CHAPTER–III

EFFECT OF FIRST ORDER CHEMICAL REACTION IN A VERTICAL DOUBLE PASSAGE CHANNEL

3.1 INTRODUCTION

Convective heat transfer in a vertical channel has practical importance in many engineering systems. Such as solar energy collectors, design of heat exchangers and cooling of electronic systems. A convection situation, in which the effects of both forced and free convection are significant is commonly referred to as mixed convection or combined convection. The effect is especially pronounced in situations where the forced fluid flow velocity is moderate and/or the temperature difference is very large. In mixed convection flows, the forced convection effects and the free convection effects are of comparable magnitude. Thus, in this situation, both forced and free convection occurs simultaneously, that is mixed convection occurs in which the effect of buoyancy forces on a forced flow or the effect of forced flow on buoyant flow is significant. Merkin (1980, 1985) has studied dual solutions occurring in the problem of the mixed convection flow over a vertical surface through a porous medium with constant temperature for the case of opposing flow. Aly et al. (2003) and Nazar and Pop (2004) have investigated the existence of dual solutions for mixed convection in porous medium.

The techniques of mixed convection to enhance heat transfer is an important objective in numerous engineering applications of the micro and nano fluids including pumping systems, chemical catalytic reactors, electronic system cooling, plate-fin heat exchangers, etc. Moreover, the arrangement of baffles may be used to cool down the
temperature in the passage for the thermally developed flow. When the channel is divided into several passages by means of plane baffles, as usually occurs in heat exchangers or electronic equipment, it is quite possible to enhance the heat transfer performance between the walls and fluid by the adjustments of each baffle position and strengths of the separate flow streams. In such configurations, perfectly conductive and thin baffles may be used to avoid significant increase of the transverse thermal resistance. Stronger streams may be arranged to occur within the passages near the channel wall surfaces in order to cool or heat the walls more effectively. Even though the subject of channel flow has been investigated extensively, few studies have so far evaluated for these effects. Recently, Prathap Kumar et al. (2011a, 2011b) and Umavathi (2011) analyzed the free convection in a vertical double passage wavy channel for both Newtonian and non-Newtonian fluids.

The growing need for chemical reactions in chemical and hydrometallurgical industries requires the study of heat and mass transfer in presence of chemical reactions. There are many transport processes that are governed by the simultaneous action of buoyancy forces due to both thermal and mass diffusion in presence of chemical reaction effect. These processes are observed in nuclear reactor safety and combustion systems, solar collectors as well as chemical and metallurgical engineering. Das et al. (1994) have studied the effects of mass transfer on the flow started impulsively past an infinite vertical plate in presence of wall heat flux and chemical reaction. Muthucumaraswamy and Ganeshan (2000, 2001) have studied the impulsive motion of a vertical plate with heat and mass flux, suction and diffusion of chemically reactive species. Seddeek (2005) has studied the finite element method for the effects of chemical reaction, variable
viscosity, thermophoresis and heat generation/absorption on a boundary layer hydromagnetic flow with heat and mass transfer over a heat surface.

When the previous studies are reviewed, one can see there was no much study performed on heat and mass transfer on a vertical enclosure with baffle. Therefore, an analysis is made to understand heat and mass transfer in a vertical channel by introducing a perfectly thin baffle for viscous fluid in the presence of first order chemical reaction.

3.2 MATHEMATICAL FORMULATION

Consider a steady, two dimensional laminar fully developed mixed convection flow in an open ended vertical channel filled with pure viscous fluid and is concentrated. The $X$ - axis is taken vertically upward and parallel to the direction of buoyancy and the $Y$ - axis is normal to it (see figure 3.1).

![Figure 3.1. Physical configuration](image)

The walls are maintained at a constant temperature. The fluid properties are assumed to be constant. The channel is divided into two passages by means of thin, perfectly conducting plane baffle and each stream will have its own pressure gradient and hence the velocity will be individual in each stream.
The governing equations for velocity, temperature and concentrations are

Stream-I

\[ \rho g \beta_1 (T_1 - T_{w_1}) + \rho g \beta_1 (C_1 - C_{w_1}) - \frac{\partial P}{\partial X} + \mu \frac{d^2 U_1}{dY^2} = 0 \]  
(3.2.1)

\[ \frac{d^2 T_1}{dY^2} + \frac{\nu}{\alpha C_p} \left( \frac{dU_1}{dY} \right)^2 = 0 \]  
(3.2.2)

\[ D \frac{d^2 C_1}{dY^2} - k_1 C_1 = 0 \]  
(3.2.3)

