CHAPTER - 4

PERISTALTIC TRANSPORT OF BIO-FLUID IN THE
PRESENCE OF VARYING TEMPERATURE WITH
A REFERENCE TO DERMAL LAYER –
A MATHEMATICAL MODEL

4.1 INTRODUCTION

The objective of the thermal modeling of milk extracted from the normal woman's breast is to study the temperature distribution among the spherical layers. Man is homeothermal; he maintains his body temperature constant in spite of wide variation in environmental temperature. The term body temperature refers to the temperature of the deeper structures such as viscera, liver and brain. The skin usually has a lower temperature than the depths of the body. This provides a temperature gradient which leads to heat loss from the depths to the surface of the body and then surface to the environment. In health, it is always to observe fairly close to the normal deep body temperature in man 36.0°C - 37.5°C by maintaining a balance between heat gain and heat loss. Heat flow within the body occurs by diffusion in tissues and by blood transport in veins and arteries. Heat energy is generated within the body in a number of ways such as gains by basal metabolism, due to muscle action, due to increased metabolism and losses due to respiratory, evaporation, ingestion. Thus the nerves carrying temperature signals are part of a multivariate negative feedback system that provides temperature regulation in the body.
It is assumed that the tissue as elastic saturated with a viscous fluid caused the viscoelastic response from the redistribution of fluid coupled with the elastic deformation.

We have modeled the flow of fluid, both milk and interstitial fluid, by considering the human teat as cylindrical with radial dimension as \( r = r_0 \) and \( z = h \) with the model geometry in non-dimensional coordinates and to compare different conditions which mimic the behavior of a teat during infant sucking. At the nipple or free end of the cylinder \( z = h \), we impose a cyclic pressure \( p = p(t) \) which mimics the pressure differential imposed by the infants sucking action by contraction and expansion. To model the compression action of the infants during sucking a compressive force \( F(z, t) \) acting radially on the surface \( r = r_0 \) which moves along the teat with a given velocity, the stripping action. Thus mimicking the stripping action, friction and elasticity of the skin are incorporated by considering the shear stress \( \sigma_a(r = r_0) \) proportional to the displacement in the axial direction.

**Physiology of Breast**

The woman's breast is composed of four essential spherical layers representing skin, fat, muscle, and core (connecting tissue). In the middle area of the skin layer there is a very highly perfused area, the areola. In the middle of areola is the nipple. The pattern of blood circulation in the breast can be divided into four main parts.

1. The arterial supply mainly dependent on subclavian and axillary arteries. The gland is entirely supplied by branches originating from the half deep arterial tree.

From the periglandular arterial tree originate: (i) superficial branches directed
towards the skin, particularly in the areolar and periareolar region: (ii) half deep branches supplying the retro glandular structures.

2. The venous drainage utilizes the routes: (i) the half deep route according to a course and a drainage which corresponds to the main preglandular arterial branches: (ii) the superficial route, originating from the periareolar ring and running in this skin layer, where it forms a tree much richer than the arterial one.

3. The group of arteries feed the deep tissues with a constant amount of blood per unit volume of tissues, while the blood supply to the outer spherical section depends on environmental temperature in two major ways:

(a) by 'vasodilation' – warm blood flows to the neighbourhood of the skin is increased when heat removal is required.

(b) by 'vasoconstriction' - the blood is by-passed from arteries to veins via deeper channels when conservation of body heat is vital.

4. The veins or the group of veins which collect the blood from the tissues utilize two different routes: the half-deep route and the superficial one.

