CHAPTER V
STEADY MHD CONVERGENT CHANNEL FLOW OF A NON-NEWTONIAN ELECTRICALLY CONDUCTING FLUID.

5.1. Introduction:

The convergent flow problem between two non-parallel planes was first solved by Jeffery\textsuperscript{109} by reducing the problem to an elliptic integral equation. Hamel\textsuperscript{110} solved this problem by calculating all three dimensional flows whose stream lines are identical with those of a potential flow. The numerical calculations of Jeffery – Hammel flows between non-parallel plane walls were studied by Millsaps and Pohlhausen\textsuperscript{111}. Srivastava\textsuperscript{112} extended Jeffery’s work to an electrically conducting fluid in the presence of a transverse magnetic field. Rosenhead studied the solution of two dimensional incompressible laminar flow in a converging channel with impermeable wall. Terril\textsuperscript{113} forwarded the solution obtained from slow laminar flow through the convergent channel with suction at one wall and blowing at the other wall. Srivastava and Hazarika\textsuperscript{114} discussed the flow between non-parallel plates with magnetic field by employing shooting method. Falkner and Skan\textsuperscript{115} first analyzed the two dimensional boundary layer flow of an incompressible, viscous, non-uniform stream past solid obstacle. A solution for the flow through convergent channel which is based on the boundary layer approximation was first obtained by Pohlhausen\textsuperscript{116}. Goldstein\textsuperscript{117} presented an interesting discussion on this problem. The hydromagnetic convergent channel flow of a Newtonian electrically conducting fluid was studied by Phukan\textsuperscript{118}.
In this chapter, we have analyzed the boundary layer flow through a convergent channel of an electrically conducting Walters fluid (Model B') in the presence of a strong transverse magnetic fluid. It is shown that the similarity solutions are possible only if the magnetic field takes a special form. The expressions for velocity and approximate skin friction at the channel wall have been obtained and numerically worked out for different values of the flow parameters involved in the solution. The effects of velocity and approximate skin friction profiles on viscoelastic parameter are discussed.

5.2 Mathematical Analysis:

The governing boundary layer equations for the steady, two-dimensional flow of an electrically conducting Walters liquid (Model B') in the presence of a magnetic field B(x) are taken as \[ \text{Djukie}^{119} \text{for Newtonian fluid} \]

\[
\begin{align*}
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= U \frac{dU}{dx} + \gamma \frac{\partial^2 u}{\partial y^2} - \frac{k_0}{p} \left[ u \frac{\partial^2 u}{\partial x \partial y^2} + ight. \\
+ v &\left. \frac{\partial^3 u}{\partial y^3} - \frac{\partial U}{\partial y} \frac{\partial^2 u}{\partial x \partial y} - \frac{\partial v}{\partial y} \frac{\partial^2 u}{\partial y^2} + \sigma B^2(x) \frac{(U - u)}{p} \right] \quad (5.2.1) \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \quad (5.2.2)
\end{align*}
\]

Subject to boundary conditions

\[ u=0, \ v=0 \text{ at } y=0, \]

\[ u=U(x), \text{ at } y\rightarrow\infty \quad (5.2.3) \]
where the x-axis coincides with the wall of the convergent channel and the y-axis is perpendicular to it. The velocity components in the x and y directions are \( u \) and \( v \) respectively, \( U \) is the main stream velocity, \( \rho \) is the fluid density, \( \nu \) is the kinematic viscosity, \( \sigma \) is the electrical conductivity of the fluid, \( k_0 \) is the non-Newtonian parameter. The induced magnetic field is negligible in comparison to the applied magnetic field, the external electrical field is zero and the electrical field due to polarization of charges is also negligible.

The potential flow near the sink is taken as [Schlichting\(^1\)]

\[
U(x) = -\frac{u_1}{x} \quad (5.2.3)
\]

which leads to similar solutions and with \( u_1 > 0 \), it represents two-dimensional motion in a convergent channel with flat walls.

We introduce the following change of variables

\[
\eta(x, y) = y \sqrt{\frac{U(x)}{-\gamma x}} = \frac{y}{x} \sqrt{\frac{u_1}{\gamma}}
\]

\[
\psi(x, y) = -\sqrt{-\frac{U(x)k^2}{x}} F(\eta) = -\sqrt{k_0 u_1} F(\eta) \quad (5.2.4)
\]

We obtain the velocity components

\[
u = \frac{\partial \psi}{\partial y} = U(x)F'(\eta), \quad v = -\frac{\partial \psi}{\partial x} = \frac{\eta}{x} \sqrt{u_1 \gamma} F'(\eta) \quad (5.2.5)
\]

It can be easily verified that the continuity equation (5.2.2) is identically verified.
Similarity solutions exist if the magnetic field has the special form [Chaim\textsuperscript{12}, a particular case]

\[ B(x) = \frac{B_0}{x} \quad (5.2.6) \]