Stream-II

\[ \rho g \beta_2 (T_2 - T_{w_2}) + \rho g \beta_2 (C_2 - C_{w_2}) - \frac{\partial P}{\partial X} + \mu \frac{d^2 U_2}{dY^2} = 0 \]  
(3.2.4)

\[ \frac{d^2 T_2}{dY^2} + \frac{\nu}{\alpha C_p} \left( \frac{dU_2}{dY} \right)^2 = 0 \]  
(3.2.5)

\[ D \frac{d^2 C_2}{dY^2} - k_2 C_2 = 0 \]  
(3.2.6)

The boundary and interface conditions on velocity, temperature and concentration are

\[ U_1 = 0, T_1 = T_{w_1}, C_1 = C_{1}' \text{ at } Y = -h \]

\[ U_2 = 0, T_2 = T_{w_2}, C_2 = \bar{C}_2 \text{ at } Y = h \]

\[ U_1 = 0, T_1 = T_2, \frac{dT_1}{dY} = \frac{dT_2}{dY}, C_1 = \bar{C}_1, C_2 = C_2' \text{ at } Y = h' \]  
(3.2.7)

Introducing the following non-dimensional variables

\[ u_i = \frac{U_i}{U_1}, \theta_i = \frac{T_i - T_{w_i}}{T_{w_i} - T_{w_2}}, Gr = \frac{g \beta_1 \Delta T h^3}{\nu^2}, G_{Cl} = \frac{g \beta_1 \Delta C h^3}{\nu^2}, Re = \frac{\bar{U}_1 h}{\nu}, Br = \frac{\bar{U}_1^2 \mu}{k \Delta T}, \]
\[ p = \frac{h^2}{\mu U_1} \frac{\partial p}{\partial X}, \quad i = 1, 2, \Delta T = T_{w_i} - T_{w_j}, \phi_1 = \frac{C - C_{01}}{C'_1 - C_{01}}, \phi_2 = \frac{C - C_{02}}{C'_2 - C_{02}}, Y = \frac{y}{h}, \]

\[ G_{c2} = \frac{g \beta_{c2} \Delta C_2 h^3}{\nu^2}, \quad \Delta C_1 = C'_1 - C_{01}, \Delta C_2 = C'_2 - C_{02}, \quad n_1 = \frac{C_1 - C_{01}}{C_1^1 - C_{01}}, \quad n_2 = \frac{C_2 - C_{02}}{C_2^1 - C_{02}}, \]

\[ \alpha_1^2 = \frac{k_1 h^2}{D}, \quad \alpha_2^2 = \frac{k_2 h^2}{D}, \quad GR_{c1} = \frac{G_{c1}}{Re}, \quad GR_{c2} = \frac{G_{c2}}{Re}, \quad GR_r = \frac{Gr}{Re} \quad (3.2.8) \]

The non-dimensional form of momentum, energy and concentration equations corresponding to stream-I and stream-II as

**Stream-I**

\[
\frac{d^2 u_1}{dy^2} + GR_c \theta_1 + GR_{c2} \phi_1 - p = 0 \quad (3.2.9)
\]

\[
\frac{d^2 \theta_1}{dy^2} + Br \left( \frac{du_1}{dy} \right)^2 = 0 \quad (3.2.10)
\]

\[
\frac{d^2 \phi_1}{dy^2} - \alpha_1^2 \phi_1 = 0 \quad (3.2.11)
\]

**Stream-II**

\[
\frac{d^2 u_2}{dy^2} + GR_c \theta_2 + GR_{c2} \phi_2 - p = 0 \quad (3.2.12)
\]

\[
\frac{d^2 \theta_2}{dy^2} + Br \left( \frac{du_2}{dy} \right)^2 = 0 \quad (3.2.13)
\]

\[
\frac{d^2 \phi_2}{dy^2} - \alpha_2^2 \phi_2 = 0 \quad (3.2.14)
\]

The boundary and interface conditions becomes,

\[ u_i = 0, \quad \theta_i = 1, \quad \phi_i = 1 \text{ at } y = -1 \]
\[ u_2 = 0, \quad \theta_2 = 0, \quad \phi_2 = n_2, \quad y = 1 \]

\[ u_1 = 0, \quad u_2 = 0, \quad \theta_1 = \theta_2, \quad \frac{d\theta_1}{dy} = \frac{d\theta_2}{dy}, \quad \phi_1 = n_1, \quad \phi_2 = 1 \text{ at } y = y^* \quad (3.2.15) \]

