Chapter 10

An Application of Information Theory for identification of 

ragas in music

“The most distinct and beautiful statement of any truth (as of music) must take at last 
the mathematical form.” ~ Henry David Thoreau

10.1 Introduction:

Chapters 8 and 9 of this thesis have been devoted to graph theoretic analysis of 
graham of a raga. In Chapter 8 graph theory is applied to understand the grammatical 
structure of raga Bhupali corresponding to its distinctive features viz. (a) Arohana 
Avarohana, (b) Catch Phrase and (c) Alap. On the other hand Chapter 9 includes graph 
theoretic comparison of the two pairs of allied ragas viz. (Bhupali, Deshkar ) and 
(Desh, Tilakkamod). The result of the comparison has revealed that estimated mean 
absolute difference of weights corresponding to alap of ragas is very small for both the 
pairs. Hence many a times when one is exposed to a musical piece, sharing same sets of 
notes with a pair of allied ragas, it is difficult to decide about the close inclination 
between the piece of music and the pair of allied ragas. In such a situation a quantity 
measuring distance between a musical piece and a particular raga might be helpful to 
determine the percentage of the raga that is contained in the musical piece. This 
Chapter addresses the problem of determining close inclination of a piece of music to a
pair of *allied ragas*. The Kullback Leibler (K-L) divergence method for measuring distance between two Markov chain models have been applied for measuring the distance mentioned above. The basic requirement of the above method is that musical pieces considered for comparisons are to be defined on identical state space. For the purpose of the experiment selected portions of *alap* of *Bhupali* and *Deshkar* and a piece of music from a musical composition of *Dr Bhupen Hazarika* have been considered. A brief review of literature on this chapter have been provided in Section 1.4 of this thesis.

Section 10.2 of this chapter includes the materials and methods that have been used in the chapter whereas Section 10.3 deals with experiments and results. Section 10.4 is about the geometrical interpretation of the Kullback Leibler distance corresponding to the *ragas* and selected piece of music.

10.2 Materials and Methods:

10.2.1 Application of Information theory:

The information regarding a musical event may be modeled by defining a random variable in terms of movement of *notes*. Then each realization of the random variable may be regarded as a *message*. Most of the work presented in this thesis is based on transition pattern of *notes* that follows a Markov chain. Thus a random variable representing the conditional event \( U_n = (X_n = j \mid X_{n-1} = i) \) may be considered as a *message*. 
Then the probability $p_{ij} = P(U_n = j \mid X_{n-1} = i)$ is regarded as transition probability where $X_n = j$ indicates the position of a note at the time of $n^{th}$ transition.

Let the amount of information gained after observing the conditional event $U_n$ with probability $p_{ij}$ be defined by

$$I(U_n) = I((X_n = j \mid X_{n-1} = i)) = \log \frac{1}{p_{ij}} = -\log p_{ij}$$

... (10.1)

The base of logarithm is arbitrary. The equation (10.1) has the following properties

(i) $I(U_n) = 0$ for $p_{ij} = 1, 0$

(ii) $I(U_n) > 0$ for $0 < p_{ij} < 1$

(iii) $I(U_n) > I(U_k)$ for $p_{ij} < p_{kr}$ where $U_k = (X_k = r \mid X_{k-1} = q)$ . (Haykin, 1999)

That is less probable an event is, the more information we gain through its occurrence. Expected value of $I(U_n) = H(U)$ is known as entropy of the random variables permitted to take a finite set of discrete values. Now let $\{p_k\}$ and $\{q_k\}$ denote any two probability distributions for a discrete random variable $X$. Then the following lemma (Gray 1990) may be used for the study of stochastic systems.

**Lemma:** Given $\{p_k\}$ and $\{q_k\}$ for a discrete random variable $X$, then

$$\sum p_k \log \left\{ \frac{p_k}{q_k} \right\} \geq 0$$

... (10.2)

which is satisfied with equality if $p_k = q_k$ for all $k$.

Again let $p_X(x)$ and $q_X(x)$ denote the probabilities that the random variable $X$ is in state $x$ under two different operating conditions. The relative entropy or Kullback-Leibler divergence (distance) between two probability mass function $p_X(x)$ and $q_X(x)$ is defined
by

$$D_{p_{\text{II}}q} = \sum p_X(x) \log \left\{ \frac{p_X(x)}{q_X(x)} \right\}$$

( Kullback 1968; Gray 1990; Cover and Thomas 1991)

where the sum is over all possible states of the system. The probability mass function $q_X(x)$ plays the role of a reference measure.

