CHAPTER-2
MICROSTRIP ANTENNAS

2.1 INTRODUCTION

A microstrip antenna, in its simplest configuration, consists of a radiating patch on one side of a dielectric substrate, which has a ground plane on the other side. The substrate used for the microstrip antenna is usually of lower relative dielectric constant and of a larger thickness than those used for microstrip lines. The relative dielectric constant, \( \varepsilon_r \), of the substrate may be typically \( 2.5 (<10) \) and the thickness may in the range of \( 0.003\lambda_s \) to \( 0.05\lambda_s \), where \( \lambda_s \) is the free space wave length. The patch dimension may be of the order of \( \lambda_d/2 \), where \( \lambda_d \) is the wavelength in the dielectric medium. In the following paragraphs some of the characteristics of the most standard shape microstrip antennas are discussed.

2.2 CIRCULAR MICROSTRIP ANTENNA

2.2.1 RESONANT FREQUENCY

The resonant frequencies of the \( TM_{nm} \) modes for the circular disc antenna Fig.2.1 are given as[68]:

\[
    f_{nm} = \frac{\alpha_{nm} c}{2\pi a_{eff}\sqrt{\varepsilon_r}}
\]

Where,
- \( \alpha_{nm} \) is the \( m\)th zero of the derivative of the bessel function of order \( 'n' \),
- \( c \) - velocity of light in free space,
- \( \varepsilon_r \) - relative dielectric constant of the substrate,
- \( a_{eff} \) - effective radius of the patch,
- \( h \) - substrate height.
- \( a \) - radius of the circular microstrip antenna

The effective radius is given by [59, 66]

\[
    a_{eff} = a \left[ 1 + \frac{2h}{\pi a\varepsilon_r} \left( \ln \frac{\pi a}{2h} + 1.7726 \right) \right]^{1/2}
\]

The above expression predicts the radius with an error of less than 2.5% for \( (a/h) \gg 1 \).
2.2.2 INPUT IMPEDANCE

At resonance the input impedance of a microstrip antenna is real. The resonant resistance \( R \) can be calculated from the total power lost in the cavity \( P_t \), using [3]

\[
R = \frac{V^2}{2P_t} \tag{2.3}
\]

If the disk is fed at an arbitrary point \((\rho_w, 0)\), the resistance at resonance is:

\[
R = \frac{h^2 E_z J_z^2(k\rho_w)}{2P_t} \tag{2.4}
\]

If \( R_o \) is the resistance at resonance for the feed location \( \rho_o = a \), then \( R \) may be written:

\[
R = \frac{R_o J_z^2(k\rho_w)}{J_z^2(ka)} \tag{2.5}
\]

The total power loss \( P_t \) includes

\( P_r \) - Power radiated,

\( P_r \) - Power dissipated in the disc resonator owing to the finite conductivity of the disc conductor and

\( P_d \) - The power dissipated in the imperfect dielectric substrate, i.e.,

\[
P_t = P_r + P_r + P_d \tag{2.6}
\]

The resonant resistance calculated above may be varied over a wide range by simply changing the feed location and the substrate thickness. It has been found that the input resistance increases when the feed is moved out towards the edge [39] and also when the substrate thickness is increased [56]. The input resistance may be adjusted by locating the feed point properly. The reactive part of the input impedance, which may be computed from the stored energy associated with the cavity field is found to change its sign as the frequency is swept through the resonant value for a particular mode, while the input resistance passes through a maximum.

An improved expression for the input impedance has been derived using the modal expansion model by Carver [8], Richards et al. [39] giving a very good agreement with the measured results for all modes and for all feed locations. It is also found that the pattern of the antenna is dependent mainly upon the field distribution of the dominant mode. Therefore, the impedance may be varied with feed location for matching purposes without affecting the pattern.
Fig. 2.1: Circular Microstrip Antenna

Fig. 2.3: Rectangular Microstrip Antenna
A detailed experimental verification of the popular theoretical models (including cavity model and moment method) given by Schaubert et al. [40] shows that for thicker substrates (>0.02λo). The agreement between the experimentally obtained resonant frequency and the input impedance of a microstrip antenna, and those calculated theoretically, in general, is poor.