The half-deep route of the returning venous system passes close to the incoming arterial system in the half-deep arterial tree and countercurrent heat exchange may take place between them

Burton [19] reported for tissue heat transfer, the laws of heat conduction through solid materials are adequate to describe the actual flow of heat in biological tissues. He considered that the heat was conducted along paths perpendicular to body surface. The coefficient of thermal conductivity was the variable depending on the blood flow near the skin. Wissler [138] studied a mathematical model with one dimensional radial heat flow.
Burns et al. [18] presented the peristaltic motion of pipe and channel flow under the assumptions of small Reynolds number. Fung et al. [41] analyzed the flow of urine associated with peristaltic action in a two dimensional channel. Shapiro et al. [114] described the peristaltic pumping using long wave length at low Reynolds number for dissipation and mechanical efficiency in relevance to ureter function. Nevins et al. [91] investigated heat transfer through subcutaneous tissue modeled as heat generating porous material. It was considered that heat transfer by conduction and also by convection due to the flow of blood the simulated porous structures of the heat and limbs by assuming a symmetrical distribution of blood flow.

Keller et al. [70] described one dimensional steady state continuum model for heat transfer through the tissue of peripheral regions in man taking into consideration the effects of tissue conduction, convection by blood flow, vascular heat exchange and tissue metabolism. Weinberg et al. [136] studied the experimental investigations on two dimensional peristaltic pumping for the measurements of physiological flow parameters at fixed locations of the tube. Heberman et al. [48] explained the effect of metabolic heat by thermal sources and solved the one dimensional energy equation into consideration, heat conduction and blood perfusion to predict the skin temperature during the thermography. Their analytical results explained the relative influences on room temperature heat transfer coefficient, blood perfusion rate, tissue thermal conductivity and metabolic heat generation rates on skin temperature. Michle [85] investigated inertial and stream line curvature effects on peristaltic pumping. Thomas et al. [128] presented the computational and experimental investigations of two dimensional non linear peristaltic flows on the assumptions of highest stress and energy exchange rights. Shukla
et al. [116] analyzed the effects of peripheral layer viscosity on peristaltic transport of bio-fluid with varying viscosity across the duct. Osman et al. [95] discussed the thermal modeling of the normal woman's breast. He developed the theoretical model by taking into consideration that metabolic heat production, tissue perfusion with capillary blood, arterial and venous blood thermal interaction and change in blood temperature with position. Liepsch [78] discussed on blood circulatory system by which the human heart operates as a double working pump for the flow of blood similar to a piston in the tube network. He also described about the contraction and expansion waves produced by the pumping action using Newtonian and non-Newtonian nature of blood. Srivastava et al. [118] explained the peristaltic transport of a particle-fluid suspension. Dalin et al. [28] presented an application of abstract differential equations to study the peristaltic transport of heat conducting viscous fluid. Basavarajappa et al. [15] studied the peristaltic transport of the two-layered viscous incompressible fluid for varying fluid behavior indices.

Mizuno et al. [87] studied the peristaltic action by the infants tongue to extract milk of women's breast. Mekheimer [84] presented the peristaltic transport of a couple-stress fluid in a uniform channel. Noble et al. [93] reported the identification factors of associated with the initiation of breast feeding in a poor urban area and recommended that breast feeding education should be started prior to the first pregnancy and tailored to concern of human beings. Frideman et al. [40] investigated the effect of prenatal consultation with a neonatologist on the incidence and duration of human milk feeding in the case of preterm infants. Mohamed [88] studied the effect of wall compliance on peristaltic transport of Newtonian fluid in an asymmetric channel and the effect of amplitude ratio. Katiyar et al. [68] explained the analytical study of Herschel-Bulkley
model of blood flow through stenosed arteries. Larry et al. [76] studied the computational model for the transition from peristaltic to pulsatile flow in the embryonic heart tube.

