CHAPTER–VI

NATURAL CONVECTION HEAT AND MASS TRANSFER OF A CHEMICALLY REACTING MICROPOLAR FLUID IN A VERTICAL DOUBLE PASSAGE CHANNEL

6.1 INTRODUCTION

The analysis of the flow properties of non-Newtonian fluids is very important in the field of fluid dynamics because of their technological application. Mechanics of non-Newtonian fluids present challenges to engineers, physicists and mathematicians, due to complex stress strain relationships of non-Newtonian fluids.

Studies of micropolar fluids have received considerable attention due to their application in a number of processes that occur in industry. Such applications include the extrusion of polymer fluids, solidifications of liquid crystals, cooling of a metallic plate in a bath, animal bloods, exotic lubricants and colloidal and suspensions solutions, for which the classical Navier stocks theory is inadequate. The essence of the theory of micropolar fluid flow lies in the extension of the constitutive equations for Newtonian fluids so that more complex fluids can be described by this theory. In this theory, rigid particles contained in a small fluid volume element are limited to rotation about the center of the volume element described by the microrotation vector. This local rotation of the particles is in addition to the usual rigid body motion of the entire volume element. In the micropolar fluid theory, the laws of classical continuum mechanics are augmented with additional equations that account for the conservation of microinertia moments and the
balance of first stress moments that arise due to consideration of the microstructure in a material and also additional local constitutative parameters are introduced.

The shear behavior of many real fluids used in the modern technology cannot be characterized by the Newtonian relationship and hence researchers have proposed diverse non-Newtonian fluid theories to explain the behavior of real fluids. The description of the motion of such fluids requires a new approach, which is different from the classical concepts. Thus, additional balance laws are needed to describe such types of complex behaviors. Physically micropolar fluids may be described as the non-Newtonian fluids consisting of dumbbell molecules or short rigid cylindrical elements, polymer fluids, fluid suspensions, animal blood, etc. The presence of dust or smoke, particularly in a gas, may also be modeled using micropolar fluid dynamics. The theory of micropolar fluids, first proposed by Eringen (1966, 1972), is capable of describing such fluids. Extensive reviews of theory of micropolar fluids and its applications can be found in review articles by Ariman et al. (1973, 1974) and recent books by Łukaszewicz (1999) and Eringen (2001). Recently Chamkha et al. (2002) have studied the fully developed free convection of a micropolar fluid in a vertical channel. Cheng (2006) has presented the fully developed natural convective flow of heat and mass transfer of a micropolar fluid in a vertical channel with asymmetric wall temperature and concentrations.

It is well known that many of the physiological fluids behave like suspensions of deformable or rigid particles in Newtonian fluid. In view of this, some researchers have used non-Newtonian fluid models for the biofluids. Dash et al. (1996) estimated the
increased flow resistance in a narrow catheterized artery using the Caisson fluid model. Banerjee et al. (1999) investigated the changes in flow and mean pressure gradient across a coronary artery with stenosis in the presence of a translational catheter using the finite element method for the Carreau model, a shear rate dependent non-Newtonian fluid model. Eringen (1966) proposed the theory of micropolar fluids to study fluids with suspension nature. In this theory, the continuum is regarded as sets of structured particles which contain not only mass and velocity, but also a substructure. That is, each material volume element contains micro volume elements that can translate and rotate independently of the motion of macro volume. In this model, two independent kinematic vector fields are introduced one representing the translation velocities of fluid particles and the other representing angular (spin) velocities of the particles, called as microrotation vector (Lukaszewicz, 1999). In many transport processes, both in nature and in industrial applications, heat and mass transfer is a consequence of the buoyancy effects caused by diffusion of heat and chemical species. The natural convective flow is often caused not entirely by temperature gradients, but also by the difference in the concentrations of dissimilar chemical species. Convection and transport processes are governed by buoyancy mechanisms arising from both thermal and species diffusion.

In particular process involving the mass transfer effects has been considered to be important in chemical engineering equipments. The other applications include solidification of binary alloys and crystal growth dispersion of dissolved materials or particulate water in flows, drying and dehydration operations in chemical and food processing plants, evaporation at the surface of water body. The order of the chemical
reaction depends on several factors. One of the simplest chemical reactions is the first
order reaction in which rate of reaction is directly proportional to the species
concentration. Das et al. (1994) have studied the effect of mass transfer on the flow
started impulsively past an infinite vertical plate in the presence of wall heat flux and
chemical reaction. Muthucumarswamy and Ganeshan (2001, 2000) have studied the
impulsive motion of a vertical plate with heat and mass flux, suction and diffusion of
chemically reactive species.

