Introduction

This thesis deals with a study of flow of dusty gases in Frenet frame field system. In recent years, the study of dusty fluid flows is receiving much attention because of its wide range applications in areas of technical importance, e.g. Aerodynamics, Gas-cooling, Combustion and fluidisation, Atmospheric fallout, Dust collection, Sedimentation problems of tidal rivers, Motion of aircraft in a dusty atmosphere, Environmental pollution, Performance of solid fuel rocket nozzles, Rain erosion, Paint spraying, Powder technology, Lunar ash flow, Flow in rocket tubes where small carbon or metallic fuel particles are present, Mass transport with chemical reaction in porous media and more recently blood flow in capillaries.

A Dusty fluid is a two-phase media with suspended small dust particles in a fluid. The study of such fluids were first handled by Landau & Lifshitz (1959) who worked on the viscosity of a dusty gas which increases by a factor proportional to the concentration by volume of the dust particles (Einstein formula).

Sproull (1961), Kazakevics & Krapkin (1961) have studied the aerodynamics resistance of a dusty turbulent flow through a system of pipes and reported that the reduction of in the resistance coefficient with the addition of dust. P.G. Saffman (1962) derived the equations of motion of dusty-gas carrying small dust particles and the equation satisfied by
small disturbances of a steady laminar flow. He also mentioned about the
effects of the mass concentration of the dust and the relaxation time $\tau$,
which depends on the size of the dust particles.

F.E. Marble (1963) studied the dynamics of gas containing small
solid particles and gave many important features and applications of dusty
gas. J.D. Murray (1963) analysed some basic aspects of one-dimensional,
imcompressible particle fluid two-phase flow.

Gorge Rudinger & Angela Chang (1964) extended the method of
characteristics to study the un-steady flows of suspensions of small dust or
liquid particles in a gas. F.E.C. Gulick (1964) applied Boltzmann equations
to study the problems of two-phase flow.

Michael. D.H. et. al. (1965) & (1967) studied Kelvin-Helmholtz
instability and the steady motion of a sphere in a dusty gas. Later Michael
& Miller (1966) have investigated the motion of a dusty gas in a semi-
infinite space about a rigid plane boundary. J.T.C. Liu (1966) & (1967)
studied the dusty-gas flow induced by an oscillating and impulsive motion
of an infinite flat plate.

Girishwaranath (1969) obtained the analytical solution of the laminar
flow of an unsteady, incompressible viscous fluid with uniform distribution
of dust particles through two rotating co-axial cylinders under the
(1970) have studied the slow motion of a sphere in a two-phased medium
and the laminar flow of a dusty gas between two rotating cylinders.
Sambashivarao (1968) was able to get the analytical expression for the velocity of dust and gas for the unsteady, viscous dusty liquid flow through a circular cylinder under the influence of the exponential pressure gradient.

P.D. Verma & A.K. Mathur (1971) considered the flow of an unsteady dusty viscous liquid through circular tube and noticed the change in velocity profiles with respect to time. S.N. Dube and L.P. Srivatsava (1972) studied the unsteady flow of a dusty viscous liquid in a channel and pipe. Y.B. Reddy (1971) & (1972) by considering the laminar flow of an unsteady liquid with uniform distribution of dust particles through a rectangular channel under the influence of exponential gradient and the flow of a dusty gas through an elliptical annulus. J.V. Healy & H.T. Yang (1972) have studied the Stokes problem for a suspension of particles. C.S. Vimla (1972) has obtained the solutions for the flow of a dusty gas between two oscillating plates.

Devisingh (1973) studied the flow of a dusty gas through the annular space between two concentric circular cylinders when the cylinders are in simple harmonic and exponential motion. M.P. Pateriya (1974) investigated the unsteady motion of a dusty viscous liquid through elliptical ducts under the influence of time varying pressure gradient. R.K. Gupta & S.C. Gupta (1976) and (1977) studied the flow of dusty gas through a channel with arbitrary time varying pressure gradient and discussed the unsteady flow of a dusty gas in cylinders of circular and sectorial cross sections using Hankel and Laplace transformations. K.K. Singh (1976) obtained the
solutions of the unsteady flow of a conducting dusty fluid through a rectangular channel with time dependent pressure gradient under the influence of a uniform magnetic field. J.P. Singh & R.S Pathak (1976) studied the unsteady flow of a dusty viscous fluid through a tube with sector of a circle as cross section. Lu & Miller (1976) have discussed the flow of a dusty gas under the influence of a constant pressure gradient in a channel with equilateral triangular cross section. E. Rukmanagadachari (1978) has obtained the solutions for unsteady laminar flow of a dusty viscous incompressible fluid through a cylindrical tube of elliptic cross section.