In this chapter, we have studied the mathematical model of the flow and deformation in a human teat is proposed to explain the physiological flow parameters in relation to contraction and expansion of axisymmetric wave propagation. During the peristaltic action the metabolic heat is being produced at different layers of the breast. Numerical results using Herschel - Bulkley fluid model have been computed using the overall heat transfer coefficient, stream function and velocity distribution. Analytical expressions are derived for various velocities, average flux with a reference to average breast radius of different layers. Thermal interaction and change in arterial blood temperature are compared by taking into consideration that the heat transfers by conduction and convection in relation to pathological conditions. Coefficient of thermal conductivity is taken as variable near the microcirculation of the blood vessels adjacent to layers of the breast. The local rise of temperature in one layer includes the sense of distribution of temperature to the adjacent layer. The basic equations of motion and continuity with a reference to cylindrical polar coordinates are considered to estimate the velocity, flux and resistance of flow for peristaltic action under the assumption of metabolic heat generated among the layers of the breast.

### 4.2 FORMULATION

To develop a mathematical model we consider the case of peristaltic motion of human milk extracted by infant due to peristaltic action of the tongue. The deformation occurs in a human teat due to contraction and expansion. The infants tongue plays an important role in sucking action. The peristaltic wave of the tongue will travel
approximately 15cm/s strips the milk from the teat. During the peristaltic action of the tongue, the teat is compressed by a further 60% in the vertical direction and it expands by 120% in the lateral direction. Despite numerous studies, the relative contribution of heat transfer through subcutaneous tissue, the heat generating through different layers of the breast. The presence of peristaltic motion in reference to milk and the heat transfer by conduction and also by convection occurs due to the flow of blood through the simulated structures of arteries. The peristaltic action of flow of blood for convection is compared by taking the non-Newtonian fluid Herschel - Bulkley fluid for the axisymmetric direction.

Consider the axisymmetric flow of Herschel - Bulkley fluid through the tube of radius ‘a’ of which the core layer (with radius h_i) is filled with 90 % of the Herschel-Bulkley fluid with another immiscible fluid as plasma in peripheral layer with radius as h. Geometry of the fluid flow is sinusoidal, traveling with wave speed ‘c’ and amplitude b_i in core layer and b in peripheral layer.

Under the peristaltic action R = H(z) be the instantaneous radius when the fluid is surrounded co-axially, (R, Z) be the fixed frame and (r', z') be the reference frame.

Then z' = Z + ct', r' = R, w_i' = w_i + c

Taking Herschel - Bulkely fluid model as,

\[ \tau = \mu e^n + \tau_0, \quad \tau \geq \tau_0 \]
\[ e = 0, \quad \tau \leq \tau_0 \quad (4.1) \]

Where \( \tau \) - shear stress, \( \tau_0 \) - yield stress, \( e \) - deformation rate, \( \mu \) - viscosity and \( n \) - fluid behavior index.
Using long wavelength approximation and neglecting wall slope and inertia forces for steady flow under lubrication theory, the equations of motion (1.22) to (1.24) for cylindrical polar coordinates \((r, \theta, z)\) to study the problem, \(w_r = 0, w_\theta = 0,\) and \(w_z = w\) then \(w_z = w(r),\)

\[
\frac{\partial p'}{\partial r'} = 0
\]  
(4.2)

\[
\frac{\partial p'}{\partial z'} = \frac{1}{r'} \frac{\partial}{\partial r'} \left\{ r' m_r \frac{\partial w_i'}{\partial r'} \left[ \frac{\partial w_i'}{\partial r'} \right]^{n-1} \right\}
\]  
(4.3)

Introducing non-dimensional quantities,

\[
r = \frac{r'}{a}, \quad z = \frac{z'}{\lambda}, \quad h_i = \frac{H_i}{c}, \quad w_i = \frac{w_i'}{a}, \quad m_r = \frac{\mu_r}{\mu_i}, \quad p = \frac{p' a^{n+1}}{m_i \lambda c^n}
\]

(4.4)

Then the non-dimensional form is given by,

\[
\frac{\partial p}{\partial z} = \frac{1}{r} \frac{\partial}{\partial \gamma} \left\{ r m_r \frac{\partial w_i}{\partial \gamma} \left[ \frac{\partial w_i}{\partial \gamma} \right]^{n-1} \right\}
\]