### 3.3 SOLUTIONS

Solutions of equations (3.2.11) and (3.2.14) can be obtained directly and are given as follows

\[ \phi_1 = B_1 \cosh(\alpha_1 y) + B_2 \sinh(\alpha_1 y) \quad (3.3.1) \]
\[ \phi_2 = B_3 \cosh(\alpha_2 y) + B_4 \sinh(\alpha_2 y) \quad (3.3.2) \]

#### Perturbation Method

Equations (3.2.9, 3.2.10, 3.2.12 and 3.2.13) are coupled nonlinear ordinary differential equations. Approximate solutions can be found by using the regular perturbation method. The Brinkman number is chosen as the perturbation parameter. Adopting this technique, solutions for velocity and temperature are assumed in the form

\[ u_i(y) = u_{i0}(y) + Br u_{i1}(y) + Br^2 u_{i2}(y) + ... \quad (3.3.3) \]
\[ \theta_i(y) = \theta_{i0}(y) + Br \theta_{i1}(y) + Br^2 \theta_{i2}(y) + ... \quad (3.3.4) \]

Substituting equations (3.3.3) and (3.3.4) in equations (3.2.9, 3.2.10, 3.2.12 and 3.2.13) and equating the coefficients of like power of \( Br \) to zero and one, we obtain the zeroth and first order equations
Stream-I

Zeroth order equations

\[ \frac{d^2 \theta_{10}}{dy^2} = 0 \]  \hspace{1cm} (3.3.5)

\[ \frac{d^2 u_{10}}{dy^2} + GR_r \theta_{10} + GR_c \phi_1 - p = 0 \]  \hspace{1cm} (3.3.6)

First order equations

\[ \frac{d^2 \theta_{11}}{dy^2} + \left( \frac{du_{10}}{dy} \right)^2 = 0 \]  \hspace{1cm} (3.3.7)

\[ \frac{d^2 u_{11}}{dy^2} + GR_r \theta_{11} = 0 \]  \hspace{1cm} (3.3.8)

Stream-II

Zeroth order equations

\[ \frac{d^2 \theta_{20}}{dy^2} = 0 \]  \hspace{1cm} (3.3.9)

\[ \frac{d^2 u_{20}}{dy^2} + GR_r \theta_{20} + GR_c \phi_2 - P = 0 \]  \hspace{1cm} (3.3.10)

First order equations

\[ \frac{d^2 \theta_{21}}{dy^2} + \left( \frac{du_{20}}{dy} \right)^2 = 0 \]  \hspace{1cm} (3.3.11)

\[ \frac{d^2 u_{21}}{dy^2} + GR_r \theta_{21} = 0 \]  \hspace{1cm} (3.3.12)

The corresponding boundary and interface conditions are

\[ u_{i0} = 0, \quad \theta_{i0} = 1, \quad \phi_i = 1 \text{ at } y = -1 \]
\[ u_{20} = 0, \ \theta_{20} = 0, \ \phi_2 = n_2 \text{ at } y = 1 \]

\[ u_{10} = 0, \ u_{20} = 0, \ \theta_{10} = \theta_{20}, \ \frac{d\theta_{10}}{dy} = \frac{d\theta_{20}}{dy}, \ \phi_1 = n_1, \ \phi_2 = 1 \text{ at } y = y^* \]

\[ u_{11} = 0, \ \theta_{11} = 0 \text{ at } y = -1 \]

\[ u_{21} = 0, \ \theta_{21} = 0 \text{ at } y = 1 \]

\[ u_{11} = 0, \ u_{21} = 0, \ \theta_{11} = \theta_{21}, \ \frac{d\theta_{11}}{dy} = \frac{d\theta_{21}}{dy}, \ \text{ at } y = y^* \quad (3.2.13) \]

The solutions of zeroth and first order equations (3.3.5) to (3.3.12) using the boundary and interface conditions in equations (3.3.13) are

Zeroth order

Stream-I

\[ \theta_{10} = l_1 y + l_2 \quad (3.3.14) \]

\[ u_{10} = A_2 + A_3 y + r_3 y^2 + r_5 y^3 + r_7 Cosh(\alpha_4 y) + r_8 Sinh(\alpha_4 y) \quad (3.3.15) \]