2.2.3 RADIATION EFFICIENCY

The antenna efficiency is defined as the ratio of the radiated power to the input power.[3]

\[ \eta\% = \frac{P_r}{P_i} \times 100 \]  

(2.7)

The efficiency increases with increasing substrate thickness and decreasing dielectric constant.

2.2.4 BANDWIDTH

The bandwidth of a microstrip antenna is defined as the frequency ranges over which the value of the input VSWR increases from unity to a tolerance limit value, s. The bandwidth of such antennas is narrow and may be expressed as [3]

\[ BW = \frac{s-1}{Q_T \sqrt{s}} \]  

(2.8)

where, \( Q_T \) is the total quality factor.

A greater bandwidth is possible by choosing a thicker substrate of low dielectric constant material.

2.2.5 DIRECTIVITY AND GAIN

The directivity 'D' of an antenna is defined as the ratio of the power density in the main beam to the average power density. From the calculated far fields, the directivity of a circular disk antenna excited in the dominant mode (n=1) may be expressed as [3]

\[ D = \frac{(1/2) \text{Re}(E_\theta H_\phi^* - E_\phi H_\theta^*)}_{|_{\theta=0}} \left( \frac{P_r}{4\pi^2} \right) \]  

(2.9)

where, \( H_\phi = \frac{E_\theta}{\eta_o} \)
A disc antenna on an alumina substrate has a directivity of about 3.5dB, which is almost independent of substrate thickness \((h \leq 0.1275 \text{ cm})\) and resonant frequency. A disc antenna designed using duriod substrate has a maximum directivity of about 5.3 dB which decreases with increasing resonant frequency and dielectric thickness. The effective gain of the antenna may be calculated from:

\[ G_r = \eta D \]  

(2.10)

2.2.6 RADIATION PATTERN

Radiation into the hemisphere above the antenna may be derived from an equivalent magnetic current distribution over the aperture and with the structure removed. That is,

\[ M = 2 \hat{E} \times \hat{n} \]  

(2.11)

Where \(\hat{n}\) is a normal unit vector pointing into the external region. The far field at resonance, in spherical polar co-ordinates, for a thin circular microstrip antenna excited in the TM-modes may be found from a potential function [64] as:

\[ E_\theta = j^n k_o \frac{e^{-ik_o r}}{r} \frac{V_o a}{2} \left[ J_{n+1}(k_o a \sin \theta) - J_{n-1}(k_o a \sin \theta) \right] \cos n \phi \]  

(2.12)

\[ E_\phi = j^n k_o \frac{e^{-ik_o r}}{r} \frac{V_o a}{2} \left[ J_{n+1}(k_o a \sin \theta) - J_{n-1}(k_o a \sin \theta) \right] \cos \theta \sin n \phi \]  

(2.13)

where the edge voltage at \(\phi = 0^\circ\) is defined as \(V_\theta = h E_o J_n(ku)\).

The far fields of the \((n=1)\)-mode have maximum along the normal to the disc, while all other modes have zero in this direction. Typical plots in the \(\phi = 0^\circ\) (E-plane) and \(\phi = 90^\circ\) (H-plane) planes as shown in the Fig. 2.2. At a frequency of 2 GHz, as compared to the rectangular patch antenna (with a 3 dB beamwidth of 111° in the E-plane and 123° in the H-plane), the circular disc microstrip antenna presents a similar 3dB beamwidth viz., 100° in the E-plane and 80° in the H-plane for a 1.59mm(h) substrate with a dielectric constant \((\varepsilon_r = 2.32)\), in other characteristics being almost same[3].

