transmission Line Model

The transmission line model is the easiest of all but it yields the least accurate results and it lacks the versatility. This model represents the microstrip antenna by two slots of width W and height h, separated by a low impedance transmission line of length L. The microstrip is a non-homogeneous line of two dielectrics; typically the substrate and air. Due to the finite length and width of the microstrip patch antenna, the fields along the edges of the patch undergo fringing as shown in the figure 2.11. The amount of fringing is a function of the dimensions of the patch and the height of the substrate. The fringing influences the resonant frequency of the microstrip patch antenna, so it must be taken into account in to the microstrip patch antennas calculation.
As seen from figure 2.11, most of the electric field lines reside in the substrate and parts of some lines exist in air. Hence the fringing makes the microstrip line look wider electrically compared to its physical dimensions. As a result, this transmission line cannot support pure transverse electric-magnetic (TEM) mode of transmission, since, the phase velocities would be different in air and the substrate. Instead, the dominant mode of propagation would be the quasi-TEM mode. Hence, an effective dielectric constant $\varepsilon_{\text{reff}}$ must be obtained in order to account for the fringing and the wave propagation in the line. For air dielectric substrate the effective dielectric constant $\varepsilon_{\text{reff}}$ has the range of $1 << \varepsilon_{\text{reff}} << \varepsilon_r$ and the value of $\varepsilon_{\text{reff}}$ will be closer to the value of the actual dielectric constant $\varepsilon_r$ of the substrate. The effective dielectric constant $\varepsilon_{\text{reff}}$ is a function of frequency. At high frequency of operation, most of the electric field lines concentrate in the substrate hence the effective dielectric constant approaches the value of the dielectric constant of the substrate. The expression for $\varepsilon_{\text{reff}}$ is given by Balanis [130] as:
\[
\varepsilon_{\text{eff}} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12\frac{h}{W}\right]^\frac{1}{2}
\]

(2.1)

Where, \(\varepsilon_{\text{eff}}\) = Effective dielectric constant.
\(\varepsilon_r\) = Dielectric constant of substrate.
\(h\) = Height of dielectric substrate.
\(W\) = Width of the patch.

Figure 2.12 shows a rectangular microstrip patch antenna of length \(L\), width \(W\), resting on a substrate of height \(h\) considering that the length is along the \(X\) direction, width is along \(Y\) direction and the height is along \(Z\) direction.

For a microstrip patch antenna to be operated in the fundamental \(\text{TM}_{10}\) mode, the length of the patch must be slightly less than \(\lambda/2\) where \(\lambda\) is the wavelength in the dielectric medium and is equal to \(\lambda_0/\sqrt{\varepsilon_{\text{eff}}}\) where, \(\lambda_0\) is the free space wavelength. The \(\text{TM}_{10}\) mode implies that the field varies one \(\lambda/2\) cycle along the length, and there is no
variation along the width of the patch. In the figure 2.13, shown below, the microstrip patch antenna is represented by the two slots, separated by a transmission line of length L and open circuited at both the ends. Along the width of the patch, the voltage is maximum and current is minimum due to open ends. The fields at the edges can be resolved into normal and tangential components with respect to the ground plane.

![Figure 2.13 Physical and effective lengths of rectangular microstrip antenna.](image)

From figure 2.14, it is seen that the normal component of the electric field at the two edges along the width are in opposite directions and thus, out of phase, since the path is $\lambda/2$ long and hence, they cancel each other in the broadside direction.

![Figure 2.14 Side View of microstrip patch with electric field component.](image)

The tangential components in phase are means that the resulting fields combine to give maximum radiated field normal to the surface of the structure. Hence the edges along the width can be represented as two radiating slots, which are $\lambda/2$ apart and excited in phase and radiating in the half space above the ground plane. The fringing
fields along the width can be modeled as radiating slots and electrically patch of the microstrip antenna looks greater than its physical dimension. The dimensions of the patch along its length have now been extended on each end by a distance \( \Delta L \), which is a function of the effective dielectric constant \( \varepsilon_{\text{reff}} \) and the width-to-height ratio \( (W/h) \).