Using the equations (5.2.3) to (5.2.6) in (5.2.1) we obtain the following similarity equations

\[ F''' + 1 - F'^2 - k\left[4F'F'' - 2F''^2\right] + M\left(l - F'\right) = 0 \quad (5.2.7) \]

Subject to boundary conditions:

\[ F' = 0 \quad \text{at } \eta = 0 \]

\[ F' = 1, F'' = 0 \quad \text{at } \eta \to \infty \quad (5.2.8) \]

Here \( M = \frac{\sigma B_0^2}{\rho u_1} \) is the magnetic parameter,

\[ k = \frac{k_0 u_1}{\rho \gamma x^2} \]

is the non-Newtonian parameter and prime denotes differentiation with respect to \( \eta \).

5.3. Analytical Solution of Similarity Equation:

In case of large magnetic parameter \( M \), we find out a solution following Sapunkov\textsuperscript{122} and Chiam,\textsuperscript{121} First, we let

\[ Z = \sqrt{M} \quad \eta, f(z) = \sqrt{M} \quad F(\eta) \quad (5.3.1) \]
which implies

\[ f'(z) = F'(\eta), \quad f''(z) = F''(\eta) / \sqrt{M}, \quad f'''(z) = F'''(\eta) / M \]  

(5.3.2)

Using these, the equation (5.2.7) can be written in the form

\[ f'''(z) - k [4 f'(z) f''''(z) - \{2 f''(z)\}^2] + \{1 - f'(z)\} \]

\[ = \varepsilon [f'(z)]^2 - 1 \]  

(5.3.3)

where \( \varepsilon = 1 / M \) and the corresponding boundary conditions are

\[ f'(z) = 0 \quad \text{at} \quad z = 0 \]

\[ f'(z) = 1, f''(z) = 0 \quad \text{at} \quad z \to \infty \]  

(5.3.4)

Expanding the unknown function \( f(z) \) in terms of powers of the small parameter \( \varepsilon \) as

\[ f(z) = f_0(z) + \varepsilon f_1(z) \]  

(5.3.5)

and using this expansion in the equation (5.3.3), we get the following sets of equation on equating like powers of \( \varepsilon \),

\[ f'''_0 - k [4 f'_0 f''''_0 - 2 f''_0^2] + (1 - f'_0) = 0 \]  

(5.3.6)

\[ f'''_1 - 4 k [f''_1 f''''_1 + f'_1 f''''_1 - f''_0 f''''_1] - f'_1 = (f'_0^2 - 1) \]  

(5.3.7)

The relevant boundary conditions are

\[ f'_0 = 0, f'_1 = 0 \quad \text{at} \quad z = 0; \]
\( f'_0 = 1, f'_1 = 0; f''_0 = 0, f''_1 = 0 \) at \( z \to \infty \) \hspace{1cm} (5.3.8)

Again to solve the equations (5.3.6) and (5.3.7) we consider \( k \) as perturbation parameter as due to small shear rate \( k \) is very small and write

\[
\begin{align*}
f_0 &= f_{00}(z) + k f_{01}(z) + O(k^2) \\
f_1 &= f_{10}(z) + k f_{11}(z) + O(k^2)
\end{align*}
\]  \hspace{1cm} (5.3.9)

Substituting (5.3.9) in the equations (5.3.6) and (5.3.7) and comparing the coefficients of like powers of \( k \) we get the following sets of ordinary differential equations:

\[ f^{\prime\prime\prime}_{00} - f^{\prime\prime}_0 + 1 = 0 \]  \hspace{1cm} (5.3.10)

\[ f^{\prime\prime\prime}_{01} - 4f^{\prime\prime}_0 f^{\prime\prime}_0 - 2f^{\prime\prime}_0 - f^{\prime}_0 = 0 \]  \hspace{1cm} (5.3.11)

\[ f^{\prime\prime\prime}_{10} - f^{\prime}_0 = f^{\prime}_0 - 1 \]  \hspace{1cm} (5.3.12)

\[ f^{\prime\prime\prime}_{11} - 4f^{\prime}_1 f^{\prime\prime}_0 - 4f^{\prime\prime}_0 f^{\prime}_1 + 4f''_0 f^{\prime}_1 - f^{\prime}_1 = 2f^{\prime}_0 f^{\prime}_0 \]  \hspace{1cm} (5.3.13)

The appropriate boundary conditions are

\[
\begin{align*}
f^{\prime}_0 &= f^{\prime}_1 = f''_0 = f''_1 = 0 \quad \text{at} \quad z = 0 \\

f^{\prime}_0 &= 1, f^{\prime}_1 = f''_0 = f''_1 = f^{\prime}_0 = f^{\prime}_1 = f^{\prime\prime}_0 = f^{\prime\prime}_1 = 0 \quad \text{at} \quad z \to \infty
\end{align*}
\]  \hspace{1cm} (5.3.14)