The rate of heat transfer in a vertical channel could be enhanced by using special
inserts. These inserts can be specially designed to increase the included angle between the
velocity vector and the temperature gradient vector, rather than to promote turbulence.
This increases the rate of heat transfer without a considerable drop in the pressure by Guo
and Wang (1998). A plane baffle may be used as an insert to enhance the rate of heat
transfer in the channel. To avoid a considerable increase in the transverse thermal
resistance into the channel, a thin and perfectly conductive baffle is used. The effect of
such baffle on the laminar fully developed combined convection in a vertical channel
with different uniform wall temperatures has been studied analytically by Salah El-Din
micropolar fluid in a vertical channel.

In this chapter, we aim to study the fully developed heat and mass transfer by
natural convection of a micropolar fluid inside a vertical channel for asymmetric wall
temperatures and concentrations. The closed form solutions are derived and the effects of
the buoyancy ratio and the vortex viscosity parameter on the flow, heat transfer and mass
transfer characteristics, such as the velocity, microrotation, total volumetric flow rates, total heat rate added to the fluid and the total species rate added to the fluid, are examined.

6.2 MATHEMATICAL FORMULATION

Consider a steady fully developed laminar natural convection flow of a micropolar fluid between two vertical plates. The vertical plates are separated by a distance \( h \). The inlet temperature is \( T_{w_2} \) and inlet concentration is \( C_0 \). The inner surface of the left plate \( (Y = -h/2) \) is kept at a constant temperature \( T_1 \) while the inner surface of the right plate \( (Y = h/2) \) is maintained at a constant temperature \( T_2 \). In addition, the concentration of a certain constituent in the solution varies from \( C_1 \) on the inner surface of the left plate to \( C_2 \) on the inner surface of the right plate. Because the flow is fully developed, the transverse velocity is zero and the flow depends only on the transverse coordinate \( Y \). The fluid properties are assumed to be constant except for density variations in the buoyancy force term.

The governing equations for velocity, temperature, microrotation velocity and concentrations are

Stream-I

\[
\rho g \beta_r \left( T_1 - T_{w_2} \right) + \rho g \beta_c (C_1 - C_0) + (\mu + \kappa) \frac{d^2 U_1}{dY^2} + \kappa \frac{dv_1}{dY} = 0
\]  

\[6.2.1\]
\[ k \frac{d^2 T}{dY^2} = 0 \]  
(6.2.2)

\[ \gamma \frac{d^2 v_1}{dY^2} - 2k v_1 - \kappa \frac{dU_1}{dY} = 0 \]  
(6.2.3)

\[ D \frac{d^2 C}{dY^2} - k C = 0 \]  
(6.2.4)

Stream-II

\[ \rho g \beta_T \left(T_2 - T_{w_2}\right) + \rho g \beta_c (C_2 - C_0) + (\mu + \kappa) \frac{d^2 U_2}{dY^2} + \kappa \frac{dv_2}{dY} = 0 \]  
(6.2.5)

\[ k \frac{d^2 T_2}{dY^2} = 0 \]  
(6.2.6)

\[ \gamma \frac{d^2 v_2}{dY^2} - 2k v_2 - k \frac{dU_2}{dY} = 0 \]  
(6.2.7)

The boundary and interface conditions on velocity, temperature, microrotation velocity and concentration are

\[ U_1 = 0, \ T_1 = T_{w_1}, \ v_1 = 0, \ C = C_1 \quad \text{at} \quad Y = -h/2 \]

\[ U_2 = 0, \ T_2 = T_{w_2}, \ v_2 = 0, \quad \text{at} \quad Y = h/2 \]

\[ U_1 = 0, \ U_2 = 0, \ T_1 = T_2, \ \frac{dT_1}{dY} = \frac{dT_2}{dY}, \quad \text{at} \quad Y = h^* \]

\[ v_1 = 0, \ v_2 = 0, \ C = C_2 \quad \text{at} \quad Y = h^* \]  
(6.2.8)
Introducing the following non-dimensional variables

\[ U_i = \frac{u_i Gr \mu}{\rho}, \quad \nu_1 = \frac{H_i Gr \mu}{h^2 \rho}, \quad \nu_2 = \frac{H_s Gr \mu}{h^2 \rho}, \quad \nu = (\mu + \frac{\kappa}{2}) j, \quad \theta_i = \frac{T_i - T_{w_i}}{T_{w_1} - T_{w_2}}, \]