To calculate the normalized extension of the length, the most popular and practical relation is given empirically by Hammerstad [131] as:

\[
\Delta L = 0.412h \frac{(\varepsilon_{\text{reff}} + 0.3)(\frac{W}{h} + 0.264)}{(\varepsilon_{\text{reff}} - 0.258)(\frac{W}{h} + 0.8)}
\]  
(2.2)

The effective length of the patch \( L_{\text{eff}} \) now becomes:

\[
L_{\text{eff}} = L + 2\Delta L
\]  
(2.3)

For a given resonance frequency \( f_r \), the length is given by [6] as:

\[
L = \frac{c}{2f_r\sqrt{\varepsilon_{\text{reff}}}} - 2\Delta L
\]  
(2.4)

Hence

\[
L_{\text{eff}} = \frac{c}{2f_r\sqrt{\varepsilon_{\text{reff}}}}
\]  
(2.5)

The resonance frequency of a rectangular microstrip patch antenna for any TM\(_{mn}\) mode is given by James and Hall [8] as:
where $m$ and $n$ are mode along $L$ and $W$ respectively.

For an efficient radiator, the practical width $W$ is given by Bahl and Bhartia [6] as:

$$w = \frac{c}{2f_r} \sqrt{\frac{2}{\varepsilon_r + 1}}$$

(2.7)

The transmission line model is successfully implemented by Pues and Van de Capelle[132]. With this method it is difficult to model the coupling between antenna elements, although it has been done successfully by A.G.Derneryd and E.H.Van Lil and A.R.Van de Capelle[133,134].

2.5.3 Cavity Model

The simplest analytical method to use in microstrip patch antenna is transmission line model. But transmission line model have numerous disadvantages like, it is useful only for patch antenna of rectangular shape, it ignores field variations along the radiating edge and is not adaptable to inclusion of the field. The cavity model for microstrip patch antennas[37,38] offers considerable improvement over the transmission line model.
The cavity model for the microstrip antennas is based on the following observations for thin substrates \((h \ll \lambda)\) [38].

i) The closed proximity between the microstrip antenna and the ground plane suggest that \(E\) has only the \(Z\)-component and \(H\) has only the \(xy\)-components in the region bound by the microstrip and the ground plane.

ii) The field in the aforementioned region is independent of the \(z\)-coordinate for all frequencies of interest.

iii) The electric current in the microstrip must have no component normal to the edge at any point on the edge, implying a negligible tangential component of \(H\) along the edge.

With this, in the cavity model, the interior region of the dielectric substrate is modeled as a cavity bounded by a magnetic wall along the edge and by electric walls on the top and bottom.

As the microstrip patch is energized, a charge distribution is seen on the upper and lower surfaces of the patch and at the bottom of the ground plane as shown in the figure 2.15.

![Figure 2.15 Charge distribution and current density creation on the microstrip patch.](Image)

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![Figure 2.15 Charge distribution and current density creation on the microstrip patch.](Image)
This charge distribution is controlled by two mechanisms: an attractive mechanism and a repulsive mechanism as discussed by Richards[135]. The attractive mechanism is between the opposite charge on the bottom side of the patch, and the ground plane. This attraction tends to keep the patch charge concentration intact at the bottom of the patch. The repulsive mechanism is between the like charges on the bottom surface of the patch. This tends to push some of the charge around the edge of the patch on to its top surface. As a result of this charge movement, currents flow at the top and the bottom surface of the patch. The cavity model assumes that the height to width ratio (i.e. height of substrate and the width of the patch) is very small and as a result of this the attractive mechanism dominates and causes most of the charge concentration and the current to be below the patch surface. Much less current would flow on the top surface of the patch and as the height to width ratio further decreases, the current on the top surface of the patch would be almost equal to zero, which would not allow the creation of any tangential magnetic field component to the patch edges. Hence, the four sidewalls could be modeled as perfectly magnetic conducting surfaces. This implies that the magnetic fields and the dielectric field distribution beneath the patch would not be disturbed. However, in practice, a finite width to height ratio would be there and this would not make the tangential magnetic fields to be completely zero, but they being very small, the sidewalls could be approximated to the perfectly magnetic conducting [7].