On substitution of the solutions of the equation (5.3.10) to (5.3.13) into the equation (5.3.5) and after differentiation w.r.t \( z \), one obtains

\[ f'(z) = \left[ (1 - e^{-z}) + e \left( \frac{2}{3} e^{-z} + \frac{1}{3} e^{-2z} - e^{-z} + ze^{-z} \right) \right] \]
\[+ k \left( -4e^{-z} + 4e^{2z} + 2ze^z \right)\]
\[+ \varepsilon \left( -\frac{1649}{100} e^{-z} + \frac{161}{12} (e^{-z} - ze^{-z}) \right)\]
\[+ \frac{5}{4} \left( 2ze^{-z} - z^2 e^{-z} \right) - e^{-2z} \]
\[+ \frac{4}{3} \left( e^{-2z} - 2ze^{2z} \right) - \frac{44}{25} e^{-3z} \]

where

\[f_{00}' = 1 - e^{-z}\]
\[f_{01}' = -4e^{-z} + 4e^{-2z} + 2ze^{-z}\]
\[f_{10}' = \frac{2}{3} e^{-z} + \frac{1}{3} e^{-2z} - e^{-z} + ze^{-z}\]
\[f_{11}' = -\frac{1649}{100} e^{-z} + \frac{161}{12} (e^{-z} - ze^{-z}) + \frac{5}{4} \left( 2ze^{-z} - z^2 e^{-z} \right)\]
\[+ \frac{9}{2} e^{-2z} + \frac{4}{3} \left( e^{-2z} - 2ze^{-2z} \right) - \frac{44}{25} e^{-3z} \quad (5.3.15)\]

From (5.3.2) and (5.3.15) we can easily find the dimensionless velocity \(f'(\eta)\) across the boundary layer.
The approximate skin friction coefficient in this case is given by

\[ \sigma = \left[ e^{-z} + \varepsilon \left( -\frac{2}{3} e^{-z} - \frac{2}{3} e^{-2z} + 2e^{-z} - ze^{-z} \right) \right] \]

\[ + k \left[ 6e^{-z} - 8e^{-2z} - 2ze^{-z} + \varepsilon \left( \frac{1649}{100} e^{z} + \right. \right. \]

\[ + \frac{161}{12} (-2e^{-z} + ze^{-z}) + \frac{5}{4} (2e^{-z} - 4ze^{-z} + z^{2}e^{-z}) \]

\[ - 9e^{-2z} + \frac{4}{3} (-4e^{2z} + 4ze^{-2z}) \right] - \frac{132}{25} e^{-2z} \]

at \( z=0 \) and presented graphically.

5.4 Results and discussions:

In this work, we find out a solution for steady MHD convergent channel flow of a visco-elastically conducting fluid in case of large magnetic parameter \( M \) following Sapunkov and Chiam. The visco-elastic effect is exhibited through the non-dimensional parameter \( k \). All the corresponding results for Newtonian fluid can be deduced by putting \( k = 0 \).

To see the influence of visco-elastic parameter \( k \), the velocity distribution \( F'(\eta) \) is plotted against \( \eta \) across the boundary layer in the figures (5.1) to (5.4). It has been observed from the figures that the values of \( F'(\eta) \) decrease with the increasing values of the visco-elastic parameter \( k \) as compared to their values for Newtonian fluid when the magnetic parameters \( M \) is fixed. Figure (5.5) depicts the approximate skin friction coefficient \( \sigma \) against the magnetic parameter \( M \) for
different values of visco-elastic parameter $k$. This figure shows that the values of the approximate skin friction $\tau$ decrease with the increasing values of the magnetic parameter $M$ in Newtonian case but the reverse pattern is observed in non-Newtonian case.

5.5 Conclusion:

The effect of visco-elastic parameter on steady MHD boundary layer flow in convergent channel has been studied.

From the present study, we can make the following conclusions:

- Velocity decreases with the increase of visco-elastic parameter when the magnetic parameter is fixed.

- Shearing stress at the wall of the convergent channel is enhanced with the increasing values of the visco-elastic parameter.

In the next chapter, we will analyze the two-dimensional MHD flow past a wedge of an electrically conducting fluid characterized by Walters liquid (model B').
Fig. 5.1: Velocity distribution $f^\prime$ vs $\eta$ for $M=2$

Fig. 5.2: Velocity distribution $f^\prime$ vs $\eta$ for $M=8$
Fig. 5.3: Velocity distribution $f'$ vs $\eta$ for $M=14$

Fig. 5.4: Velocity distribution $f'$ vs $\eta$ for $M=20$
Fig. 5.5: Approximate skin friction coefficient $\sigma$ vs $M$. 

- Dotted line: $k = 0$
- Dashed line: $k = 0.05$
- Solid line: $k = 0.10$