P. Mitra (1979) obtained solutions for the flow of an incompressible dusty gas in the annulus of two concentric co-axial circular cylinders.


Ghosh and Mitra (1984) studied the flow of dusty fluids through horizontal pipes when the pressure gradient is an arbitrary function of time. Osiptosov. A.N. (1985) studied the boundary layer on a blunt body in the flow of dusty gas, which has applications in motion of aircraft in a dusty atmosphere. Kurochkina. E.P. & Strongin M.P. (1985) investigated the non-linear stability of two-phase jets and has given the influence of dust
particles present in the flow on the macro structures and microstructures of the jet flows modifying the rate of turbulent momentum, heat and mass transfer.

Bestman A.R. (1986) has studied the radioactive heat transfer in a dusty gas in a vertical channel, which has applications in hypersonic and re-entry problems. He also studied the thermal stability of a dusty gas in a hydromagnetic rotating flow. Krishna, Rajamani (1987) have simplified the procedure for the solutions of dusty gas model equations for steady state transport in non-reacting systems by constructing the dusty gas model equations for diffusion of \( n \) component mixtures in porous media by giving the dusty gas model equations for diffusions by casting into \( n \)-dimensional matrix notations. Wang. B.Y and Glass I.I. (1986) have obtained the solutions for the compressible laminar boundary layer flows of a dilute dusty gas over a semi-infinite flat plate through finite difference methods.

Snigdha Saxena and Sharma (1987) noted the changes in the velocity profiles of a dusty liquid under the influence of uniform magnetic field with time-varying pressure gradient by considering two-dimensional laminar flow of a conducting dusty, viscous, incompressible liquid. Ramamurthy.V. and Rao U.S. (1987) have discussed the steady stream generated by a vibrating plate parallel to a fixed plate in a dusty fluid. Hastaoglu & Hassan (1988) constructed the dusty gas flux model to describe the transport of the diffusing gasses. Mayura H & Glass I.I.(1988) studied steady supersonic flows around a sharp corner for a dusty gas in which the gas and
the particles make a significant exchange of moment and heat. Debnath, Lokenath and Gosh (1988) obtained the solutions for unsteady hydromagnetic conducting dusty fluid flows between two oscillating plates in presence of an external transverse magnetic field. Numerical study on gas-solid two-phase nozzle and jet flows (1988) was studied by Hayashi. A. et.al.


Shriram, Gupta and Singh have obtained velocities of the dust particles and fluid in the presence of varying electro-magnetic field by considering unsteady flow of a dusty viscous stratified fluid through an inclined open rectangular channel with varying permeable bed using Beavers and Joseph boundary conditions. Calmelet & Eluhu (1993) studied the flow of a conducting dusty gas through a channel of square cross section in the presence of applied magnetic field. Ramrao B.V. (1993) studied the behaviour of the concentration fields in dusty gas flows. Chamka Ali. J. (1994) obtained analytical solutions for the flow of a dusty fluid between two porous flat plates.

Rosenberg and Mendis (1995) investigated the conditions for forming a coulomb lattice of dusty grains, which are charged by the ultra violet induced photoemissiori of electrons. Monaghan J.J. & Kocharayan. A. (1995) have given how to formulate the two-phase flow of a dusty gas using S.P.H. The formulation is general and can be extended to deal with
gas, solid and liquid phases in each of which there may be several species. Datta N. and Dallal D.C. (1995) have studied the flow and heat transfer of a dusty fluid within the annulus of circular cylinders under a pulsatile pressure gradient. Veldsink, J.W. et. al. (1995) used dusty gas models for the description of mass transport with chemical reaction in porous media.

Chamka Ali. J (1996) studied the flow of a compressible dusty gas boundary layer flow over a flat surface and he also constructed the continuous two-phase model allowing particle phase trusses and magnetic field effects developing and applying to the problem of flow of a dusty gas past an infinite porous flat plate.