Stream-II

\[ \theta_{20} = l_3 y + l_4 \quad (3.3.16) \]

\[ u_{20} = A_4 + A_5 y + r_5 y^2 + r_7 y^3 + r_9 Cosh(\alpha_5 y) + r_8 Sinh(\alpha_5 y) \quad (3.3.17) \]

First order

Stream-I

\[
\begin{align*}
\theta_{11} &= E_2 + E_3 y + p_{11} y^2 + p_{21} y^3 + p_{31} y^4 + p_{41} y^5 + p_{51} y^6 + p_{61} Cosh(2\alpha_1 y) \\
&+ p_{71} Sinh(2\alpha_1 y) + p_{81} Cosh(\alpha_1 y) + p_{91} Sinh(\alpha_1 y) + p_{101} y Cosh(\alpha_1 y) \\
&+ p_{111} y Sinh(\alpha_1 y) + p_{121} y^2 Cosh(\alpha_1 y) + p_{131} y^3 Sinh(\alpha_1 y) \\
\end{align*}
\quad (3.3.18)
\]
\[ u_{11} = E_6 + E_5 y + R_1 y^2 + R_2 y^3 + R_3 y^4 + R_4 y^5 + R_5 y^6 + R_6 y^7 + R_7 y^8 \]
\[ + R_8 \text{Cosh}(2\alpha_1 y) + R_9 \text{Sinh}(2\alpha_1 y) + R_{10} \text{Cosh}(\alpha_1 y) + R_{11} \text{Sinh}(\alpha_1 y) \]
\[ + R_{12} y \text{Cosh}(\alpha_1 y) + R_{13} y \text{Sinh}(\alpha_1 y) + R_{14} y^2 \text{Cosh}(\alpha_1 y) + R_{15} y^2 \text{Sinh}(\alpha_1 y) \] (3.3.19)

Stream-II

\[ \theta_{21} = E_4 + E_5 y + s_1 y^2 + s_2 y^3 + s_3 y^4 + s_4 y^5 + s_5 y^6 + s_6 \text{Cosh}(2\alpha_2 y) \]
\[ + s_7 \text{Sinh}(2\alpha_2 y) + s_8 \text{Cosh}(\alpha_2 y) + s_9 \text{Sinh}(\alpha_2 y) + s_{10} \text{Cosh}(\alpha_2 y) \]
\[ + s_{11} \text{Sinh}(\alpha_2 y) + s_{12} y^2 \text{Cosh}(\alpha_2 y) + s_{13} y^2 \text{Sinh}(\alpha_2 y) \] (3.3.20)

\[ u_{21} = E_8 + E_7 y + w_1 y^2 + w_2 y^3 + w_3 y^4 + w_4 y^5 + w_5 y^6 + w_6 y^7 + w_7 y^8 \]
\[ + w_8 \text{Cosh}(2\alpha_2 y) + w_9 \text{Sinh}(2\alpha_2 y) + w_{10} \text{Cosh}(\alpha_2 y) \]
\[ + w_{11} \text{Sinh}(\alpha_2 y) + w_{12} y \text{Cosh}(\alpha_2 y) + w_{13} y \text{Sinh}(\alpha_2 y) \]
\[ + w_{14} y^2 \text{Cosh}(\alpha_2 y) + w_{15} y^2 \text{Sinh}(\alpha_2 y) \] (3.3.21)

The constants appeared in all the above equations are:

\[ B_1 = \frac{\text{Sinh}(\alpha_1 y^*) + n_1 \text{Sinh}(\alpha_1)}{\text{Sinh}(\alpha_1 y^*) \text{Cosh}(\alpha_1) + \text{Sinh}(\alpha_1) \text{Cosh}(\alpha_1 y^*)}, \]

\[ B_2 = \frac{n_1 \text{Cosh}(\alpha_1) - \text{Cosh}(\alpha_1 y^*)}{\text{Sinh}(\alpha_1 y^*) \text{Cosh}(\alpha_1) + \text{Sinh}(\alpha_1) \text{Cosh}(\alpha_1 y^*)}, \]

\[ B_3 = \frac{\text{Sinh}(\alpha_2) - n_2 \text{Sinh}(\alpha_2 y^*)}{\text{Sinh}(\alpha_2) \text{Cosh}(\alpha_2 y^*) - \text{Sinh}(\alpha_2 y^*) \text{Cosh}(\alpha_2)}, B_4 = \frac{1 - B_2 \text{Cosh}(\alpha_2 y^*)}{\text{Sinh}(\alpha_2 y^*)}, \]