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Fig. 2.2: Typical Radiation Patterns in the E and H-plane of the circular Patch Antenna

(a) TM\textsubscript{11} -mode  
(b) TM\textsubscript{12} -mode
2.3 RECTANGULAR MICROSTRIP ANTENNA

2.3.1 ELEMENT WIDTH

For a dielectric substrate of thickness $h$, an antenna operating frequency of $f_c$, and for an efficient radiator (Fig.2.3), a practical width [3] is

$$W = \frac{c}{2f_c \sqrt{\varepsilon_r}} \left( \frac{\varepsilon_r + 1}{2} \right)^{1/2}$$

(2.14)

where, $c$ is the velocity of light.

For the widths smaller than those selected according to equation (2.14), radiator efficiency is lower, while for larger widths, the efficiency is greater. But higher order modes may result, causing field distortions.

2.3.2 ELEMENT LENGTH

Once $W$ is known, the length of the resonant element is then obtained from [3].

$$L = \frac{c}{2f_c \sqrt{\varepsilon_r}} - 2\Delta l$$

(2.15)

where, $\varepsilon_r = $ effective dielectric constant.

$$\varepsilon_r = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left( 1 + \frac{12h}{W} \right)^{1/2}$$

(2.16)

and

$$\frac{\Delta l}{h} = 0.412 \left( \frac{(\varepsilon_r + 0.3)(W_h + 0.264)}{(\varepsilon_r - 0.258)(W_h + 0.8)} \right)$$

(2.17)

Because of the inherent narrow bandwidth of the resonant element, length is a critical parameter, and equation (2.15) should be used to obtain an accurate value for the line length ‘$L$’.

2.3.3 INPUT IMPEDANCE

The input impedance for a microstrip radiator is an essential parameter; it should be accurately known so as to provide a good match between the element and the feed [3].
\[ Z_{in} = -j \frac{\hbar \omega}{\varepsilon L W} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\varepsilon_m^2 \cos^2 \left( \frac{m \pi x}{L} \right) \cos^2 \left( \frac{n \pi y}{W} \right)}{\omega^2 - \omega_m^2} G_{mn} \]  

where, \( \omega_m \) is the complex resonant frequency [3]

\[ G_{mn} = \frac{\sin \left( \frac{m \pi l}{2W} \right) \sin \left( \frac{n \pi l}{2L} \right)}{m \pi l \sin \left( \frac{m \pi l}{2W} \right) n \pi l \sin \left( \frac{n \pi l}{2L} \right)} \]  

\( d_x, d_y \) are the widths of the feed in the \( x \) and \( y \) directions, respectively.

\( L \)- length of the patch, \( W \)-width of the patch, \( (x, y) \)- feed point co-ordinates, \( h \)-height of the substrate and \( \varepsilon_r \)-permittivity of the substrate.

### 2.3.4 EFFICIENCY

The antenna efficiency, defined as the ratio of the radiated power to the input power, may be expressed as [3]

\[ \eta \% = \frac{P}{P_i} \times 100 \]  

### 2.3.5 BAND WIDTH

The bandwidth of a microstrip antenna for a feed line \( VSWR < s \) can be shown to be [3]

\[ BW = \frac{s - 1}{Q_r \sqrt{s}} \]  

The larger bandwidth is possible by choosing the thicker substrate of lower \( \varepsilon_r \). The bandwidth may also be increased by increasing the inductance of the radiator, by cutting holes or slots in it or by adding reactive components to improve the match of the radiator to the feed line.

### 2.3.6 DIRECTIVITY AND GAIN

The directivity and gain of the antenna is defined as the ratio of the maximum power density in the main beam to the average radiated power density. For a single slot, the directivity may be expressed as [3]
Where \( I \) is defined in [3]. For the case of a microstrip antenna comprising two slots at a spacing of \( L \), in the E-plane, the directivity expression becomes

\[
D_z = \frac{2D}{1 + g_{12}}
\]

(2.23)

where \( g_{12} \) is the normalised mutual conductance defined in [3]. For \( g_{12} << 1 \),

\[
D_z = 2D
\]

(2.24)

and for,

\[
\omega << \lambda_0, \quad D_z = 6.6
\]

(2.25)

\[
\omega >> \lambda_0, \quad D_z = \frac{8w}{\lambda_0}
\]

(2.26)

The effective gain is

\[
G_e = \eta D_z
\]

(2.27)

It may be seen from [3], that for a substrate with given \( \varepsilon_r \), the gain increases with substrate thickness 'h' and decreases with increase in \( \varepsilon_r \).