\[ Gr = \frac{g \beta_T \Delta T h^3}{\nu^2}, \quad G_c = \frac{g \beta_c \Delta C h^3}{\nu^2}, \quad \phi = \frac{C - C_0}{C_1 - C_0}, \quad \Delta T = T_{w_i} - T_{w_2}, \quad \Delta C = C_1 - C_0, \]

\[ Y = \frac{y}{h}, \quad Y^* = \frac{y^*}{h}, \quad \alpha^2 = \frac{k h^2}{D}, \quad N = \frac{\beta_T (C_1 - C_0)}{\beta_c (T_1 - T_{w_2})} \quad (6.2.9) \]

where \( U_i \) is the velocity component in the stream wise direction. \( T_i \) and \( C \) are the fluid temperature and species concentration, respectively. \( \nu_i \) is the angular velocity of the micropolar fluid, \( \kappa \) is the vortex viscosity and \( j \) is the microinertia density. Here \( \gamma \) is the spin gradient viscosity and we assume that \( \nu = \left( \mu + \frac{\kappa}{2} \right) j \). Property \( \mu \) is the dynamic viscosity of the fluid and \( \rho \) is the fluid density. \( \beta_T \) and \( \beta_c \) are the coefficients for thermal expansion and for concentration expansion, respectively and \( g \) is the gravitational acceleration. Note that the boundary condition for the microrotation at the fluid solid interface is \( \nu_i = 0 \), the condition of zero spin, as used by Takhar et al. (1998). The microstructure does not rotate relative to the surface.

By using these parameters one obtains the momentum, energy, microrotation velocity and concentration equations corresponding to stream-I and stream-II as
Stream-I

\[
(1 + K) \frac{d^2u_1}{dy^2} + \theta_1 + N \phi_1 + K \frac{dH_1}{dy} = 0 \quad (6.2.10)
\]

\[
\frac{d^2\theta_1}{dy^2} = 0 \quad (6.2.11)
\]

\[
\left(1 + \frac{K}{2}\right) \frac{d^2H_1}{dy^2} - BK \left(2H_1 + \frac{dU_1}{dy}\right) = 0 \quad (6.2.12)
\]

\[
\frac{d^2\phi}{dy^2} - \alpha^2 \phi = 0 \quad (6.2.13)
\]

Stream-II

\[
(1 + K) \frac{d^2u_2}{dy^2} + \theta_2 + K \frac{dH_2}{dy} = 0 \quad (6.2.14)
\]

\[
\frac{d^2\theta_2}{dy^2} = 0 \quad (6.2.15)
\]

\[
\left(1 + \frac{K}{2}\right) \frac{d^2H_2}{dy^2} - BK \left(2H_2 + \frac{dU_2}{dy}\right) = 0 \quad (6.2.16)
\]

The boundary and interface conditions becomes,

\[
u_1 = 0, \theta_1 = 1, H_1 = 0, \phi = 1 \quad \text{at} \quad y = -1/4
\]

\[
u_2 = 0, \theta_2 = 0, H_2 = 0, \quad \text{at} \quad y = 1/4
\]
\[ u_1 = 0, u_2 = 0, \theta_1 = \theta_2, \frac{d\theta_1}{dy} = \frac{d\theta_2}{dy} \quad \text{at} \quad y = y^* \]

\[ H_1 = 0, H_2 = 0, \phi = n \quad \text{at} \quad y = y^* \]  
(6.2.17)

where \( n = \frac{C_2 - C_0}{C_1 - C_0} \) is the wall concentration ratio. Here we also define the material parameters \( B = \frac{h^2}{j} \) and \( K = \frac{\kappa}{\mu} \).

For Newtonian fluid (\( \kappa = 0 \))

The governing equations for velocity, temperature and concentrations are

Stream-I

\[ \rho g \beta_r (T_1 - T_{w_1}) + \rho g \beta_c (C_1 - C_0) + \mu \frac{d^2 U_1}{dY^2} = 0 \]  
(6.2.18)

\[ k \frac{d^2 T_1}{dY^2} = 0 \]  
(6.2.19)

\[ D \frac{d^2 C}{dY^2} - k C = 0 \]  
(6.2.20)

Stream-II

\[ \rho g \beta_r (T_2 - T_{w_2}) + \mu \frac{d^2 U_2}{dY^2} = 0 \]  
(6.2.21)

\[ k \frac{d^2 T_2}{dY^2} = 0 \]  
(6.2.22)
Using the non-dimensional parameters in equations (6.2.18) to (6.2.22) we obtain the non-dimensionalised momentum, energy and concentration equations corresponding to stream-I and stream-II as