(4.5)

Boundary conditions are,

\[
w_i = -1 \quad \text{for} \quad r = h, \quad \text{(at the wall)}
\]

\[
\frac{\partial w_i}{\partial \gamma} = 0 \quad \text{for} \quad r = 0, \quad \text{(at the axis)}
\]

(4.6)

Geometry of the tube wall is taken as,

\[
H(z) = a + b \sin \left( \frac{2 \pi z}{\lambda} \right)
\]

\[
H(z) = 1 + \varepsilon \sin \left( \frac{2 \pi z}{\lambda} \right)
\]

(4.7)

Where \(\varepsilon = b/a\) is the amplitude ratio
\[
\frac{\partial p}{\partial z} = p \text{ (constant) Then, } \\
\frac{\partial}{\partial r} \left\{ r \frac{\partial w_i}{\partial r} m, \left| \frac{\partial w_i}{\partial r} \right|^{n_i - 1} \right\} = pr
\]  
(4.8)

On integrating with respect to \( r \),

\[
\frac{\partial w_i}{\partial r} \left| \frac{\partial w_i}{\partial r} \right|^{n_i - 1} = \frac{pr}{2m} + \frac{A}{r m_r} \quad \text{by taking} \quad m_r = m = \frac{m_1}{m_2}
\]

Where \( A \) is the constant of integration.

at \( \frac{\partial w_i}{\partial r} = 0 \) at \( r = 0 \), \( A = 0 \)

\[
\frac{\partial w_i}{\partial r} \left| \frac{\partial w_i}{\partial r} \right|^{n_i - 1} = \frac{pr}{2m}
\]  
(4.9)

For power low fluids in a circular tube for immissible fluids, replacing

\[
\left| \frac{\partial w_i}{\partial r} \right| = \frac{pr}{2m} \left| \frac{1}{n_i} \right|
\]  
(4.10)

Rearranging the equation (4.9) as,

\[
\frac{\partial w_i}{\partial r} \left| \frac{pr}{2m} \right|^{1 - \frac{1}{n_i}} = \frac{pr}{2m}, \quad \text{since} \quad \frac{1}{n_i} = j_i
\]

\[
\frac{\partial w_i}{\partial r} \left| \frac{pr}{2m} \right|^{-1} = \frac{pr}{2m}
\]

\[
\frac{\partial w_i}{\partial r} = \frac{pr}{2m} \left| \frac{pr}{2m} \right|^{-1}
\]

\[
\frac{\partial w_i}{\partial r} = \frac{p}{2m} \left| \frac{p}{2m} \right|^{-1} r^{j_i}
\]  
(4.11)

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The overall heat transfer coefficient from arterial blood tissue space can be estimated as the ratio of the computed thermal conductivity to the varying height through the artery assuming the effective resistance (outside radius $R$ which is half spacing blood vessels and $r_0$ the average radius of blood vessel)

$$u = \frac{K}{r_0 \log(R/r_0)} \quad (4.12)$$

Where $K$ - Thermal conductivity, $u$ - over all heat transfer coefficient from arterial blood to the tissue space and to the veins and $r_0$ - average radius of blood vessels,

$$R = r = r_0 e^{-u r_0}$$

Replacing $K$ by $K^*$,

where $k^*$ is the equilibrium constant for $K$ associated with countercurrent heat exchange due to peristaltic action and it is the ratio of temperature volume $\varphi_v$ minus the temperature and the arterial temperature $\varphi_a$, then

$$u = \frac{K K^*}{r_0 \log(R/R^*)} \quad (4.13)$$

Here the value of $K^*$ is estimated using Taylor's series with the initial value as 0.65 and for further approximation defining $K^*$ as,

$$K^* = \frac{\varphi_v - \varphi}{\varphi_a - \varphi}$$

$$= 0.65 + 0.633 \left( \frac{z}{R^*} \right) - 2.84 \left( \frac{z}{R^*} \right)^2 + 2.48 \left( \frac{z}{R^*} \right)^3 + 0.97 \left( \frac{z}{R^*} \right)^4 - 0.896 \left( \frac{z}{R^*} \right)^5$$