Hajj et.al. (1998) have given a general variational formalism for the solution of steady flow of dusty fluids. Goel and Agrawal (1998) studied numerically the thermal convection in a visco-elastic dusty fluid in a porous medium in presence of a horizontal magnetic field. Chakraborty & Borkakati (1998) analysed unsteady laminar convection flow of an incompressible electrically conducting visco-elastic fluid through an inclined parallel plate channel in porous medium in presence of a uniform magnetic field applied normal to the plates.

Most of the authors work mentioned above is based on suitable boundary conditions in dusty fluid flows. However, we have studied the dusty fluid flows in anholonomic co-ordinate system, i.e. Frenet frame field system in $E^3$ without using any boundary conditions. The introduction of geometric theories in the study of fluid flows has simplified the
mathematical complexities to a greater extent and gives the possible information regarding the flow in a more general way.

During the second part of twentieth century, the authors Barron (1977) Trusdell (1960), Kanwal (1957), Purushotham (1965), Indrasena (1978) and Bagewadi, Prasanna Kumar & Siddabasappa (1997) have tried to study the fluid flows (by using the techniques of differential geometry in $E^3$). In this work, we have used the same approach to study the flow of dusty fluid by considering the basic equations of motion given by P.G. Saffman (1962).

**Frenet-Serret Formulae:**

These formulae help to measure the turning and twisting of a curve in $E^3$. If $\beta:I\rightarrow E^3$ define a unit speed curve, then $\beta'=t$ defines a unit tangent vector function to the curve $\beta$ because the magnitude of $t=1$. The derivative of $t$, i.e. $\dot{t}=\beta''$ measures the turning of the curve $\beta$ in $E^3$ and is called the curvature vector function of $\beta$. The length of the curvature vector $\dot{t}$ gives a numerical measure of turning of the curve $\beta$ which is a real value function usually denoted by $k(\dot{t'})=k$ for all $s\in I$ and is called the curvature of $\beta$. Thus $k \geq 0$. Larger the value of $k$, sharper the turning of the curve $\beta$.

Now we impose that $k$ never be zero, so $k > 0$. Then, $(t'/k)=n$ on $\beta$ represents a unit vector function in the direction of the curve $\beta$ is turning at each point. $n$ is called the principal normal vector function to the curve.
The vector function $b = txn$ on $\beta$ is called the bi-normal vector function. These vector functions $t, n$ and $b$ are unit vector functions of $s$ mutually perpendicular to each other at each point on $\beta$. The vector functions $t, n$ and $b$ are called the Frenet Frame Field on $\beta$.

In 1847, Frenet, expressed the derivatives $t', n'$ and $b'$ in terms of $t$, $n$ and $b$ as given below:

$$t' = k n, \quad n' = -k t + \tau b \quad \text{and} \quad b' = -\tau n.$$ 

where $\tau$ is called the torsion of the curve.

In 1851, Serret independently proved the above results. Hence they are named as Frenet-Serret Formulae. The success of Frenet approach to curves led mathematicians like Darboux (1880) to adopt the method of moving frames to the study of surfaces. It was Cartan (1933) who first used the method to study of the geometric properties of the surfaces. His idea was to assign a frame to each point on the object under study, i.e. a curve, surface or Euclidean space itself. Then using orthonormal expansion, express the rate of change of the frame in terms of the frame itself. This is what the Frenet formulae do in the case of a curve.

**Definitions:**

Vector fields $E_1$, $E_2$ and $E_3$ on $E^3$ constitute a frame field on $E^3$ provided same $E_i.E_j = \delta_{ij}$ $(1 \leq i, j \leq 3)$. Eg. $V_1 = (1, 0, 0)$, $V_2 = (0, 1, 0)$, $V_3 = (0, 0, 1)$ is a natural frame field on $E^3$. 

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i) If \( r, \theta, z \) are the cylindrical co-ordinate functions on \( E^3 \), then \( E_1 = \cos \theta v_1 + \sin \theta v_2 \). \( E_2 = -\sin \theta v_1 + \cos \theta v_2 \). \( E_3 = v_3 \) forms a cylindrical frame field.

ii) If \( \beta: I \to E^3 \), is a unit curve in \( E^3 \) then, \( t, n \) and \( b \) forms a Frenet frame field.

iii) The plane containing the tangent and the principal normal is called the osculating plane, the plane containing the tangent and the bi-normal is called the rectifying plane and the plane containing the principal normal and the bi-normal is called the normal plane.

