CHAPTER 2

Toyoki Koga’s “Foundations of Quantum Physics”

“If a spinning particle is not quite a point particle, nor a solid three dimensional top, what can it be? What is the structure which can appear under probing with electromagnetic fields as a point charge, but as far as spin and wave properties are concerned exhibits a size of the order of the Compton wave length?”

A. O. Barut [3]

“We have perhaps forgotten that there was a time when we wanted to be told what an electron is. The question was never answered. No familiar conceptions can be woven round the electron, it belongs to the waiting list.”

2.1. Introduction

In this chapter we give a brief introduction to the work of Toyoki Koga on the foundations of quantum physics. Koga has as his aim the development of a field theory of the electron and other particles that includes gravitation but in which the equations do not contain Planck’s constant and the mass and charge of the electron. The “fundamental equations” are covariant in a non-Minkowski sense and their solutions are expected not to have singularities.

Koga accepts the Schrödinger equation for a single electron because of its many successful predictions. But he does not accept the superposition principle. He obtains a new solution with a deterministic interpretation. He shows that a conventional de Broglie wave represents an ensemble of free electrons. Thus, a de Broglie wave does not describe an individual electron but merely an “average” electron.

Koga starts with an analysis of the Schrödinger equation which has a solution (he calls it an elementary field) representing a field that is stable and localised in space. A conventional wave function can be obtained by averaging over an ensemble of elementary fields.

Koga’s solution of the Schrödinger equation starts with a modification of old work of de Broglie. It is a field with a
singularity. He interprets it as meaning that the electron is a localised field centred around a point. However, he believes that the singularity should not really be there and that it can be removed by considering a suitable nonlinear equation rather than the linear Schrödinger equation.

Koga’s book [27] contains detailed arguments against the Copenhagen interpretation of Quantum Mechanics and its explanation in terms of his theory. I omit these completely as I have not done any work on this aspect.

It seems to me that Koga’s work substantiates Einstein’s view that the electron is a localised field and there is a deeper underlying theory. This theory implies that the conventional eigenvalue solutions of Schrödinger’s equation stand for ensembles of electrons in static states, rather than individual electrons.

Einstein believed that Quantum Mechanics, namely the Schrödinger equation, was a purely statistical theory which applied only to ensembles. But Koga exhibits a solution (for a single electron) which represents a localised field.

The Schrödinger equation does not describe the electron completely. It is not consistent with Special Relativity. In fact, what Schrödinger first obtained (and discarded) was the Klein-Gordon equation (as it was later called). Later Dirac, by
factoring the Klein-Gordon equation, obtained an equation that described the electron relativistically and happened to require electron spin.

Next, Koga makes a similar analysis of the Dirac equation. Here the elementary field consists of four scalar complex valued functions on spacetime. The Dirac matrices transform as a 4-vector. They reflect the anisotropy of the electron. (In chapter 3 we use Geometric Algebra to study the elementary Dirac field in detail.)

Koga uses the formula he obtained earlier (in connection with the Schrödinger equation) to also solve the Klein-Gordon equation which, as is well known, leads to a solution of the Dirac equation. He then interprets it deterministically and re-interprets electron spin as a real phenomenon in physical space rather than just an abstract mathematical property.

The Dirac equation, like the Schrödinger equation, is linear and does not completely reveal the electron structure (its elementary field solution has a singularity, just like the Schrödinger field) Hence Koga develops a set of nonlinear equations including gravitational effects, for a collection of functions that, under certain assumptions of approximation, lead to the concepts of mass and charge. These equations reduce to the Maxwell-Lorentz equations and to the Dirac equation under different
limiting assumptions. Hence he calls them the fundamental equations. He uses the same equations, with modified boundary conditions, to study the photon and neutron and the mechanism of strong interaction. We describe in this chapter only the Schrödinger and Dirac elementary fields. For completeness, we give a brief account of the general relativistic theory.

An excerpt from the preface of [28] summarises Koga’s goals and philosophy. These matters are not discussed in this thesis. They are covered in Chapters VI-IX of [27] (Chapter VI is based on the paper [26]) along with his ideas on the photon, quantum-electrodynamical phenomena and other elementary particles (which are all fields according to Koga).