\[ l_1 = -\frac{1}{2}, l_2 = \frac{1}{2}, l_3 = -\frac{1}{2}, l_4 = \frac{1}{2}, r_1 = \frac{(p - GR_r l_2)}{2}, r_2 = -\frac{GR_r l_2}{6}, r_3 = -\frac{GR_c l_4}{\alpha_1^2}, \]

\[ r_4 = -\frac{GR_c l_2}{\alpha_1^2}, r_5 = -\frac{GR_l l_4 + p}{2}, r_6 = \frac{GR_l l_3}{6}, r_7 = -\frac{GR_c B_1}{\alpha_2^2}, r_8 = -\frac{GR_c B_4}{\alpha_2^2}, \]

\[ A_1 = -\frac{(r_1 (y^*^2 - 1) + r_2 (y^*^3 + 1) + r_3 (\text{Cosh}(\alpha_1 y^*) - \text{Cosh}(\alpha_1)) + r_4 (\text{Sinh}(\alpha_1 y^*) + \text{Sinh}(\alpha_1)))}{1 + y^*}, \]

\[ A_2 = A_1 - r_1 + r_2 - r_3 \text{Cosh}(\alpha_1) + r_4 \text{Sinh}(\alpha_1), \]
$$A_3 = \frac{r_5(-y^{*2}+1)+r_6(-y^{*3}+1)+r_7(\text{Cosh}(\alpha_2 y^*)-\text{Cosh}(\alpha_2))+r_8(\text{Sinh}(\alpha_2 y^*)-\text{Sinh}(\alpha_2))}{y^{*1}}$$

$$A_4 = -A_3 - r_5 - r_6 - r_7 \text{Cosh}(\alpha_2) - r_8 \text{Sinh}(\alpha_2), \quad \rho_i = -\frac{(2A_i^2 + \alpha_i^2 r_i^2 - \alpha_i^2)}{4}$$

$$p_2 = -\frac{2A_4 r_4}{3}, \quad p_3 = -\frac{(4r_1^2 + 6A_4 r_2)}{12}, \quad p_4 = -\frac{3r_1 r_2}{5}, \quad p_5 = -\frac{3r_2^2}{10}, \quad p_6 = -\frac{(r_2^2 + r_4^2)}{8},$$

$$p_7 = -\frac{r_3 r_4}{4}, \quad p_8 = -\frac{(2A_4 r_4 \alpha_1^2 - 8r_1 r_4 \alpha_1 + 36r_2 r_4)}{\alpha_i^3}, \quad p_9 = -\frac{(2A_4 r_3 \alpha_1^2 - 8r_1 r_4 \alpha_1 + 36r_2 r_3)}{\alpha_i^3},$$

$$p_{10} = -\frac{(4r_1 r_4 \alpha_1^2 - 24r_2 r_3)}{\alpha_i^2}, \quad p_{12} = -\frac{6r_2 r_4}{\alpha_1}, \quad p_{11} = -\frac{(4r_2 r_3 \alpha_1^2 - 24r_2 r_3)}{\alpha_i^2}, \quad p_{13} = -\frac{6r_2 r_3}{\alpha_1},$$

$$s_1 = -\frac{(2A_i^2 + r_i^2 \alpha_i^2 - r_i^2 \alpha_i^2)}{4}, \quad s_2 = \frac{-2A_4 r_3}{3}, \quad s_3 = \frac{-4r_1^2 + 6A_4 r_2}{12}, \quad s_4 = -\frac{3r_6^2}{5}, \quad s_5 = -\frac{3r_6^2}{10},$$

$$s_6 = -\frac{r_3^2 + r_4^2}{8}, \quad s_7 = -\frac{r_3 r_6}{4}, \quad s_8 = -\frac{(2A_4 r_6 \alpha_2^2 - 8r_1 r_6 \alpha_2 + 36r_6 r_8)}{\alpha_2^3},$$

$$s_9 = -\frac{(2A_4 r_6 \alpha_2^2 - 8r_1 r_6 \alpha_2 + 36r_6 r_8)}{\alpha_2^3}, \quad s_{10} = -\frac{(4r_5 r_6 \alpha_2^2 - 24r_6 r_8)}{\alpha_2^2},$$