10.2.2 Raga identification in a piece of music using Markov chain:

Assuming that each of the raga $A$ and $B$ has a unique style that can be recognized in a certain piece of music, one can select a feature (such as movement of notes) that looks interesting for both the raga $A$ and $B$ and the piece of music respectively. The movement of notes in both the raga $A$ and $B$ and also the piece of music can be modeled as a Markov chain. Then based on these three Markov models, for each of the raga and the piece of music $U$, a two-way music identification can be achieved by the following hypothesis test.

The piece of music $U$ is closer to raga $A$, if

$$D( P_{UI} P^A) < D( P_{UI} P^B)$$

$B$, if

$$D( P_{UI} P^A) > D( P_{UI} P^B)$$

Note: The condition of equality has not been considered.

where $P^U$ is the transition probability matrix of the piece of music $U$, and $D$ is a distance metric on the space of $N \times N$ probability matrices. In this research we use the Kullback-Leibler distance metric.
\[ D(P^{(1)} \prod P^{(2)}) = \alpha \sum_{i=1}^{N} \sum_{j=1}^{N} p_{ij}^{(1)} \log \left( \frac{p_{ij}^{(1)}}{p_{ij}^{(2)}} \right) \] … (10.4)

Although the Kullback-Leibler distance metric is not symmetric and does not obey the triangular inequality, it is useful to interpret the metric as the distance between distributions (Cover and Thomas 1991). Kullback-Leibler distance metric may be used to measure the difference between the distances of a musical piece from two different ragas.

**Theorem:** For ragas A and B defined under irreducible chain with t.p.m \( P^A \) and \( P^B \), both having identical state space, the statistic 

\[ \log \left( \frac{P_{ij}^{(A)}}{P_{ij}^{(B)}} \right) \] is a measure of difference between distances \( D(P^U \prod P^A) \) and \( D(P^U \prod P^B) \), where \( P^U \) is a transition probability matrix corresponding to a piece of music U.

**Proof:** Since the ragas corresponding to North Indian classical music produce irreducible state space, it may be performed by starting at any note. The initial probabilities \( W_i = P(X_0 = i) \) are same for all the states belonging to the state space. Hence the result defined in section 10.3 may be applied to transition probabilities \( p_{ij} \) by multiplying it by \( W_i \) and summing over \( i,j \) for all \( i,j \) belonging to the state space.

This happens because \( \sum_i \sum_j W_i p_{ij} = 1 = \sum_i W_i p_i \), where \( W_i = 1/s \) with \( s = \text{size of the state space} \) and \( p_i = \sum_j p_{ij} \). \( W_i p_i \) plays the role of \( p_k \) defined in Section 10.2.
Now \( D(P^U \prod P^A) = \sum_i \sum_j W_{ij} p_{ij}^{(U)} \log \left( \frac{p_{ij}^{(U)}}{p_{ij}^{(A)}} \right) \) … (10.5)

\[ D(P^U \prod P^B) = \sum_i \sum_j W_{ij} p_{ij}^{(U)} \log \left( \frac{p_{ij}^{(U)}}{p_{ij}^{(B)}} \right) \] … (10.6)

Hence the difference \( D(P^U \prod P^A) - D(P^U \prod P^B) \) is

\[ = \sum_i \sum_j W_{ij} p_{ij}^{(U)} \log \left( \frac{p_{ij}^{(U)}}{p_{ij}^{(A)}} \right) - \sum_i \sum_j W_{ij} p_{ij}^{(U)} \log \left( \frac{p_{ij}^{(U)}}{p_{ij}^{(B)}} \right) \]

\[ = \sum_i \sum_j W_{ij} p_{ij}^{(U)} \log \frac{p_{ij}^{(U)}}{p_{ij}^{(A)}} - \sum_i \sum_j W_{ij} p_{ij}^{(U)} \log \frac{p_{ij}^{(U)}}{p_{ij}^{(B)}} \]

\[ = \sum_i \sum_j W_{ij} p_{ij}^{(U)} \log \left( \frac{p_{ij}^{(U)}}{p_{ij}^{(A)}} \right) - \sum_i \sum_j W_{ij} p_{ij}^{(U)} \log \left( \frac{p_{ij}^{(U)}}{p_{ij}^{(B)}} \right) \]