“If the principle of general relativity is upheld as fundamental in physics, it appears, according to Einstein, that the unification of theories of matter is to be made in a theory of fields. The governing partial-differential equations are to be non-linear and inhomogeneous, and to contain no symbols representing quantum, mass and charge. These constants are to be interpreted only as almost-invariant integrals of fields, and the concept of action at a distance is an auxiliary device compensating their deficiencies in representing the reality. Thus, the foundations of physics may conceptually be purified and simplified. At the same time, however, it is recognized that those fields cannot be
defined in any directly operational sense, and their connections to observed phenomena are to be made only after a process of reorganizing the conceptual structure; that is to introduce again those old and once abandoned concepts such as quantum, mass and charge, on the understanding that the use of them is merely tentative for ad hoc and pragmatic purposes.”

2.2. The Schrödinger equation

In 1926 Schrödinger gave the following equation for the electron

$$i\hbar \frac{\partial \psi}{\partial t} + \frac{\hbar^2}{2m} \nabla^2 \psi - U\psi = 0$$

where $U$ is the potential energy and $m$ the mass of the electron (assumed to be a particle for the time being). Substituting

$$\psi = a \exp(iS/\hbar)$$

where $a$ and $S$ are real functions of $t$ and the position vector $r = (x, y, z)$, one gets, on taking real and imaginary parts,

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + U - \frac{\hbar^2 \nabla^2 a}{2ma} = 0$$

and

$$\frac{\partial a^2}{\partial t} + \text{div} \left( \frac{a^2 \nabla S}{m} \right) = 0.$$ 

The fourth term of the real part above was called the quantum potential by David Bohm. If it is absent, that equation becomes
the Hamilton-Jacobi equation with trajectories given by

\[ \frac{dr}{dt} = \frac{p}{m}, \quad \frac{dp}{dt} = -\text{grad } U, \]

where \( p \), the momentum, is defined to be \( \nabla S \) and the energy of the electron is \( E = -\frac{\partial S}{\partial t} \) which yields

\[ E = \frac{p^2}{2m} + U. \]

In the general case (\( \hbar \neq 0 \)) we get

\[
\left( \frac{\partial}{\partial t} + \frac{p}{m} \cdot \frac{\partial}{\partial r} \right) - \left( \nabla U - \frac{\hbar^2}{2m} \nabla \left( \frac{\nabla^2 a}{a} \right) \cdot \frac{\partial}{\partial p} \right) \right) a^2 = 0
\]

where the independent variables are \( r, p \) and \( t \). Koga calls this the Liouville equation of a quantum mechanical particle. Trajectories are obtained by taking \( p = md\dot{r}/dt \). The energy is \( E = -\partial S/\partial t \) which yields

\[ E = \frac{p^2}{2m} + U - \frac{\hbar^2 \nabla^2 a}{2ma}. \]

This is invariant on a trajectory.

Koga points out that there is some resemblance between the present theory of the Schrödinger equation and the theory of de Broglie and Bohm [19]. But in Koga’s theory, the de Broglie wave, rather than being a real wave guiding the electron, is a fictitious wave constructed by superposing an ensemble of similar and independent elementary fields.
I do not go into these aspects in the thesis.

Now consider a free electron: $U = 0$ up to a constant which we ignore. By a suitable choice of inertial frame, we take $p = 0$, and assume $\partial a^2 / \partial t = 0$. Then the Liouville equation gives

$$\nabla (\nabla^2 a / a) = 0$$

and so

$$\nabla^2 a / a = K \quad \text{(constant)}$$

which has a solution (assuming $0 < K = \kappa^2$, $\kappa > 0$)

$$a = \frac{\exp(-\kappa r)}{r}$$

where $r = \sqrt{x^2 + y^2 + z^2}$. In the 1920s, de Broglie worked on these lines (but took $K < 0$).

Here $S = -Et$, $E = -\hbar^2 \kappa^2 / 2m$. In general, if the electron has velocity $v$, we have

$$E = \frac{mv^2}{2} - \frac{\hbar^2 \kappa^2}{2m}, \quad S = -Et + mv \cdot r.$$

It should be kept in mind that this solution to the Schrödinger equation is not considered a quantum-mechanical state but is supposed to give a pointwise description of the electron field. There is no superposition principle.

On reading the argument above, it may appear that the energy of a free electron at rest, according to this theory, is negative. But Koga explains that the expression for energy here
(in his theory of the Schrödinger equation) is obtained by ignoring the relativistic rest-mass energy in the Dirac equation. In Koga’s interpretation, the free electron is a localised field (which he calls an elementary field) rather than a point particle. The function that represents it has a singularity. Presumably, if in reality there is no singularity (the view of Einstein and Koga) then the representation is an approximate one. The reason given by Koga is that a linear equation like the Schrödinger equation cannot perfectly describe reality.