Stream-I

\[ \frac{d^2 u_1}{dy^2} + \theta_1 + N\phi_1 = 0 \]  \hspace{1cm} (6.2.23)

\[ \frac{d^2 \theta_1}{dy^2} = 0 \]  \hspace{1cm} (6.2.24)

\[ \frac{d^2 \phi_1}{dy^2} - \alpha^2 \phi_1 = 0 \]  \hspace{1cm} (6.2.25)

Stream-II

\[ \frac{d^2 u_2}{dy^2} + \theta_2 = 0 \]  \hspace{1cm} (6.2.26)

\[ \frac{d^2 \theta_2}{dy^2} = 0 \]  \hspace{1cm} (6.2.27)

6.3 SOLUTIONS

Solutions of equations (6.2.10) to (6.2.16) using the boundary conditions in equation (6.2.17) are

\[ \theta_1 = c_1 y + c_2 \]  \hspace{1cm} (6.3.1)
\[
\phi = c_5 \cosh(\alpha y) + c_6 \sinh(\alpha y) \quad (6.3.2)
\]

\[
H_1 = c_7 \cosh(\sqrt{\tau} y) + c_8 \sinh(\sqrt{\tau} y) + l_5 \sinh(\alpha y) + l_6 \cosh(\alpha y) + l_7 y^2 + l_8 y + l_9
\quad (6.3.3)
\]

\[
u_i = \frac{-1}{K+1} \left( \frac{Kc_i}{\sqrt{\tau}} \sinh(\sqrt{\tau} y) + \frac{Kc_i}{\sqrt{\tau}} \cosh(\sqrt{\tau} y) + \frac{Kl_5}{\alpha} \cosh(\alpha y) \right.

+ \frac{Kl_6}{\alpha} \sinh(\alpha y) + \frac{Kl_7 y^3}{3} + \frac{Kl_8 y^2}{2} + c_1 y^3 + \frac{c_2 y^2}{2} + \frac{N c_5 \cosh(\alpha y)}{\alpha^2}

\left. + \frac{N c_6 \sinh(\alpha y)}{\alpha^2} + c_9 y + c_{10} \right)
\quad (6.3.4)
\]

Stream-II

\[
\theta_2 = c_3 y + c_4
\quad (6.3.5)
\]

\[
H_2 = d_1 \cosh(\sqrt{\tau} y) + d_2 \sinh(\sqrt{\tau} y) + p_3 y^2 + p_6 y + p_7
\quad (6.3.6)
\]

\[
u_2 = \frac{-1}{K+1} \left( \frac{Kd_1}{\sqrt{\tau}} \sinh(\sqrt{\tau} y) + \frac{Kd_2}{\sqrt{\tau}} \cosh(\sqrt{\tau} y) + \frac{Kp_3 y^3}{3} + \frac{Kp_6 y^2}{2} \right.

+ \frac{c_1 y^3}{6} + \frac{c_2 y^2}{2} + F_1 y + F_2 \left. \right)
\quad (6.3.7)
\]

**Newtonian fluid** \((\kappa = 0)\)

Solving equations (6.2.23) to (6.2.27) with their corresponding boundary and interface conditions (6.2.17) we obtain the solutions corresponding to stream-I and stream-II as follows
Stream-I

\[ \theta_1 = c_1 y + c_2 \]  
\[ (6.3.8) \]

\[ \phi = b_1 \text{Cosh}(\alpha y) + b_2 \text{Sinh}(\alpha y) \]  
\[ (6.3.9) \]

\[ u_1 = c_3 y + c_4 - y^2 \left( \frac{c_2}{2} \right) - \frac{c_1}{6} y^3 - \frac{N b_1 C \text{osh}(\alpha y)}{\alpha^2} - \frac{N b_2 S \text{inh}(\alpha y)}{\alpha^2} \]  
\[ (6.3.10) \]

Stream-II

\[ \theta_2 = c_3 y + c_4 \]  
\[ (6.3.11) \]

\[ u_2 = c_5 + c_7 y + p_5 y^2 + p_6 y^3 \]  
\[ (6.3.12) \]

The dimensionless total volumetric flow rate is given by

\[ Qv = Qv_1 + Qv_2 \]  
\[ (6.3.13) \]

where \( Qv_1 = \int_{-0.25}^{0} u_1 \, dy \), \( Qv_2 = \int_{0}^{0.25} u_2 \, dy \)