The impedance function for the microstrip antenna has complex poles. The imaginary parts of these poles account for the power lost by radiation and by dielectric and conduction losses. The microstrip antenna is modeled to make it more...
resemble with cavity by addition of loss to the cavity dielectric by appropriately adjusting the loss tangent of the cavity dielectric. Though impedance function for the ideal cavity has only real poles, now in microstrip antenna modeling the imaginary parts of the poles of the cavity filled with the lossy dielectric will no longer be zero. A lossy cavity would now represent an antenna and for the cavity with perfectly conducting electric and magnetic walls the loss is taken into account by the effective loss tangent $\delta_{\text{eff}}$.

At any frequency $f$ near a resonance, the quality factor is given by \[ Q = \frac{2\pi f (\text{average total stored energy})}{(\text{average power dissipated})} = \frac{1}{\delta_{\text{eff}}} \] (2.8)

Hence, $\delta_{\text{eff}} = \frac{1}{Q_T}$ (2.9)

$Q_T$ is the total antenna quality factor and has been expressed by [136]

\[ \frac{1}{Q_T} = \frac{1}{Q_d} + \frac{1}{Q_c} + \frac{1}{Q_r} \] (2.10)

$Q_d$ represent the quality factor of the dielectric and is given as:

\[ Q_d = \frac{\omega_s w}{P_d} = \frac{1}{\tan \delta} \] (2.11)
Where, \( \omega_r \) is the angular resonant frequency.

\( W_r \) is the total energy stored in the patch at resonance.

\( P_d \) is the dielectric loss.

tan\( \delta \) is the loss tangent of the dielectric.

\( Q_c \) represents the quality factor of the conductor and is given as:

\[
Q_c = \frac{\omega_r W_r}{P_c} = \frac{h}{\Delta}
\]  
(2.12)

Where, \( P_c \) is the conductor loss.

\( \Delta \) is the skin depth of the conductor.

\( h \) is the height of the substrate.

\( Q_r \) represents the quality factor for radiation and is given as:

\[
Q_r = \frac{\omega_r W_r}{P_r}
\]  
(2.13)

Where \( P_r \) is the power radiated from the patch.

Substituting equations (2.11), (2.12) and (2.13) in equation (2.10), we get

\[
\delta_{eff} = \tan\delta + \frac{\Delta}{h} + \frac{P_r}{\omega_r W_r}
\]  
(2.14)

Thus, equation (2.14) describes the total effective loss tangent for the microstrip patch antenna.
Once "Q" is known, the antenna can be analyzed as if it is a lossy cavity. This is significant since the most commonly used patch antenna shapes corresponds to cavities having a separable geometry amenable to simple analytical treatment. This is the basic idea used in the cavity model approximation.

2.5.4 Full Wave Method – Moment Method

The most popular method, that provides the full wave analysis for the microstrip patch antenna, is the moment method. In mathematical literature, Moment Method is known as Weighted residuals and can be applied to the solution of both differential and integral equations. The method owes its name to the process of taking moments by multiplying the function with an appropriate weighting function and integrating. On microstrip antenna analysis with this method, the surface currents are used to model the microstrip patch and the volume polarization currents are used to model the fields in the dielectric slab. It has been shown by Newman and Tulyathan (30) how an integral equation is obtained for these unknown currents and using the method of moments, these electric field integral equations are converted into matrix equations which can then be solved by various techniques of algebra to provide the result. In electromagnetic theory, the method became popular after the pioneering work done by R.F.Harrington in 1967. Since then it has been one of the most popular methods for solving the electromagnetic boundary value problem. A brief overview of the moment method described by Harrington (137) is given below:
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The basic form of the equation to be solved by the method of moment is:

\[ F(g) = h \]  \hspace{1cm} (2.15)

Where, \( F \) is a known linear operator, \( g \) is an unknown function, and \( h \) is the source or excitation function. The aim here is to find \( g \), when \( F \) and \( h \) are known. The unknown function \( g \) can be expanded as a linear combination of \( N \) terms to give:

\[ g = \sum_{n=1}^{N} \alpha_n g_n = \alpha_1 g_1 + \alpha_2 g_2 + \ldots + \alpha_n g_n \]  \hspace{1cm} (2.16)

Where, \( \alpha_n \) are unknown constants and \( g_n \) are known functions usually called a basis functions or expansion functions.