\[ = \sum_i \sum_j W_{ij} p_{ij}^{(U)} \left( \frac{p_{ij}^{(U)}}{p_{ij}^{(A)}} \right) - \sum_i \sum_j W_{ij} p_{ij}^{(U)} \left( \frac{p_{ij}^{(U)}}{p_{ij}^{(B)}} \right) \]

\[ = \sum_i \sum_j W_{ij} p_{ij}^{(U)} \left( \frac{p_{ij}^{(U)}}{p_{ij}^{(A)}} \right) = \sum_i \sum_j W_{ij} p_{ij}^{(U)} \left( \frac{p_{ij}^{(U)}}{p_{ij}^{(B)}} \right) = E[Y] = \mu \ldots (10.7) \]

where \( Y_i = \log \left( \frac{p_{ij}^{(U)}}{p_{ij}^{(A)}} \right) \cdot \frac{p_{ij}^{(U)}}{p_{ij}^{(B)}} \).

Now \( \hat{p}_{ij} = \frac{n_{ij}}{n_i} \) is a maximum likelihood estimate where \( n_{ij} \) is the number of times the system visited the state \( j \) provided it entered state \( i \) initially and \( n_i \) is the total number of times the system is in state \( i \). Hence \( Y_i \) is a function of maximum likelihood estimates and a sufficient statistic for finite \( n \), where \( n = \sum n_i \). However for large \( n \), mean of \( Y_i \), \( \bar{Y} \) is asymptotically normal with mean \( \mu \) and variance \( \sigma^2/n \).
Hence the statistic \( \frac{Y - \mu}{\sigma n^{1/2}} \) can be used to test the significant difference between the distances \( D(P^U \prod P^A) \) and \( D(P^U \prod P^B) \) for large sample size.

### 10.3 Experiment and Result:

For our experiment we have selected two ragas viz. Bhupali and Deshkar as A and B respectively. A piece of music U from a musical composition of Dr Bhupen Hazarika has been considered to verify whether the piece of music U is closer to any one of the ragas A and B. It has already been mentioned in chapter 8 that alap of a raga is a better representative for exhibiting the characteristics of a raga. Hence for the experiment, compositions of portions of alap of A and B are taken under consideration. The piece of music U is taken from the Hindi song “Naino me darpan...”, the Assamese version of the same song is “Tomar uxah kahua komal...”. The selected piece of music and the portion of alaps corresponding to Bhupali and Deshkar have identical state space. The weight (transition probability) matrices of U and alap corresponding to A and B are given below along with their digraphs. The nodes of the digraphs are taken to be \( (1 \ 3 \ 5 \ 8 \ 10 \ 13) \).
Graph $U^1$ is defined by

$V(U^1) = \{1,3,5,8,10,13\}$

$E(U^1) = \{e_{11}, e_{13}, e_{15}, e_{18}, e_{33}, e_{35}, e_{3\ 13}, e_{31}, e_{55}, e_{88}, e_{8\ 10}, e_{8\ 13}, e_{8\ 5}, e_{10\ 10}, e_{10\ 8}, e_{13\ 13}, e_{13\ 8}\}$

$\psi(U^1) = \{(1,1)\ (1,3)\ (1,5)\ (1,8)\ (3,3)\ (3,5)\ (3,13)\ (3,1)\ (5,5)\ (5,13)\ (5,3)\ (8,8)\ (8,10,)\ (8,13)\ (8,5)\ (10,10)\ (10,8)\ (13,13)\ (13,8)\}$

$C(U^1) = \{\{1(e_{11}, e_{13}, e_{15}, e_{18})\ 1,3,5,8\} \{3(e_{33}, e_{35}, e_{3\ 13}, e_{31})\ 3,5,13,1\} \{5(e_{55}, e_{5\ 13}, e_{53})\ 5,13,3\} \{8(\ e_{88}, e_{8\ 10}, e_{8\ 13}, e_{8\ 5})\ 8,10,13,5\} \{10(e_{10\ 10}, e_{10\ 8})\ 10,8\} \{13(e_{13\ 13}, e_{13\ 8})\ 13,8\}\}$