### 2.3.7 RADIATION PATTERN

The far field radiation pattern of a rectangular microstrip patch operating in the TM_{10} mode is broad in both the E- and H-planes. The pattern of a patch over a large ground plane may be calculated by modeling the radiator as either two parallel uniform magnetic line sources of length 'L' separated by distance 'w', or as two equivalent electric current sources as suggested in Fig: 2.4. The effect of the ground plane and substrate is loaded by imaging the slot at an electrical distance 'kh'. If the slot voltage across either radiating is taken as \( V_o \), The calculated fields are [9]

\[
E_\theta = -\frac{jV_o k_o \varepsilon_0 e^{-j\beta r}}{\sin\left(2\frac{L}{k_o}\sin\theta\sin\phi\right)} \left[\cos(k_o \cdot \cos\theta)\right] \left[\cos\left(2\frac{w}{k_o}\sin\theta\cos\phi\right)\right] \cos\phi
\]

(2.28)
Fig. 2.4: Geometry for Far-Field Pattern of Rectangular Microstrip Antenna
where, \( k = k_0 \sqrt{\varepsilon_r} \)

### 2.4 ANNULAR RING MICROSTRIP ANTENNA

There are several interesting features associated with this patch. First, for a given frequency, the size is substantially smaller than that of the size of the circular patch when both are operated in the lower mode. In application to arrays, this allows the elements to be more densely situated, thereby reducing the grating lobe problem. Secondly, it is possible to combine the annular ring with the second microstrip element, such as the circular disc within its aperture, to form a compact dual band antenna system. Thirdly, the separation of the modes can be controlled by the ratio of outer to inner radii. Finally, it has been found that, by operating in one of the higher-side broadside modes, i.e. \( TM_{12} \), the impedance bandwidth is several times larger than is achievable in other patches of comparable dielectric thickness.

#### 2.4.1 RESONANT FREQUENCY

Consider an annular ring patch Fig.2.5 with outer radius \( b \) and inner radius \( a \). Assuming that only TM modes exist, the resonant frequencies are determined by [23]

\[
f_{mn} = \frac{k_m c}{2\pi \sqrt{\varepsilon_r}}
\]

where, \( c \) - velocity of light,

\( k_m \) - are the roots of the characteristic equation and \( \varepsilon_r \) - effective dielectric constant.

\[
\varepsilon_r = \frac{\varepsilon_r + 1}{2} + \frac{(\varepsilon_r - 1)}{2} \left(1 + \frac{10h}{w}\right)^{-1/2}
\]

where, \( w = (b-a) \)

To account for the fringing fields along the curved edges of the ring, it has been suggested that the inner and outer radii be modified according to,

\[
b_r = b + (w_c f_r) \cdot w
\]

\[
a_r = a + (w_c f_r) \cdot w
\]
Fig. 2.5: Annular Ring Microstrip Antenna
where,  
\[ w_c(f) = w + \frac{w_c(o) - w}{1 + (f/f_p)^2} \]  
(2.34)  
\[ w_c(o) = \frac{120 \pi h}{z_o \sqrt{\mu}} \]  
(2.35)  
\[ f_p = \frac{z_o}{2 \mu} \]  
(2.36)  
where, \( \mu \) is the permeability and \( z_o \) is the quasi-static characteristic impedance of a microstrip line of width 'w'.