The dimensionless total heat rate added to the fluid is given by

\[ E = E_1 + E_2 \]  
\[ (6.3.14) \]

where \( E_1 = \int_{-0.25}^{0} u_1 \, \theta_1 \, dy \), \( E_2 = \int_{0}^{0.25} u_2 \, \theta_2 \, dy \)

The dimensionless total species rate added to the fluid is given by

\[ Cs = Cs_1 \]  
\[ (6.3.15) \]

where \( Cs_1 = \int_{-0.25}^{0} u_1 \, \phi \, dy \)
The constants appeared in all the above equations are:

\[ c_1 = -2, c_2 = \frac{1}{2}, c_3 = -2, c_4 = \frac{1}{2}, c_5 = \frac{nS \sinh \frac{\alpha}{4} + S \sinh \alpha y^*}{C \cosh \frac{\alpha}{4} S \sinh \alpha y^* + C \cosh \alpha y^* S \sinh \frac{\alpha}{4}}, \]

\[ c_6 = \frac{nC \cosh \frac{\alpha}{4} - C \cosh \alpha y^*}{C \cosh \frac{\alpha}{4} S \sinh \alpha y^* + C \cosh \alpha y^* S \sinh \frac{\alpha}{4}}, l_5 = \frac{-Nc_5 \tau}{\alpha(2 + K)(\alpha^2 - \tau)}, \tau = \frac{2KB}{2 + K} \]

\[ l_6 = \frac{-Nc_5 \tau}{(2 + K)(\alpha^2 - \tau)}, l_7 = \frac{c_1}{\alpha(2 + K)}, l_8 = \frac{c_2}{(2 + K)}, l_9 = l_{91} + B_1 l_{92}, l_{91} = \frac{-c_1}{\tau(2 + K)} \]

\[ l_{92} = \frac{1}{(2 + K)}, c_7 = c_{71} + B_1 c_{72}, \]

\[ l_3(S \sinh \frac{\alpha}{4} S \sinh \sqrt{\tau} y^* - S \sinh \alpha y^* S \sinh \frac{\sqrt{\tau}}{4}) - l_6(C \cosh \frac{\alpha}{4} S \sinh \sqrt{\tau} y^* + C \cosh \alpha y^* S \sinh \frac{\sqrt{\tau}}{4}) \]

\[ + C \cosh \alpha y^* S \sinh \frac{\sqrt{\tau}}{4} - l_7(S \sinh \sqrt{\tau} y^* + y^2 S \sinh \frac{\sqrt{\tau}}{4}) + l_8(S \sinh \sqrt{\tau} y^* - y^* S \sinh \frac{\sqrt{\tau}}{4}) \]

\[ c_{71} = \frac{-l_9(S \sinh \sqrt{\tau} y^* + S \sinh \frac{\sqrt{\tau}}{4})}{C \cosh \frac{\sqrt{\tau}}{4} S \sinh \sqrt{\tau} y^* + C \cosh \sqrt{\tau} y^* S \sinh \frac{\sqrt{\tau}}{4}} \]

\[ c_{72} = \frac{-l_{92}(S \sinh \sqrt{\tau} y^* + S \sinh \frac{\sqrt{\tau}}{4})}{C \cosh \frac{\sqrt{\tau}}{4} S \sinh \sqrt{\tau} y^* + C \cosh \sqrt{\tau} y^* S \sinh \frac{\sqrt{\tau}}{4}}, c_8 = c_{81} + B_1 c_{82}, \]

\[ c_{81} = \frac{c_{71} C \cosh \frac{\sqrt{\tau}}{4} - l_5 S \sinh \frac{\alpha}{4} + l_6 C \cosh \frac{\alpha}{4} + \frac{l_7}{16} - \frac{l_8}{4} + l_{91}}{S \sinh \frac{\sqrt{\tau}}{4}}, c_{82} = \frac{c_{72} C \cosh \frac{\sqrt{\tau}}{4} + l_{92}}{S \sinh \frac{\sqrt{\tau}}{4}} \]

\[ c_9 = c_{91} + B_1 c_{92} \]
\[
c_{91} = \frac{4}{4y^*+1} \left( -\frac{Kc_{71}}{\sqrt{\tau}} (S \sinh \sqrt{\tau} y^*) + \frac{Kc_{31}^1}{\sqrt{\tau}} (C \cosh \sqrt{\tau} y^*) - C \sinh \sqrt{\tau} y^* \right) + \frac{Kl_5}{\alpha} (C \cosh \sqrt{\tau} y^*) - \frac{Kl_k}{\alpha} (S \sinh \sqrt{\tau} y^*) - Kl_4 \left( \frac{1}{192} + \frac{y^*3}{3} \right) + \frac{Nb_2}{\alpha^2} (C \cosh \sqrt{\tau} y^*) - \frac{Nb_2}{\alpha} (S \sinh \sqrt{\tau} y^*) \right)
\]