Koga explains how an ensemble of free elementary fields is represented by a de Broglie wave. He considers the case when all have the same velocity, but his result can be extended to the case of several velocities.

According to Koga, a de Broglie wave only represents an ensemble of electrons, not a single particle. For a system of several electrons, one must consider a separate Schrödinger equation for each of them. There is no such thing as the wave function of a system.

Wave-Partice Duality has the following meaning in Koga’s theory: an electron looks like a particle in some experiments and like a wave in others. But it is neither; it is a localised field. Koga explains these things in [21, 22] and Chapter IV of [27]. See also [28].

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According to Koga’s theory of the Schrödinger equation, a free electron has a spherically symmetric amplitude and has a singularity at its centre. The size is not given by the theory; Koga takes it to be about $1/\kappa$, and assumes $1/\kappa = \hbar/mc$. In the presence of an external potential, the elementary field deforms and there is a “tunnel effect” arising from the interaction between the elementary field, the external electromagnetic field and the internal gravitational field of the electron. This is related to the stability of atoms. (The latter topic is taken up by him later.)

It should be noted that the energy of the (free) electron and $\kappa$ are supposed to be constants of nature. Koga assumes that $\kappa$ is closely related to the size of the electron field. As $r$ increases, $a$ decreases to 0. From some point onwards, $a$ can be considered “negligible”; this can be taken to be a bound on the electron’s radius. The larger the value of $\kappa$, the smaller the radius.

Koga argues that from these considerations, it follows that an electron in an atom is at rest relative to the nucleus, and this “tunnel effect” is complete or maximal when the electron is in an energy eigenstate of the atom. We do not go into details in this brief introduction.

Although, for the purpose of determining the electron radius, we consider $a$ to be negligible beyond some distance from
the centre, it should be noted that at any point, the Maxwell field (which Koga obtains from the Dirac field) is the sum of the fields due to individual particles. For this purpose the value of \( a \) (of each particle) is not negligible.

### 2.3. The Dirac equation

In this section we describe Koga’s treatment of the Dirac equation. In Chapter 3 we first review the history of the spin concept and then study Koga’s theory using Geometric Algebra, which reveals some new information.

Around 1930 several researchers (e.g., Schrödinger, Fock, Ivanenko) tried to modify the Dirac equation to satisfy General Relativity. They realised that a change was needed in the concept of spinor. According to their theory, the components of a spinor field at each point depend only on the point and are invariant under coordinate transformations. But at each point a collection of four independent vectors, called a tetrad, is defined and is a continuous function of spacetime. The spinor transformation law at each point is valid with respect to tetrad rotations rather than coordinate transformations. (See the books by Anderson [1] or Lord [29].) Remarkably, Koga’s ideas on the Dirac equation can be considered a special case of this approach for flat space and tetrads consisting of orthogonal unit
vectors related by Lorentz transformations. Koga gives no indication that he was aware of the old work mentioned above. It seems he was not.

As in the Schrödinger case, in Koga’s theory of the Dirac equation the solution is not a quantum-mechanical state but gives the properties of the electron field at each point of spacetime.

Koga shows that at points far from the electron the (Dirac) $\psi$ field reduces to an electromagnetic field, satisfying Maxwell’s equations, if properly interpreted (see [25] or [27], Chapter V).

For an electron in an (external) electromagnetic field, the Dirac equation is, in modern notation,

$$i\hat{\gamma}^\mu(\partial_\mu - ieA_\mu)|\psi\rangle = m|\psi\rangle$$

where, in the Copenhagen interpretation, $|\psi\rangle$ is considered a 4-spinor and the matrices $\hat{\gamma}^\mu$ are invariant under Lorentz transformations. Koga argues that $|\psi\rangle$ must be a collection of four scalar functions of spacetime and the Dirac matrices must transform as the components of a 4-vector (a generalisation of this was used by Fock and others in the 1920s in an attempt to make the Dirac equation general relativistic). This will ensure that the anisotropy embodied in the equation (his words) does not
rotate together with the coordinate system when we make a coordinate transformation.

Koga gives a solution to the Dirac equation for a free electron \((A_\mu = 0)\) starting with a solution to the Klein-Gordon equation

\[
\left( \hbar^2 \frac{\partial^2}{\partial t^2} - \hbar^2 c^2 \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) + m^2 c^4 \right) \varphi = 0
\]

where we work in Minkowski space: a point is given by \((ct, x, y, z) = (x^0, x^1, x^2, x^3)\) and the metric is \(\eta_{ij}dx^i dx^j = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2\).