Where $\varphi$ - temperature, $\varphi_v$ - temperature volume, $\varphi_a$ - arterial temperature, $z$ - axial distance, $R^*$ - breast layer radius (for different layers)
Then equation (4.13) gives

\[
u = \frac{K}{r_0 \log (r/r_0)} \left\{ 0.65 + 0.633 \left( \frac{Z}{R'} \right) - 2.84 \left( \frac{Z}{R'} \right)^2 + 2.48 \left( \frac{Z}{R'} \right)^3 + 0.97 \left( \frac{Z}{R'} \right)^4 - 0.896 \left( \frac{Z}{R'} \right)^5 \right\}
\]

(4.14)

The ratio of volume temperature with total temperature and the arterial temperature with total temperature and during the peristaltic transport of bio fluid both depend on assigning the breast layers radius and at different axial distances for countercurrent heat exchange, then setting \( \frac{K}{r_0} = K_1 \) and using equation (4.13),

\[
K_1 K = K_1 \left( \frac{\phi_v - \phi}{\phi_a - \phi} \right)
\]

(4.15)

\[
K_1 K = K_1 \left\{ 0.65 + 0.633 \left( \frac{Z}{R'} \right) - 2.84 \left( \frac{Z}{R'} \right)^2 + 2.48 \left( \frac{Z}{R'} \right)^3 + 0.97 \left( \frac{Z}{R'} \right)^4 - 0.896 \left( \frac{Z}{R'} \right)^5 \right\}
\]

Then taking \( K = r_0 \log \left( \frac{R = r}{r_0} \right) \) and using equation (4.12),

\[
\frac{1}{u} = \frac{r_0}{K} \log \left( \frac{r}{r_0} \right)
\]

Equation (4.11) becomes,

\[
\left[ \frac{-k}{u^2} e^{u_0} \right] \frac{dw_i}{du} = \frac{p}{2m} \left( \frac{p}{2m} \right)^{j_i-1} \left\{ \frac{K_{ij}}{r_0 e^{u_0}} \right\}^{j_i}
\]

\[
\frac{dw_i}{du} = -\frac{K}{u^2} e^{u_0} \frac{p}{2m} \left( \frac{p}{2m} \right)^{j_i-1} \frac{K_{ij}}{r_0^{j_i}} e^{u r_0}
\]
\[
\frac{dW_j}{du} = -\frac{P}{u^2} \frac{r_0}{2m} \frac{J_i}{2m} e^{(1+J_i)K}\left(\frac{1}{u}e^{u_0} \frac{1}{u} \right) \\
\frac{dW_i}{du} = -K \frac{J_i}{2m} \left\{ \frac{1}{u^2} e^{u_0} \frac{1}{u} \right\} \\
\frac{dW_i}{du} = -C_1 \frac{C_2}{u^2} e^{u} \\
(4.16)
\]

\[
C_1 = \frac{P}{2m} J_i \frac{r_0}{J_i} \quad \text{and} \quad C_2 = \frac{(1+J_i)K}{r_0}
\]

4.3 ANALYSIS

In this investigation, the tissue perfusion with capillary blood, arterial and venous blood thermal interaction and change of arterial blood temperature were combined in one blood effect (\(B\ L\ D\)), mathematically it can be expressed as,

\[
B\ L\ D = C_b \ w_b + u (1 + K)
\]