\[
c_{92} = \frac{4}{4y^*+1} \left( -\frac{Kc_{71}}{\sqrt{\tau}} (S \sinh \sqrt{\tau} y^*) + \frac{Kc_{31}^1}{\sqrt{\tau}} (C \cosh \sqrt{\tau} y^*) - C \sinh \sqrt{\tau} y^* \right), c_9 = B_i + Kl_9
\]

\[
B_i = \frac{c_{91} - Kl_{91}}{1 + Kl_{92} - c_{92}}, p_3 = \frac{c_3}{2(2 + K)}, p_6 = \frac{c_4}{2 + K}, d_i = d_{11} + B_2 d_{12},
\]

\[
p_7 = p_{71} + B_2 p_{72}, p_{71} = \frac{c_3}{\tau(2 + K)}, p_{72} = \frac{1}{(2 + K)},
\]

\[
p_5 \left( \frac{S \sinh \sqrt{\tau} y^*}{16} - y^* \cosh \sqrt{\tau} y^* \right) + p_6 (S \sinh \sqrt{\tau} y^*) - y^* S \sinh \sqrt{\tau} y^*)
\]

\[
d_{11} = \frac{p_{71} (S \sinh \sqrt{\tau} y^* - S \sinh \sqrt{\tau} y^*)}{C \cosh \sqrt{\tau} y^* - C \sinh \sqrt{\tau} y^* S \sinh \sqrt{\tau} y^*)}
\]

\[
d_{12} = \frac{p_{72} (S \sinh \sqrt{\tau} y^* - S \sinh \sqrt{\tau} y^*)}{C \cosh \sqrt{\tau} y^* - C \sinh \sqrt{\tau} y^* S \sinh \sqrt{\tau} y^*)}
\]

\[
F_{11} = \frac{4}{4y^*+1} \left( \frac{Kd_{11}^1}{\sqrt{\tau}} (S \sinh \sqrt{\tau} y^*) - S \sinh \sqrt{\tau} y^* \right) + \frac{Kd_{21}^1}{\sqrt{\tau}} (C \cosh \sqrt{\tau} y^*) - C \cosh \sqrt{\tau} y^*) + Kp_5 \left( \frac{1}{192} - \frac{y^*3}{3} \right)
\]

\[
+ Kp_6 \left( \frac{1}{32} - \frac{y^*2}{2} \right) - c_3 \left( \frac{1}{384} - \frac{y^*3}{6} \right) + c_4 \left( \frac{1}{32} - \frac{y^*2}{2} \right)
\]

\[
F_{12} = \frac{4}{4y^*+1} \left( \frac{Kd_{12}^1}{\sqrt{\tau}} (S \sinh \sqrt{\tau} y^*) - S \sinh \sqrt{\tau} y^* \right) + \frac{Kd_{22}^1}{\sqrt{\tau}} (C \cosh \sqrt{\tau} y^*) - C \cosh \sqrt{\tau} y^*)
\]
\[ B_2 = F_1 - Kp_7, \quad B_2 = \frac{F_1 - Kp_7}{Kp_7 + 1 - F_2}, \quad c_{10} = c_{31} + B_1 c_{32}, \]

\[ c_{32} = \frac{c_{02}}{4} + \frac{Kc_{72}}{\sqrt{\tau}} S \sinh \frac{\sqrt{\tau}}{4} - \frac{Kc_{82}}{\sqrt{\tau}} C \cosh \frac{\sqrt{\tau}}{4}, \quad c_8 = c_{81} + B_1 c_{82}, \quad F_2 = F_{21} + B_2 F_{22}, \]

\[ c_{31} = \frac{c_{01}}{4} + \frac{Kc_{71}}{\sqrt{\tau}} S \sinh \frac{\sqrt{\tau}}{4} - \frac{Kc_{81}}{\sqrt{\tau}} C \cosh \frac{\sqrt{\tau}}{4} - \frac{Kl_5}{\alpha^2} C \cosh \frac{\sqrt{\tau}}{4} + \frac{Kl_6}{\alpha} S \sinh \frac{\sqrt{\tau}}{4} - \frac{Kl_7}{192} \]