$$s_{11} = -\frac{(4r_5 r_6 \alpha_2 - 24r_6 r_8)}{\alpha_2^2}, \quad s_{12} = -\frac{6r_6 r_8}{\alpha_2}, \quad s_{13} = -\frac{6r_6 r_8}{\alpha_2},$$

$$G_9 = -(p_1 - p_2 + p_3 + p_4 + p_5 + p_6 \text{Cosh}(2\alpha_1) - p_7 \text{Sinh}(2\alpha_1) + p_8 \text{Cosh}(\alpha_1) - p_9 \text{Sinh}(\alpha_1) - p_{10} \text{Cosh}(\alpha_1) + p_{11} \text{Sinh}(\alpha_1) + p_{12} \text{Cosh}(\alpha_1) - p_{13} \text{Sinh}(\alpha_1)),$$

$$G_{10} = -(s_1 + s_2 + s_3 + s_4 + s_5 + s_6 \text{Cosh}(2\alpha_2) + s_7 \text{Sinh}(2\alpha_2) + s_8 \text{Cosh}(\alpha_2) + s_9 \text{Sinh}(\alpha_2) + s_{10} \text{Cosh}(\alpha_2) + s_{11} \text{Sinh}(\alpha_2) + s_{12} \text{Cosh}(\alpha_2) + s_{13} \text{Sinh}(\alpha_2)),$$

$$G_{11} = (s_{10} y^{*2} + s_{12} y^{*3} + s_{13} y^{*4} + s_4 y^{*5} + s_5 y^{*6} + s_6 \text{Cosh}(2\alpha_2 y^*)+s_7 \text{Sinh}(2\alpha_2 y^*) + s_8 \text{Cosh}(\alpha_2 y^*) + s_9 \text{Sinh}(\alpha_2 y^*) + s_{10} \text{Cosh}(\alpha_2 y^*) + s_{11} \text{Sinh}(\alpha_2 y^*) + s_{12} \text{Cosh}(\alpha_2 y^*) + s_{13} \text{Sinh}(\alpha_2 y^*) - p_1 y^{*2} - p_2 y^{*3} + p_3 y^{*4} - p_4 y^{*5} - p_5 y^{*6} - p_6 \text{Cosh}(2\alpha_2 y^*) - p_7 \text{Sinh}(2\alpha_2 y^*) - p_8 \text{Cosh}(\alpha_1 y^*) - p_9 \text{Sinh}(\alpha_1 y^*) - p_{10} y^{*2} \text{Cosh}(\alpha_1 y^*) - p_{11} y^{*2} \text{Sinh}(\alpha_1 y^*))$$

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\[ G_{12} = (2s_1y^* + 3s_2y^{*2} + 4s_3y^{*3} + 5s_4y^{*4} + 6s_5y^{*5} + 2\alpha_2s_6 Sinh(2\alpha_2y^*) + 2\alpha_3s_7 Cosh(2\alpha_2y^*) + s_8\alpha_2 Sinh(\alpha_2y^*) + \alpha_2s_9 Cosh(\alpha_2y^*) + s_{10}(y^*\alpha_2 Sinh(\alpha_2y^*) + Cosh(\alpha_2y^*)) + s_{11}(y^*\alpha_2 Cosh(\alpha_2y^*) + Sinh(\alpha_2y^*)) + s_{12}(2y^*\alpha_2 Cosh(\alpha_2y^*) + 2\alpha_2y^{*2} Sinh(\alpha_2y^*)) + s_{13}(2y^*\alpha_2 Sinh(\alpha_2y^*) + \alpha_2y^{*2} Cosh(\alpha_2y^*)) - 2p_1y^* - 3p_2y^{*2} - 4p_3y^{*3} - 5p_4y^{*4} - 6p_5y^{*5} - 2\alpha_1p_6 Sinh(2\alpha_1y^*) - 2\alpha_2p_7 Cosh(2\alpha_1y^*) - p_8\alpha_2 Sinh(\alpha_1y^*) - \alpha_2p_9 Cosh(\alpha_1y^*) - p_{10}(y^*\alpha_2 Sinh(\alpha_1y^*) + Cosh(\alpha_1y^*)) - p_{11}(y^*\alpha_1 Cosh(\alpha_1y^*) + Sinh(\alpha_1y^*)) - p_{12}(2y^*\alpha_1 Cosh(\alpha_1y^*) + \alpha_1y^{*2} Sinh(\alpha_1y^*)) - p_{13}(2y^*\alpha_1 Sinh(\alpha_1y^*) + \alpha_1y^{*2} Cosh(\alpha_1y^*)) ) \]