$U^1$ is a musical graph.
Transition probability matrix $P^U$ corresponding to Graph $U^1$:

$$P^U = \begin{bmatrix}
1 & 3 & 5 & 8 & 10 & 13 & \text{Outdegree} \\
1 & 0.33 & 0.20 & 0.34 & 0.14 & 0 & 0 & 4 \\
3 & 0.47 & 0.32 & 0.16 & 0 & 0 & 0.05 & 4 \\
5 & 0 & 0.74 & 0.21 & 0 & 0 & 0.055 & 3 \\
8 & 0 & 0 & 0.16 & 0.58 & 0.16 & 0.05 & 4 \\
10 & 0 & 0 & 0 & 0.56 & 0.44 & 0 & 2 \\
13 & 0 & 0 & 0 & 0.14 & 0 & 0.82 & 2 \\
\text{Indegree} & 2 & 3 & 4 & 4 & 2 & 4 \\
\text{Polarity} & -2 & -1 & 1 & 0 & 0 & 2 \\
\text{Strength} & 6 & 7 & 7 & 8 & 4 & 6 & \text{Total} \\
\text{Strength} & & & & & & & = 38
\end{bmatrix}
Graph $A^1$ is defined by,

$V(A^1) = \{1,3,5,8,10,13\}$

$E(A^1) = \{e_{11}, e_{13}, e_{15}, e_{18}, e_1, 13, e_{35}, e_{31}, e_{55}, e_{58}, e_5, 10, e_{513}, e_5, 3, e_5, e_{1310}, e_{138}, e_{135}, e_{133}, e_{131} \}$

$\psi(A^1) = \{(1,1) (1,3) (1,5) (1,8) (1,13) (3,5) (3,1) (5,5) (5,8) (5,10) (5,13) (5,3) (5,1) (8,10) (8,13) (8,5) (8,3) (10,13) (10,8) (13,13) (13,10) (13,8) (13,5) (13,3) (13,1) \}$
A^1 is a musical graph.

**Transition probability matrix P^A corresponding to Graph A^1:**

\[
P^A =
\]

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>8</th>
<th>10</th>
<th>13</th>
<th>Outdegree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.32</td>
<td>0.40</td>
<td>0.04</td>
<td>0.04</td>
<td>0</td>
<td>0.04</td>
<td>5</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
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<td>0.36</td>
<td>0.11</td>
<td>0.03</td>
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<tr>
<td>10</td>
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<td>0</td>
<td>0.5</td>
<td>0</td>
<td>0.46</td>
<td>2</td>
</tr>
<tr>
<td>13</td>
<td>0.04</td>
<td>0.02</td>
<td>0.04</td>
<td>0.05</td>
<td>0.38</td>
<td>0.13</td>
<td>6</td>
</tr>
</tbody>
</table>

| Indegree | 4  | 4  | 5  | 4  | 3  | 5 |
| Polarity | -1 | 2  | -1 | 0  | 1  | -1 |
| Strength | 9  | 6  | 11 | 8  | 5  | 11 |

**Total strength = 50**
Graph $B^1$ is defined by

$$V(B^1) = \{1,3,5,8,10,13\}$$

$$E(B^1) = \{e_{11},e_{13},e_{15},e_{18},e_{110},e_{113},e_{31},e_{55},e_{58},e_{53},e_{88},e_{810},e_{813},e_{85},e_{81},e_{1010},$$

$$\quad e_{1013},e_{108},e_{1313},e_{1310},e_{138}\}$$

$$\psi(B^1) = \{(11),(13),(15),(18),(110),(113),(31),(55),(58),(53),(88),$$

$$(810),(813),(85),(81),(1010),(1013),(108),(105),(1313),(1310),(138)\}$$

$$C(B^1) = \{\{1(e_{11},e_{13},e_{15},e_{18},e_{110},e_{113})1,3,5,8,10,13\},\{3(e_{31})1\},\{5(e_{55},e_{58},$$

$$e_{510},e_{53})5,8,10,3\},\{8(e_{88},e_{810},e_{813},e_{85},e_{81})8,10,13,5,1\},\{10(e_{1010},e_{1013},e_{108},$$

$$e_{105})10,13,8,5\},\{13(e_{1313},e_{1310},e_{138})13,10,8\}\}$$

$B^1$ is a musical graph.
Transition probability matrix $P^B$ corresponding to Graph $B'$:

$$p^B =$$

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>8</th>
<th>10</th>
<th>13</th>
<th>Outdegree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.18</td>
<td>0.04</td>
<td>0.18</td>
<td>0.29</td>
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<td>0.04</td>
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<tr>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<tr>
<td>5</td>
<td>0</td>
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<td>0.07</td>
<td>0.49</td>
<td>0.02</td>
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<tr>
<td>8</td>
<td>0.01</td>
<td>0</td>
<td>0.39</td>
<td>0.23</td>
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<td>0.02</td>
<td>5</td>
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<tr>
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<td>0</td>
<td>0.02</td>
<td>0.75</td>
<td>0.04</td>
<td>0.19</td>
<td>4</td>
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<tr>
<td>13</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.14</td>
<td>0.55</td>
<td>0.24</td>
<td>3</td>
</tr>
<tr>
<td>Indegree</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Polarity</td>
<td>-3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Strength</td>
<td>9</td>
<td>3</td>
<td>8</td>
<td>10</td>
<td>9</td>
<td>7</td>
<td>Total strength= 46</td>
</tr>
</tbody>
</table>

The above graphs and tables shows that Total strength of Bhupali > Total strength of Deshkar > Total strength of Song for the states {1,3,5,8,10,13} under consideration.

Hence Bhupali and Deshkar may be considered as a reference measure for the piece of music mentioned above. It is to be noted that calculation of the distances mentioned above are computed for $p_{ij}^A > 0$ and $p_{ij}^B > 0$. This experiment is conducted in three stages viz.

1) Testing the significant differences between the distances $D(P^A \big| \prod P^A)$ and

\[ 266 \]
2) Computing the distances \( D(P^U \prod P^A) \), \( D(P^U \prod P^B) \).

3) Obtaining the distance between reference measures A and B i.e. \( D(P^A \prod P^B) \).

**Results:**

1) Applying the theorem stated above, the statistic \( \bar{Y} = \frac{\mu}{\sigma n}^{-1/2} \) is asymptotically normal with mean zero and variance 1. Therefore \( Z \) test has been applied to test the hypothesis \( H_0: \) There is no significant difference between the distances \( D(P^U \prod P^A) \) and \( D(P^U \prod P^B) \) corresponding to the states \{1, 3, 5, 8, 10, 13\}. From the above tables of transition probabilities we have calculated

\[
\bar{Y} = \frac{\sum Y_i}{36}
\]

and

\[
\text{var}(Y) = \frac{\sum Y_i^2}{36} - \bar{Y}^2
\]

Hence \( Z = 1.09 \).

Since the calculated value of \( Z \) is smaller than the tabulated value at 5% level of significance we may accept the null hypothesis \( H_0 \) and thus we may conclude that there exists no significant difference between the distances \( D(P^U \prod P^A) \) and \( D(P^U \prod P^B) \).

2) Even if there is no significant difference between the distances \( D(P^U \prod P^A) \) and \( D(P^U \prod P^B) \), there is a possibility that the piece of music U may be closer to either of Bhupali(A) or Deshkar(B). For the purpose of verification we computed the statistics
corresponding to the distances $D(P^U \prod P^A)$ and $D(P^U \prod P^B)$. The results are tabulated in the following table.

<table>
<thead>
<tr>
<th>Statistic corresponding to Distance</th>
<th>Mean value</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D(P^U \prod P^A)$</td>
<td>20%</td>
<td>36%</td>
</tr>
<tr>
<td>$D(P^U \prod P^B)$</td>
<td>14%</td>
<td>25%</td>
</tr>
</tbody>
</table>

This shows that the piece of music is closer to Deshkar with less variance as compared to Bhupali.

4) In this Section we have also computed the Kullback Leibler distance between a portion of alap of A and B. The mean of the statistic corresponding to $D(P^A \prod P^B)$ is 2% and variance is equal to 3%.

10.4 Geometrical representation of the problem:

The closeness between a piece of music and two ragas may also be presented geometrically as follows.