The effects of blood perfusion rate and arterial thermal interactions are considered using equivalent values for physical parameters involved. For deep tissue inner layer from the surface the physical parameters are considered to be uniform then,

\[
u = \frac{B\ L\ D - C_b \ w_b}{1 + K} = \frac{B\ L\ D - C_b \ w_b}{1 + \left( \phi_v - \phi \right) / (\phi_a - \phi)}
\]

\[
u = \frac{B\ L\ D - C_b \ w_b}{1 + 0.65 + 0.633 \left( \frac{Z}{R'} \right) - 2.84 \left( \frac{Z}{R'} \right)^2 + 2.48 \left( \frac{Z}{R'} \right)^3 + 0.97 \left( \frac{Z}{R'} \right)^4 - 0.896 \left( \frac{Z}{R'} \right)^5}
\]

(4.17)

The expression for velocity in terms of \(B\ L\ D\) is given by introducing equations (4.14) and (4.16) in equation (4.17), we obtain

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\[ w_i = w_0 + \frac{1}{1!} \left[ \frac{K}{\tau_0 \log(r/\rho_0)} \left\{ \begin{array}{l} 0.65 + 0.633 \left( \frac{z}{R'} \right) - 2.84 \left( \frac{z}{R'} \right)^2 + 2.48 \left( \frac{z}{R'} \right)^3 \\ + 0.97 \left( \frac{z}{R'} \right)^4 - 0.896 \left( \frac{z}{R'} \right)^5 \end{array} \right\} - u_0 \right] \]

\[ \times \left[ -4.2 \times 10^{-7} \text{C}_1 e^{0.00065 \text{C}_2} \right] \]

\[ + \frac{1}{2!} \left[ \frac{K}{\tau_0 \log(r/\rho_0)} \left\{ \begin{array}{l} 0.65 + 0.633 \left( \frac{z}{R'} \right) - 2.84 \left( \frac{z}{R'} \right)^2 + 2.48 \left( \frac{z}{R'} \right)^3 \\ + 0.97 \left( \frac{z}{R'} \right)^4 - 0.896 \left( \frac{z}{R'} \right)^5 \end{array} \right\} - u_0 \right]^2 \]

\[ \times \left[ 2.75 \times 10^{-10} \text{C}_1 e^{0.00065 \text{C}_2} (0.00065 \text{C}_2 + 2) \right] + \]

\[ + \frac{1}{3!} \left[ \frac{K}{\tau_0 \log(r/\rho_0)} \left\{ \begin{array}{l} 0.65 + 0.633 \left( \frac{z}{R'} \right) - 2.84 \left( \frac{z}{R'} \right)^2 + 2.48 \left( \frac{z}{R'} \right)^3 \\ + 0.97 \left( \frac{z}{R'} \right)^4 - 0.896 \left( \frac{z}{R'} \right)^5 \end{array} \right\} - u_0 \right]^3 \]

\[ \times \left[ -1.8 \times 10^{-12} \text{C}_1 e^{0.00065 \text{C}_2} (4.2 \times 10^{-7} \text{C}_2^2 + 3.9 \times 10^{-3} u_1 + 6) \right] + \]

\[ + \frac{1}{4!} \left[ \frac{K}{\tau_0 \log(r/\rho_0)} \left\{ \begin{array}{l} 0.65 + 0.633 \left( \frac{z}{R'} \right) - 2.84 \left( \frac{z}{R'} \right)^2 + 2.48 \left( \frac{z}{R'} \right)^3 \\ + 0.97 \left( \frac{z}{R'} \right)^4 - 0.896 \left( \frac{z}{R'} \right)^5 \end{array} \right\} - u_0 \right]^4 \]

\[ \times \left[ 1.16 \times 10^{-16} \text{C}_1 e^{0.00065 \text{C}_2} (2.75 \times 10^{-10} \text{C}_2^3 + 5.06 \times 10^{-6} \text{C}_2^2 + 0.023 \text{C}_2 + 24) \right] \]

\[ + \frac{1}{5!} \left[ \frac{K}{\tau_0 \log(r/\rho_0)} \left\{ \begin{array}{l} 0.65 + 0.633 \left( \frac{z}{R'} \right) - 2.84 \left( \frac{z}{R'} \right)^2 + 2.48 \left( \frac{z}{R'} \right)^3 \\ + 0.97 \left( \frac{z}{R'} \right)^4 - 0.896 \left( \frac{z}{R'} \right)^5 \end{array} \right\} - u_0 \right]^5 + \ldots \]