\[ E_4 = -\frac{(y^*G_{12} + G_9 - G_{10} - G_{11} - G_{12})}{2}, \quad E_2 = \frac{(G_9 + G_{10} + G_{11} + G_{12} (1 - y^*))}{2}, \]

\[ E_3 = \frac{(-G_9 + G_{10} + G_{11} - G_{12} (1 + y^*))}{2}, \quad E_4 = G_{10} - E_3, \]

\[ R_1 = -\frac{GR_7 E_2}{2}, \quad R_2 = -\frac{GR_7 E_1}{6}, \quad R_3 = -\frac{GR_7 p_1}{12}, \quad R_4 = -\frac{GR_7 p_2}{20}, \quad R_5 = -\frac{GR_7 p_3}{30}, \]

\[ R_6 = -\frac{GR_7 p_4}{42}, \quad R_5 = -\frac{GR_7 p_5}{56}, \quad R_9 = -\frac{GR_7 p_6}{4\alpha_1^2}, \quad R_9 = -\frac{GR_7 p_7}{4\alpha_1^2}, \quad R_{15} = -\frac{GR_7 p_{13}}{\alpha_1^2}, \]

\[ R_{10} = -\frac{(p_6\alpha_1^3 - 2p_1\alpha_1 + 6p_{12})GR_7}{\alpha_1^4}, \quad R_{11} = -\frac{(p_6\alpha_1^3 - 2p_1\alpha_1 + 6p_{12})GR_7}{\alpha_1^4}, \]

\[ R_{12} = -\frac{(p_{10}\alpha_1 - 4p_{13})GR_7}{\alpha_1^3}, \quad R_{13} = -\frac{(p_{10}\alpha_1 - 4p_{13})GR_7}{\alpha_1^3}, \quad R_{14} = -\frac{GR_7 p_{12}}{\alpha_2^2}, \]

\[ W_1 = -\frac{GR_7 E_4}{2}, \quad W_2 = -\frac{GR_7 E_1}{6}, \quad W_3 = -\frac{GR_7 s_1}{12}, \quad W_4 = -\frac{GR_7 s_2}{20}, \quad W_5 = -\frac{GR_7 s_3}{30}, \]

\[ W_6 = -\frac{GR_7 s_4}{42}, \quad W_7 = -\frac{GR_7 s_5}{56}, \quad W_8 = -\frac{GR_7 s_6}{4\alpha_2^2}, \quad W_9 = -\frac{GR_7 s_7}{4\alpha_2^2}, \]

\[ W_{10} = -\frac{GR_7 (s_6\alpha_2^2 - 2s_1\alpha_2 + 6s_{12})}{\alpha_2^4}, \quad W_{14} = -\frac{GR_7 s_{12}}{\alpha_2^2}, \]

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The velocity, temperature, concentration fields for a viscous fluid in a vertical channel in the presence of first order chemical reaction containing a thin conducting baffle is studied analytically. The nonlinear coupled ordinary differential equations governing the flow have been solved by regular perturbation method. The thermal Grashof number $GR_T$, mass Grashof numbers $GR_c (= GR_{c1} = GR_{c2})$, pressure gradient $p$, chemical reaction parameter $\alpha$ is fixed as 5, 5, -5, &1 respectively, for all the graphs except for varying one.

### 3.4 RESULTS AND DISCUSSION

The velocity, temperature, concentration fields for a viscous fluid in a vertical channel in the presence of first order chemical reaction containing a thin conducting baffle is studied analytically. The nonlinear coupled ordinary differential equations governing the flow have been solved by regular perturbation method. The thermal Grashof number $GR_T$, mass Grashof numbers $GR_c (= GR_{c1} = GR_{c2})$, pressure gradient $p$, chemical reaction parameter $\alpha$ is fixed as 5, 5, -5, &1 respectively, for all the graphs except for varying one.
The effect of thermal Grashof number $GR_t$ on the velocity and temperature is shown in figures 3.2a, b, c and 3.3a, b, c respectively at different baffle positions. As the thermal Grashof number increases, the velocity increases near the left (hot) wall and also at the right (cold) wall at all the baffle positions in both the streams. The maximum point of velocity is in the stream-II for the baffle position at $y^* = -0.8$, where as it is in stream-I for the baffle position at $y^* = 0$ and 0.8. From figure 3.3a, b, c it is seen that when the baffle position is near the hot wall the temperature increases in stream-I where as in stream-II it increases up to $y = 0.5$ and decreases from $y = 0.5$ as seen in figure 3.3a. The temperature increases continuously in both the streams when the baffle is at the center of the channel and at the right wall as seen in figure 3.3b and 3.3c respectively. The increase in temperature with increase in thermal Grashof number is obvious. The increase in thermal Grashof number increases buoyancy force and hence increases the velocity and temperature fields.