\[ = \frac{c_{01}}{4} + \frac{Kc_{71}}{\sqrt{\tau}} S \sinh \frac{\sqrt{\tau}}{4} - \frac{Kc_{81}}{\sqrt{\tau}} C \cosh \frac{\sqrt{\tau}}{4} + \frac{Kl_6}{\alpha} S \sinh \frac{\sqrt{\tau}}{4} - \frac{Kl_7}{192} \]

\[ c_{81} = \frac{c_{01}}{4} + \frac{Kc_{71}}{\sqrt{\tau}} S \sinh \frac{\sqrt{\tau}}{4} - \frac{Kc_{81}}{\sqrt{\tau}} C \cosh \frac{\sqrt{\tau}}{4} + \frac{Kl_6}{\alpha} S \sinh \frac{\sqrt{\tau}}{4}, \quad c_{82} = \frac{c_{01}}{4} + \frac{Kc_{71}}{\sqrt{\tau}} S \sinh \frac{\sqrt{\tau}}{4} - \frac{Kc_{81}}{\sqrt{\tau}} C \cosh \frac{\sqrt{\tau}}{4} + \frac{Kl_6}{\alpha} S \sinh \frac{\sqrt{\tau}}{4} \]

\[ d_2 = d_{21} + B_2 d_{22}, d_{21} = \frac{-d_{11} C \cosh \frac{\sqrt{\tau}}{4} - \frac{p_5}{16} - \frac{p_6}{4} - p_{71}}{S \sinh \frac{\sqrt{\tau}}{4}}, \quad d_{22} = \frac{-d_{12} C \cosh \frac{\sqrt{\tau}}{4} - p_{72}}{S \sinh \frac{\sqrt{\tau}}{4}} \]

\[ F_{22} = -\frac{F_{12}}{4} - \frac{Kd_{22}}{\sqrt{\tau}} C \cosh \frac{\sqrt{\tau}}{4} - \frac{Kd_{12}}{\sqrt{\tau}} S \sinh \frac{\sqrt{\tau}}{4} \]

\[ F_{21} = -\frac{F_{11}}{4} - \frac{c_4}{32} - \frac{c_3}{384} - \frac{Kp_5}{192} - \frac{Kp_6}{32} - \frac{Kd_{21}}{\sqrt{\tau}} C \cosh \frac{\sqrt{\tau}}{4} - \frac{Kd_{11}}{\sqrt{\tau}} S \sinh \frac{\sqrt{\tau}}{4} \]

\[ p_1 = \frac{-c_2}{2}, \quad p_2 = \frac{-c_1}{6}, \quad p_3 = \frac{-b_3}{\alpha^2}, \quad p_4 = \frac{-N b_2}{\alpha^2}, \quad p_5 = \frac{c_4}{2}, \quad p_6 = \frac{-c_3}{6}, \quad c_8 = \frac{-c_7}{4} - \frac{p_5}{16} - \frac{p_6}{64} \]

\[ c_5 = \frac{4}{1 + 4 y^*} (p_1 \frac{1}{16} - y^{*2}) - \frac{p_2}{164} (\frac{1}{16} + y^{*3}) + p_3 (C \cosh \frac{\alpha}{4} C \cosh \alpha y*) - p_4 (S \sinh \frac{\alpha}{4} + S \sinh \alpha y*) \]

\[ p_6 = \frac{c_4}{4} - \frac{p_2}{64} - \frac{p_3}{4} C \cosh \frac{\alpha}{4} + p_4 S \sinh \frac{\alpha}{4}, \quad c_7 = \frac{4}{1 + 4 y^*} (p_3 \frac{1}{16} - y^{*2}) + p_6 \frac{1}{64} - y^{*3} \]
6.4 RESULTS AND DISCUSSION

The heat and mass transfer by natural convection of chemically reacting micropolar fluid flowing in a vertical double passages channel has been analyzed. The analytical solution obtained for fluid flow as well as heat and mass transfer have been obtained and the results are presented graphically.

Figures 6.1 a, b, c and 6.2 a, b, c are the velocity and microrotation velocity profiles for various vortex viscosity parameters $K = 0, 0.5, 1$ and $1.5$, $n = 1$, $N = 2, \alpha = 2$, $h = 1$ and $B = 1$. Increasing the vortex viscosity parameter tends to decreasing the fluid velocity in the double passage channel at all baffle positions $y^* = -0.2, y^* = 0, y^* = 0.2$. The magnitude of micro rotation tends to increase as the vortex viscosity parameter is increased.