Koga studies the Klein-Gordon equation like he did the Schrödinger equation. He gives the following solution:

\[
\varphi = a \exp \left( \frac{iS}{\hbar} \right)
\]

where \(S = -Ect + \mathbf{p} \cdot \mathbf{r} \)

and \(a = \frac{\exp(-\kappa |\mathbf{r}'|)}{|\mathbf{r}'|} \).

Here \(\mathbf{r}' = \frac{\mathbf{r} - \mathbf{u}t}{\sqrt{1 - w^2/c^2}}\) (\(\mathbf{r} = xi + yj + zk = \text{position vector}\)),

\(\mathbf{p} = \frac{\mathbf{u}E}{c}\) (\(\mathbf{u} = 3\)-velocity of the electron),

\(E^2 = \frac{m^2 c^2 - \hbar^2 \kappa^2}{1 - w^2/c^2}\) (\(cE = \text{energy}\)).
As in the case of the Schrödinger equation, here also $E$ and $\kappa$ are closely related. But the energy of the electron is now $cE$.

The energy $cE$ is defined by $E = -\partial S / \partial (ct)$. The momentum $p$ is $\nabla S$ (here $\nabla$ is the 3-dimensional gradient).

Koga gives a justification for considering $E$ to be independent of $t$ and $p$ independent of $r$. I omit it here.

For a free electron at rest, we have $E^2 = m^2 c^2 - \hbar^2 \kappa^2$. Since $\kappa$ is a constant of nature, so is $E^2$.

The Klein-Gordon equation yields the Dirac equation as follows: it can be written as

$$D_0 D_1 \varphi = 0 = D_1 D_0 \varphi.$$

In Koga’s notation, we take

$$D_0 = \beta (\hbar \frac{\partial}{\partial t}) + \beta \alpha \cdot i \hbar c \frac{\partial}{\partial \mathbf{r}} - mc^2,$$

$$D_1 = \beta (\hbar \frac{\partial}{\partial t}) + \beta \alpha \cdot i \hbar c \frac{\partial}{\partial \mathbf{r}} + mc^2.$$

Here $\beta$ is a $4 \times 4$ matrix and $\alpha$ is a triple of $4 \times 4$ matrices satisfying well-known commutation relations.

Then, if $\varphi$ is a solution of the Klein-Gordon equation (more precisely, a 4-tuple of scalar solutions) and $|\psi\rangle = D_1 \varphi$ then $|\psi\rangle$ satisfies $D_0 |\psi\rangle = 0$, which is the Dirac equation.
It should be noted that although Koga uses $D_0$ to get the Dirac equation it would be equally justified to use $D_1$ to write an equation for the electron. For each of these “Dirac equations”, there are two possible energies: positive and negative.

Koga takes $\varphi_j = a \exp(iS/\hbar)A_j \exp(i\theta_j)$ for $j = 1, 2, 3, 4$ as four solutions to the Klein-Gordon equation, and

$$\langle \psi \rangle = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = D_1 \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{pmatrix}.$$

Koga concludes that the field representing a free electron (as above) is circularly symmetric only about the axis which passes the point $r = ut$ and is parallel to the $z$-axis. We omit the details. His arguments are mathematically not complete or satisfactory [32] as will be explained shortly.

Koga’s solution to the Dirac equation, as I understand it, is not meant to be a replacement of the work done by Dirac in the 1920s, which culminated in the prediction of the positron.

Rather, it is an improvement of Koga’s solution of the Schrödinger equation and is a step towards his goal of a theory including gravitation.
It gives information about the electron field, including the field in the very small region which many consider a point. He wanted, among other things, to prove that a free electron has an axis of symmetry and spins around it like a top.

For this purpose, Koga considers a rotation of the coordinate system about the $z$ axis by an angle $\varphi$. He looks at what happens to the four complex components of the solution: $\psi_1$, $\psi_2$, $\psi_3$, $\psi_4$.

For instance, the expression for $\psi_1$ has four terms, each containing only one of the angles $\theta_j$ in its argument.

After the rotation, he shows that in the case of $\psi_1$, $\theta_4$ gets replaced by $\theta_4 - \varphi$.

But at this point, he asserts that since the angles $\theta_j$ are arbitrary, we can simply ignore the effect of $\varphi$!

I consider this unsatisfactory.

In chapter 3 we analyse the Dirac equation and Koga’s solution using geometric algebra. Our analysis suggests that the electron spins and shudders. However, there is no interference between states; in fact there are no states in Koga’s theory, only pointwise descriptions of the electron field.