The effect of mass Grashof number $GR_m (= GR_{c1} = GR_{c2})$ on the flow is shown in figures 3.4a, b, c and 3.5a, b, c at all the baffle positions. The effect of $GR_m$ is to increase the velocity and temperature in both the streams at all baffle positions. When the baffle position is near the left and right walls, there is no much effect of mass Grashof number on the flow field. There is a distinct increase in velocity and temperature field in stream-II when the baffle position is near the left wall in stream-I and stream-II when the baffle position is in the middle of the channel and in stream-I when the baffle position is near the right wall. From figure 3.5a, b, c it is seen that there is reversal of temperature field in stream-II when the baffle position is near the left and middle of channel, where as there is
no such reversal of the temperature field when the baffle position is at the right wall. This is due to the fact that the left wall is at higher temperature when compared to right wall. Further increase in mass Grashof number implies increase in concentration buoyancy force and hence increases the flow field. The similar result was also observed by Muturaj and Srinivas (2010) without inserting the baffle in a vertical wavy channel.

The effect of Brinkman number $Br$ on the flow field is shown in figures 3.6a, b, c and 3.7a, b, c. As the Brinkman number increases both the velocity and temperature increases in both the streams at all the baffle positions. However the increases in velocity and temperature is highly significant in stream-I when the baffle position is near the left wall in both the streams when the baffle position is middle of the channel and in stream-I when the baffle position is right wall. This is due to the fact that as the Brinkman number increases the viscous dissipation also increases which results in increase of flow field.

The effect of first order chemical reaction parameter $\alpha = \alpha_1 = \alpha_2$ on the velocity, temperature and concentration is shown in figures 3.8a, b, c, 3.9a, b, c and 3.10a, b, c respectively. As $\alpha$ increases velocity, temperature and concentration decreases in both the streams at all the baffle positions. This is due to the facts that increase in $\alpha$ increases the concentration of the fluid which results in the reduction of heat and mass transfer. The increase of chemical reaction parameter was to reduce the velocity, concentration field as observed by Shivaiah and Anand Rao (2012) for unsteady MHD free convection flow past a vertical porous plate.
Figure 3.2. Velocity profiles for different values of thermal Grashof number $GR_T$ at (a) $y^* = -0.8$ (b) $y^* = 0.0$ (c) $y^* = 0.8$
Figure 3.3. Temperature profiles for different values of thermal Grashof number $GR_T$ at (a) $y^* = -0.8$ (b) $y^* = 0$ (c) $y^* = 0.8$
Figure 3.4. Velocity profiles for different values of mass Grashof number $GR_c$ at (a) $y^* = -0.8$ (b) $y^* = 0$ (c) $y^* = 0.8$
Figure 3.5. Temperature profiles for different values of mass Grashof number $GR_c$ at (a) $y^* = -0.8$ (b) $y^* = 0$ (c) $y^* = 0.8$
Figure 3.6. Velocity profiles for different values of Brinkman number Br at (a) $y^* = -0.8$ (b) $y^* = 0$ (c) $y^* = 0.8$
Figure 3.7. Temperature profiles for different values of Brinkman number $Br$ at (a) $y^* = -0.8$ (b) $y^* = 0$ (c) $y^* = 0.8$
Figure 3.8. Velocity profiles for different values of chemical reaction parameter $\alpha$ at (a) $y^* = -0.8$ (b) $y^* = 0$ (c) $y^* = 0.8$
Figure 3.9. Temperature profiles for different values of chemical reaction parameter $\alpha$ at (a) $y^* = -0.8$ (b) $y^* = 0$ (c) $y^* = 0.8$
Figure 3.10. Concentration profiles for different values of chemical reaction parameter $\alpha$ at (a) $y^* = -0.8$ (b) $y^* = 0$ (c) $y^* = 0.8$.