If the number of terms in equation (2.16) is infinite, we shall obtain an exact solution. But for computational purposes, the number of terms is finite and we obtain an approximate solution.

Substituting equation (2.16) in (2.15) and using the linear property of the operator \( F \), we can rewrite (2.15) as:

\[ \sum_{n=1}^{N} a_n F(g_n) = h \]  \hspace{1cm} (2.17)

The basis function \( g_n \) must be selected in such a way that each \( F(g_n) \) in the above equation can be calculated. The unknown constant \( \alpha_n \) cannot be determined
directly because there are \( N \) unknowns, but only one equation. One method of finding these constant is the method of weighted residuals. In this method, a set of trial solution is established with one or more variable parameters. The residuals are a measure of the difference between the trial solution and the true solution. The variable parameters are selected in a way which guarantees a best fit of the trial functions based on the minimization of the residuals. This is done by defining a set of weighting (or testing) functions \( \{W_m\} = W_1, W_2, \ldots, W_N \) in the domain of the operator \( F \).

Taking the inner product of equation (2.17) with each weighting functions \( W_m, m = 1, 2, 3, \ldots, N \), this lead to:

\[
\sum_{n=1}^{N} a_n \langle w_m, F(g_n) \rangle = \langle w_m, h \rangle
\]  

(2.18)

Where \( m = 1, 2, \ldots, N \).

Writing in matrix form the set of equations (2.18) may be written as:

\[
[F_{mn}] [a_n] = [h_m]
\]  

(2.19)

Where,

\[
[F_{mn}] = \begin{bmatrix}
\langle w_1, F(g_1) \rangle & \langle w_1, F(g_2) \rangle & \cdots & \langle w_1, F(g_N) \rangle \\
\langle w_2, F(g_1) \rangle & \langle w_2, F(g_2) \rangle & \cdots & \langle w_2, F(g_N) \rangle \\
\vdots & \vdots & \ddots & \vdots \\
\langle w_N, F(g_1) \rangle & \langle w_N, F(g_2) \rangle & \cdots & \langle w_N, F(g_N) \rangle 
\end{bmatrix}
\]  

(2.20)
The unknown constants can now be found using algebraic techniques such as LU decomposition or Gaussian elimination. It must be remembered that the weighting functions must be selected appropriately so that elements of \( \{W_n\} \) are not only linearly independent but they also minimize the computations required to evaluate the inner product. One such choice of the weighting functions may be to let the weighting and the basic function be the same, that is, \( W_n = g_n \). This is called as the Galerkin’s method as described by Kantorovich and Akilov [138].

From the antenna theory point of view, we can write the electric field integral equation as:

\[
E = f_e (J)
\]

Where, E is the known incident electric field.
\( J \) is the unknown induced current.

\( f_e \) is the linear operator.

The first step in the moment method solution process would be to expand \( J \) as a finite sum of basis function given as:

\[
J = \sum_{i=1}^{M} J_i b_i
\]  

(2.24)

Where \( b_i \) is the \( i^{th} \) basis function and \( J_i \) is an unknown coefficient.

The second step involves the defining of a set of \( M \) linearly independent weighting functions, \( w_j \). Taking the inner product on both sides and substituting equation (2.24) in equation (2.23) we get:

\[
\langle w_j, E \rangle = \sum_{i=1}^{M} \langle w_j, f_e(J_i, b_i) \rangle
\]

(2.25)

Where \( j = 1, 2, \ldots, M \).

Writing in matrix form as:

\[
[Z_y][J] = [E_j]
\]

(2.26)

Where \( Z_y = \langle w_j, f_e(b_i) \rangle \)

\( E_j = \langle w_j, H \rangle \)

\( J \) is the current vector containing the unknown quantities.
The vector $E$ contains the known incident field quantities and the terms of the $Z$ matrix are functions of geometry. The unknown coefficients of the induced current are the terms of the $J$ vector. Using any of the algebraic schemes mentioned earlier, these equations can be solved to give the current and then the other parameters such as the scattered electric and magnetic fields can be calculated directly from the induced currents. Thus, the moment method has been briefly explained for use in antenna problems. The software used in this thesis, Zeland Inc's IE3D[139] is a moment method simulator.