(4.18)
The flux across the layers and through the arterial with thermal interaction is

\[ Q = 2\pi \int_{\eta}^{\eta} w_{r} \, dr \]

\[ Q = 2\pi \int_{\eta}^{\eta} \left[ w_{0} + \frac{1}{1!} \frac{K}{r_{0}\log(r/r_{0})} \left\{ \frac{0.65 + 0.633}{R'_{r_{0}}} - 2.84\left(\frac{Z}{R'}\right)^{2} + 2.48\left(\frac{Z}{R'}\right)^{3} \right\} - u_{0} \right] \]

\[ + \frac{1}{2!} \frac{K}{r_{0}\log(r/r_{0})} \left\{ \frac{0.65 + 0.633}{R'_{r_{0}}} - 2.84\left(\frac{Z}{R'}\right)^{2} + 2.48\left(\frac{Z}{R'}\right)^{3} \right\} - u_{0} \]

\[ + \frac{1}{3!} \frac{K}{r_{0}\log(r/r_{0})} \left\{ \frac{0.65 + 0.633}{R'_{r_{0}}} - 2.84\left(\frac{Z}{R'}\right)^{2} + 2.48\left(\frac{Z}{R'}\right)^{3} \right\} - u_{0} \]

\[ + \frac{1}{4!} \frac{K}{r_{0}\log(r/r_{0})} \left\{ \frac{0.65 + 0.633}{R'_{r_{0}}} - 2.84\left(\frac{Z}{R'}\right)^{2} + 2.48\left(\frac{Z}{R'}\right)^{3} \right\} - u_{0} \]

\[ + \frac{1}{5!} \frac{K}{r_{0}\log(r/r_{0})} \left\{ \frac{0.65 + 0.633}{R'_{r_{0}}} - 2.84\left(\frac{Z}{R'}\right)^{2} + 2.48\left(\frac{Z}{R'}\right)^{3} \right\} - u_{0} \]

\[ \left[ -4.2 \times 10^{-7} C_{1} e^{0.00065 C_{2}} \right] \]

\[ + \frac{1}{2} \frac{K}{r_{0}\log(r/r_{0})} \left\{ \frac{0.65 + 0.633}{R'_{r_{0}}} - 2.84\left(\frac{Z}{R'}\right)^{2} + 2.48\left(\frac{Z}{R'}\right)^{3} \right\} - u_{0} \]

\[ \left[ 2.75 \times 10^{-10} C_{1} e^{0.00065 C_{2}} (0.00065 C_{2} + 2) \right] \]

\[ + \frac{1}{3} \frac{K}{r_{0}\log(r/r_{0})} \left\{ \frac{0.65 + 0.633}{R'_{r_{0}}} - 2.84\left(\frac{Z}{R'}\right)^{2} + 2.48\left(\frac{Z}{R'}\right)^{3} \right\} - u_{0} \]

\[ \left[ -1.8 \times 10^{-12} C_{1} e^{0.00065 C_{2}} (4.2 \times 10^{-7} C_{2}^{2} + 3.9 \times 10^{-3} u_{1} + 6) \right] \]

\[ + \frac{1}{4} \frac{K}{r_{0}\log(r/r_{0})} \left\{ \frac{0.65 + 0.633}{R'_{r_{0}}} - 2.84\left(\frac{Z}{R'}\right)^{2} + 2.48\left(\frac{Z}{R'}\right)^{3} \right\} - u_{0} \]

\[ \left[ 1.16 \times 10^{-16} C_{1} e^{0.00065 C_{2}} (2.75 \times 10^{-10} C_{2}^{3} + 5.06 \times 10^{-6} C_{2}^{2} + 0.023 C_{2} + 24) \right] \]