The theory discussed above is, of course, only approximate. It suggests a single spin frequency although, in reality, there may be terms with several frequencies.
Koga shows that the Dirac equation implies the Maxwell-Lorentz equation of the electromagnetic field, provided the comparison of the two is restricted to their time-independent (or slowly varying) solutions. The reason for this restriction is that the Dirac field may contain some high frequency terms which are averaged out in the Maxwell fields (he uses the term “coarse-grained”). He derives the correct value of the magnetic moment of the electron (which, as is well known, equals the Bohr magneton). The only assumption he makes here is that the Dirac field is localised, and no specific solution is used.

2.4. Koga’s general relativistic theory of the electron

Koga considers the Schrödinger equation, and also the Dirac equation, inadequate to describe the structure of the electron. For him these theories are merely pointers to a theory that includes gravitation: specifically, the gravitational field of the electron itself. Here I am only trying to give a very brief glimpse of Koga’s work. Hence there is no complete explanation of the notation, etc. A reader unfamiliar with general relativity can omit this section.

Koga assumes a geometry that is more restricted than general Riemannian geometry but more general than the geometry of Minkowski spacetime (which he calls Euclidean geometry).
For the matter field he gives the following two sets of non-linear equations:

\[ \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^j} (\sqrt{-g} F^{ij}) - g^{ij} \frac{\partial \eta}{\partial x^j} = 0 \]  
\[ (1) \]

\[ \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^j} (\sqrt{-g} F^{*ij}) - g^{ij} \frac{\partial \xi}{\partial x^j} = 0 \]  
\[ (2) \]

Here \( g \) is the determinant of the metric tensor \( g^{ij} \); \( F^{ij} \) is an anti-symmetric tensor and \( F^{*ij} \) is conjugate of the \( F^{ij} \); \( \xi \) and \( \eta \) are scalars.

Koga further considers that Einstein’s equations:

\[ R_{ij} - \frac{1}{2} g_{ij} R = -K T_{ij} \]

are not useful to study the internal field of the electron because they are quite complicated and have not been verified on a microscopic scale. He replaces them by a set of four equations containing only first order derivatives of the metric tensor,

\[ \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^i} g = a g_{ij} (F^{*jk} - g^{jk} \xi) \frac{\partial \eta}{\partial x^k} + b g_{ij} (F^{jk} - g^{jk} \eta) A_k \]  
\[ (3) \]

(Here \( a \) and \( b \) are constants and \( A_k \) represents an external electromagnetic field of macroscopic scale.)

Koga calls all these the fundamental equations. The fundamental equations given by (1) and (2) reduce to the Dirac
equation (as an approximation) and to the Maxwell-Lorentz
equation (in another approximation).

In order to get the Dirac equation or the Maxwell-Lorentz
equation from (1) and (2), the equation for the metric tensor
field given by (3) is used to linearise the nonlinear terms occurring in (1) and (2).

Koga further notes that none of the above fundamental equa-
tions (1), (2) and (3) contain the constants such as mass $m$,
charge $e$ and the Planck’s constant $h$. By substituting the sym-
bol of mass for a certain function which is assumed to be almost
invariant, and making other approximations he gets the Dirac
equation for the electron. By changing the boundary condi-
tions, similar equations are obtained for nucleons. Similarly,
by substituting the symbol of electric charge for another func-
tion and applying an averaging process the Maxwell-Lorentz
equations are obtained.

Koga rejects Mach’s principle (which, according to many
physicists, states that the mass of a body is entirely due to its
interaction with the rest of the universe, but mainly nearby mat-
ter) on the ground that the concepts of mass, force and action at
a distance have no place in the fundamental equations and the
Mach principle is only conceivable in terms of such classical
mechanical concepts and is hence not compatible with relativity. We will see more of Mach’s principle in Chapter 4.

An examiner has pointed out that the statement above is not Mach’s principle, but only a misunderstanding of it that is widespread among physicists including Sachs (who I got it from). A correct version (given by the examiner) is that inertial forces (and hence inertia, not mass) arise due to interaction with other matter.

Actually, Koga states (and rejects) the Mach principle as “the inner structure of the electron is a reflection of the external universe”. He does not give any reference for this statement. This seems just an example of the misunderstanding mentioned above; it is even possible that Koga got it from Sachs.

Koga gives arguments for rejecting his version of Mach’s principle. I don’t go into this in detail.

All this is the subject of the paper [26] and Chapter 6 of the book [27]. In the book he also studies the photon, quantum electrodynamical phenomenon and other particles like the proton, neutron and pions using the same ideas.