Since triangular inequality is not satisfied corresponding to the distances viz. $D(P^A \prod P^B) = 2\%$, $D(P^U \prod P^A) = 20\%$ and $D(P^U \prod P^B) = 14\%$, the following geometrical construction is performed.
Fig 10.4: Geometrical representation of angle of inclination of U with A and B

where A represents *raga Bhupali*, B represents *raga Deshkar* and U represents the piece of music considered for verification. Here C is a point obtained by extending the line AB towards right hand side where UA = 0.20, UB = 0.14, AB = 0.02 and AC = 0.20. The angle of inclination between A and U is $\alpha$ and between U and C is $\beta$. From the simple geometric construction of the triangle AUC it may be observed that the line parallel to CU drawn at the point B makes an angle $\beta$ at the point B. Though triangular inequality may not be satisfied for K-L distance a similar triangle AOB may be constructed to obtain the angle of inclination at B also. If UM is the perpendicular distance between U and AC then $\alpha = \tan^{-1}(UM/AM)$ and $\beta = \tan^{-1}(UM/MC)$. 
From triangle AUM and CUM we have
\[ UM^2 = AU^2 - AM^2 = CU^2 - CM^2 \]
=> \[ AC^2 - AM^2 = CU^2 - (AC-AM)^2 \]
=> \[ 2 AC^2 - CU^2 = 2 AC\times AM \] ... (10.8)
=> \[ AM = 2 AC^2 - CU^2 / 2 AC \] ... (10.9)

Again
\[ UM^2 = (AU-AM)(AU+AM) \]
\[ UM = [(AU-AM)(AU+AM)]^{1/2} \] ... (10.10)

Thus \( \alpha \) and \( \beta \) may be obtained from equations 10.8, 10.9 and 10.10 respectively. The area of the triangle AUC is given by \( AC\times UM/2 \). The image of the triangle is given by the triangle \( AU/C \). It is clear from the above that as the distance between the \textit{raga} and piece of music \( U \) increases, the angle of inclination between them decreases. It is known that if two angles and two sides of a triangle is equal to the corresponding angles and sides of another triangle then the triangles are equal to each other. The position of \( U \) depends on the position of \( B \) relative to \( A \). When \( B \) lies in circumference of the circle with center \( A \) and radius \( AB \), \( U \) lies on the circumference of the circle with center \( A \) and radius \( AU \) (if \( AU > BU \)).

**Discussion:**

The results of the experiment reveals that apart from the distance between a musical composition and a \textit{raga}, their angle of inclination may also play a crucial role in determining the percentage of a \textit{raga} that is contained in the musical piece. Instead of two \textit{allied ragas} one can consider two similar types of compositions to measure their
close inclination with a piece of music. Moreover the above experiment can be further extended for measuring distances between musical compositions based on First Passage Time Distribution and Limiting Probabilities of note transitions.

**Scope for Future work:**

While carrying out the present research work it has been felt that there is immense scope for further research on application of statistics in musicology. However it is not possible to incorporate everything in a single thesis and therefore the following areas of possible future research work on this topic have been suggested.

(1) In the chapters where Markov chain have been used for modeling transitions of different musical events it was assumed that

\[ X_n = i, \text{ for } i=1,2,...,l \text{ and } i \in S, \ S \text{ being the state space of the chain. Here the musical event 'i' has been treated as a single entity. However the Markov chain theory may be extended for modeling event vectors. In this case we write } X_n = E_i \text{ where } E_i = (e_{i1}, e_{i2},..., e_{ik}) \]

\[ \forall \ i=1,2,...,l \text{ is a set of controlling parameters } e_{ij} \text{'s where } E_i \in S \text{ such that } S=\{(e_{11},e_{12},...,e_{1k}),(e_{21},e_{22},...,e_{2k}),..., (e_{l1},e_{l2},...,e_{lk})\}. \text{Again the values of the controlling parameters } e_{ij} \text{'s are defined by a set of event -parameter spaces.} \]

(2) Like raga Bhupali the grammatical structure of other ragas can be similarly analyzed using graph theory.

(3) The musical compositions of three great composers of Assam viz. Dr Bhupen
Hazarika, Jyoti Prasad Agarwalla and Bishnu Rabha may be classified according to different genre like patriotic, romantic, nature, folk etc. Then for each genre, information theory may be employed for analyzing the works of the above composers statistically.

(4) Non-parametric methods may be used to study the significant differences between the above mentioned compositions corresponding to different genres.

(5) Problem 3 and 4 can be extended to graph theoretic approach for pattern classification.

(6) Based on 3 and 5 any composition of one of the composers mentioned above may be compared with that of the other two using information theory. A measure of dissimilarity between the musical compositions may then be obtained using Kullback Leibler distance function as discussed in Chapter 10.

(7) A problem of composer identification and style discrimination may be undertaken for analyzing the works of Mahapurush Shankardeva and Madhavdeva. A similar methodology may also be adopted to study the folksongs of Assam.