\[ + \frac{1}{5} \frac{K}{r_{0}\log(r/r_{0})} \left\{ \frac{0.65 + 0.633}{R'_{r_{0}}} - 2.84\left(\frac{Z}{R'}\right)^{2} + 2.48\left(\frac{Z}{R'}\right)^{3} \right\} - u_{0} \]

\[ \left[ r \, dr + ... \right] \]

\[ (4.20) \]
Stream function with thermal interaction is given by,

\[ d\psi = r\psi dr \]

\[ \psi = \int_{0.2}^{0.4} r\psi dr + A \quad (4.21) \]

Integrating and finding the values of \( A \) with 4.4988 then we consider the closeness of the approximation is computed by introducing the percentage error given by,

\[ \text{percentage error} = \left( \frac{\text{Max}|q - \psi|}{\psi} \right) \times 100 \]

where \( \psi = \text{Max}|\psi| \) for \( 0 \leq r \leq 1 \)

The peristaltic transport of fluid with thermal interaction for the amplitude range \( \varepsilon: 0.2 - 0.8 \) is found to be the positive pumping. Then the resistance of flow \( R_f \) the ratio of average pressure rise and the time averaged flux is observed in the direction of the sinusoidal wave.

\[ R_f = \left[ \frac{-8Q\left[2 + 3\varepsilon^2\right]}{\left[1 - \varepsilon^2\right]^2 \left(Q + 1 + \frac{\varepsilon^2}{2}\right)} - \frac{8}{\left[1 - \varepsilon^2\right]^{3/2} \left(Q + 1 + \frac{\varepsilon^2}{2}\right)} \right] \quad (4.22) \]

4.4 RESULTS AND DISCUSSION

Numerical results for the temperature effect during the peristaltic transport of milk shows that the upper layer causing relatively higher blood perfusion rate than the lower layer. The effect of increasing blood flow rate to 20\%, 40\% and 60\% over the normal temperature distribution is noticed by the plots of velocity distribution and flow rates against various values of thermal interaction and viscosity of Herschel-Bulkely fluid. The trend is similar to [87] that showed the blood flow rates during menstrual
period increase the temperature level of mammotherms of women breast. This will help the medical researchers to investigate the clinical features for identifying the tumour detection due to increased temperature with increased metabolic heat rate. Thus they can record the time taken for the growth of tumour due to metabolic heat rate and the size of the tumour. Resistance of flow gives the variation of average pressure rise to the time averaged flux due to thermal interaction.

Fig. 4.1 shows the peristaltic motion with sinusoidal wave for long wavelength approximation (maintaining the balance between heat loss and heat gain)

Fig. 4.2 gives the velocity versus various values of amplitude ratio of peristaltic transport of fluid with thermal interaction for maximum $|\psi|$. 

Fig. 4.3 describes the flux due to positive pumping under the normal deep body temperature of 37.875°C (average values with 10 different calculations).

Fig. 4.4 depicts the resistance of flow for positive pumping under the increased temperature with increased metabolic heat rate.

4.5 CONCLUSION

Due to Peristaltic action of the milk extracted from the breast, we can further analyze that, the variation of temperature at different roots (vessels that milk is flowing as sinusoidal wave). Also the amount of milk and the resistance of flow can be mathematically tractable, so that the amount of yield of milk depends on the amount that flows into one teat followed by the temperature variations at different stages.
Fig. 4.1 Peristaltic transport in axisymmetric tube
(For heat balance between heat gain and heat loss)

Fig. 4.2 Velocity Vs radius
(For thermal interaction for max. |ψ|)
Fig. 4.3  Flux Vs amplitude ratio (in response to 10 different temperature calculations leading to 37.875°C temperature)

Fig. 4.4  Resistance of flow Vs amplitude ratio (With increased metabolic heat rate)