A pair of empirical formulas for the modified radii, sufficient for many engineering purposes, are given by,  
\[ a_c = a - (3/4)h \quad \text{and} \quad h_c = h + (3/4)h \]  
(2.37)

2.4.2 INPUT IMPEDANCE

The input impedance of the co-axial feed annular ring is given by [23]  
\[ Z_{in} = jw \mu_c h \sum \sum \left( \frac{\pi k^2}{2n_k a} \right)^2 \left( J_n(k_m d) - J_n'(k_m d) \right) \left( 1 - \frac{n^2}{k_m^2 a^2} \right) \]  
(2.38)

where,  
\[ k_{eff} = k_o \sqrt{\varepsilon_r (1 - j \delta_{eff})} \quad \text{and} \quad \delta_{eff} \quad \text{effective loss tangent} \]

'a' and 'b'- are inner and outer radius and 'd'- feed probe distance from the center.

2.4.3 RADIATION PATTERN

The far zone electric field is [23]  
\[ E_r = \frac{j^n 2h k_o E_o e^{-jk_o r}}{\pi k_{nm}} \frac{\cos(n \phi)}{r} \left[ J_n'(k_o a \sin(\theta)) - J_n'(k_o b \sin(\phi)) \right] \frac{J_n'(k_{nm} a)}{J_n'(k_{nm} b)} \]  
(2.39)  
\[ E_o = -\frac{j^n 2h k_o E_o e^{-jk_o r}}{\pi k_{nm}} \frac{\cos(\theta) \sin(n \phi)}{\sin(\theta)} \left[ J_n(k_o a \sin(\theta)) - J_n(k_o b \sin(\phi)) \right] a \frac{J_n(k_{nm} a)}{b J_n(k_{nm} b)} \]  
(2.40)

Using the equations (2.39) and (2.40), the relative radiation patterns for the various modes can be plotted. It is seen that, for the \( TM_{11} \) and \( TM_{12} \) modes, the strongest radiation occurs in the broadside direction \( (\theta = 0^\circ) \). On the other hand, radiation patterns for the \( TM_{21} \) and \( TM_{22} \) modes have nulls in the broadside direction, with the strongest radiation occurring at oblique angles.
Fig. 2.6(a): Electric Field Distribution On Annular Ring Microstrip Antenna (TM_{11}-mode)

Fig. 2.6(b): Electric Field Distribution On Circular Microstrip Antenna (TM_{11}-mode)
2.5 DISCUSSION (CIRCULAR M.S.A)

It is found that (the feed-probe position) the permittivity of the substrate, thickness and also the ground plane size affect considerably the radiation pattern, mode excitation, resonant frequency and the input impedance of the microstrip antennas.

2.5.1 PERMITTIVITY OF THE SUBSTRATE

Increasing the substrate permittivity reduces the patch size and subsequently [61] the beam width increases, but the effect is stronger in the E-plane than in the H-plane. As a result, the symmetry of the radiation patterns deteriorates and the cross-polar level is increased.

2.5.2 SUBSTRATE THICKNESS

The bandwidth of the microstrip antenna normally increases by increasing the substrate thickness. For the TM_{11}-mode circular patch, it is seen [61] that increasing the substrate thickness h, increases the beam width in the E-plane but reduces it in the H-plane, until 'h' reaches 0.06λ, after which the relationship reverses. It is found that the substrate thickness generally has a greater effect [63] on the cross-polarization level, which increases with increase in the substrate thickness, although the cross-polar patterns do not get affected much.

2.5.3 SIZE OF THE GROUND PLANE

The size of the ground plane has a pronounced effect on the far field patterns [50] and the input resonant resistance of the antenna [48]. The resonant frequency increases by about 4% but the input resistance increases dramatically when the ground plane and the patch diameters become equal. Again, increasing the ground plane radius beyond 1.3 times the patch radius, has negligible effect both on the input resistance and the resonant frequency.

Thus, to summarize,

1. It becomes imperative, for a successful design, all the inter-related features of the circular microstrip antenna discussed above, like the dependence of bandwidth, Q-factor and gain on the dielectric constant / substrate thickness and the resultant surface
wave effects /cross polarization, have to be carefully taken into account, especially for the stacked electro-magnetically coupled microstrip antennas.

2. Although the theoretical models do not predict the resonant frequency and the input impedance accurately [40] for thicker substrates, they do give a reasonable starting point for the design.