The effect of buoyancy ratios $N$ on velocity and microrotation velocity are shown in figures 6.3 a, b, c and 6.4 a, b, c respectively. It is seen that the magnitude of microrotation velocity is enhanced with increase in the buoyancy ratio. Moreover, increasing the buoyancy ratio tends to accelerate the fluid flow in the vertical double passage channel, which is the similar result found by Cheng (2006).

The effect of first order chemical reaction parameter $\alpha$, on velocity, microrotation velocity and concentration fields are seen in figures 6.5 a, b, c, 6.6 a, b, c and 6.7 a, b, c respectively. As $\alpha$ increases the velocity, microrotation velocity and concentration decreases in stream-I and remains constant in stream-II at all baffle positions. The similar result was also obtained by Srinivas and Muturajan (2011) for mixed convective flow in a
vertical channel. This is due to the fact that the fluid in stream-I is concentrated. The maximum value of velocity and microrotation velocity is seen in stream-II for the baffle position at $y^* = -0.2$ and in stream-I at baffle position at $y^* = 0$ and 0.2.

Figure 6.8a, b, c are the plots for the variation of the dimensionless volumetric flow rate $Q_v$ with the buoyancy ratio $N$ for various vortex viscosity parameters $K = 0, 0.5, 1, 1.5, \alpha = 2, n = 1$, $h = 1$ and $B = 1$. Increasing the buoyancy ratio tends to accelerate the fluid flow, thus raising the volume flow rate of the fluid flowing through the vertical double passage channel. Moreover, the dimensionless volume flow rate flowing through the vertical channel tends to decreases as the vortex viscosity parameter is increased at all baffle positions.

Figure 6.9a, b, c shows the variation of the dimensionless total species rate added to the fluid $C_s$ with the buoyancy ratio $N$ for various vortex viscosity parameters $K = 0, 0.5, 1, 1.5, \alpha = 2, n = 1$, $h = 1$ and $B = 1$. Increasing the buoyancy ratio accelerates the fluid flow, thus enhancing the mass transfer rate between the wall and the fluid flowing through the vertical double passage channel. Moreover, increasing the vortex viscosity parameter tends to decreases the dimensionless total species rate added to the fluid in the vertical double passage channel at all baffle positions.

The dimensionless total heat rate added to the fluid $E$ is plotted as functions of the buoyancy ratio $N$ for various vortex viscosity parameters $K = 0, 0.5, 1, 1.5, n = 1$, $h = 1$ and $B = 1$, as shown in figure 6.10a, b, c. Increasing the buoyancy ratio tends to
accelerate the fluid flow, raising the heat transfer rate between the wall and the fluid, thus increasing the total heat rate added to the fluid in the vertical double passage channel at all baffle positions
Figure 6.1. Velocity profiles for different values of vortex viscosity parameter $K$ at (a) $y^* = -0.2$ (b) $y^* = 0$ (c) $y^* = 0.2$
Figure 6.2. Microrotation velocity profiles for different values of vortex viscosity parameter $K$ at (a) $y^* = -0.2$ (b) $y^* = 0$ (c) $y^* = 0.2$
Figure 6.3. Velocity profiles for different values of buoyancy ratio $N$ at (a) $y^* = -0.2$ (b) $y^* = 0$ (c) $y^* = 0.2$
Figure 6.4. Microrotation velocity profiles for different values of buoyancy ratio $N$ at (a) $y^* = -0.2$ (b) $y^* = 0$ (c) $y^* = 0.2$
Figure 6.5. Velocity profiles for different values of chemical reaction parameter $\alpha$ at (a) $y^* = -0.2$ (b) $y^* = 0$ (c) $y^* = 0.2$
Figure 6.6. Microrotation velocity profiles for different values of chemical reaction parameter $\alpha$ at (a) $y^* = -0.2$ (b) $y^* = 0$ (c) $y^* = 0.2$
Figure 6.7. Concentration profiles for different values of chemical reaction parameter $\alpha$ at (a) $y^* = -0.2$ (b) $y^* = 0$ (c) $y^* = 0.2$
Figure 6.8. Effect of buoyancy ratio on volumetric flow rate for different values of vortex viscosity parameter at
(a) $y^* = -0.2$ (b) $y^* = 0$ (c) $y^* = 0.2$
Figure 6.9. Effect of buoyancy ratio on species concentration rate for different values of vortex viscosity parameter at (a) $y^* = -0.2$, (b) $y^* = 0$ (c) $y^* = 0.2$
Figure 6.10. Effect of buoyancy ratio on total energy flow rate for different values of vortex viscosity parameter at (a) $y^* = -0.2$, (b) $y^* = 0$ (c) $y^* = 0.2$