Chapter 1 consists of the solutions of two types of dusty fluid flows.

1. One-dimensional steady flow of a dusty gas in an anholonomic co-ordinate system.

2. Steady one-dimensional dusty phase and three-dimensional fluid flows of a dusty fluid in Frenet frame field.

In both the types of flows the basic equations of P.G. Saffman were intrinsically decomposed and solved for the velocities of dust phase and fluid phase and also for the pressure gradient. In the first case, the velocities of the dust and fluid phase are assumed to be uni-directional in Frenet frame field system and the expressions for these velocities and also for the fluid pressure were obtained in terms of the spin coefficients, i.e. the geometrical parameters like curvatures and torsions of the streamlines when the flow is 1) parallel straight line, i.e. \( k_s = 0 \), 2) parallel and \( k_s \neq 0 \) under the assumption that the sum of the deformations of the streamlines
along the principal normal and bi-normal is constant. In addition, the following theorem has been proved.

**Theorem:** If the dusty fluid flow is steady, viscous and incompressible in Frenet frame field system, then the flow is parallel straight line or no radial flow exists for which $N$ is constant on the stream lines provided the paths of the dust and fluid particles is uni-directional.

This theorem is a generalisation of the following two theorems given by R.M. Barron.

**Theorem A:** In the steady plane flow of an incompressible, viscous dusty fluid, if the velocity of the dust particles is every where parallel to the velocity of the fluid particles, then the flow must be radial or in parallel straight lines.

**Theorem B:** No steady plane radial flow of the viscous, incompressible dusty fluid exists for which $N$ is constant on the streamlines or constant throughout the flow.

The graphs are plotted for the velocities of both the phases and for the pressure of the fluid against the parameters $s$ under suitable assumptions.

In the case second type, the equations of continuity and linear momentum are used in which the dust phase velocity is in one direction and the fluid phase velocity in all the three directions. Intrinsic decompositions of these basic equations are carried out and the resulting equations are solved for the velocities of dust phase and fluid phase and it
is found that the component of the fluid phase velocity in binormal direction to be zero which suggests the flow is in only one plane, i.e. s-n plane (Osculating plane). Further, the pressure of the fluid phase is computed by assuming the mean curvature of the streamline to be constant in dust phase and the number density of the dust phase to be uniform. The graphs are plotted for the components of fluid phase i.e. $u_s$ against the component $u_n$ by taking values of the relaxation time $\tau = m/K$ and for the mean curvature of the streamline of the dust phase. In addition, the curvature of the fluid phase is considered as unity. The graphs obtained are parabolic in nature and hence the fluid path is also parabolic which is in accordance with the quadratic approximation of a function of two variables in three-dimensional Euclidean space (Barret ‘O neil).

Second chapter deals with the study of one-dimensional steady/unsteady dusty fluid flows and mathematical models are constructed in Frenet frame field system. The expressions for the pressure gradient amenable to determine the velocity of the dust and fluid phase are determined analytically in three-dimensional Euclidean space. We also derived the expressions for fluid phase velocity and for dust phase velocity when the pressure gradient is linear, periodic and exponential. The velocities of the dust and fluid phase were calculated numerically for all the different pressure gradients mentioned above. The graphs are plotted and discussed for all the velocities corresponding to the different pressure gradients given above.
In chapter three, we have studied steady dusty fluid flow by considering the effect of the temperature of one phase on the other phase. The basic equations of motion are decomposed intrinsically in Euclidean space $E^3$ using Frenet frame field system and flow analysis is carried out by considering the different conditions on the velocity of the gas and temperature of the dust phases. It is shown that the two-phase fluid flow behaves almost like a perfect gas flow in almost all the cases. Because it is found that the relaxation zones of both velocity and temperatures of the dusty gas are same. Graphs are drawn for the dust particle velocity for the different pressure gradients. Further the numerical values for the velocities are calculated and the discussion is carried out on the graphs.

The flow analysis for unsteady flow of dusty gas is also carried out in chapter 4, by varying the velocity and temperature of dust and gas respectively. In addition, in this case, it was found that the paths of velocities and temperatures behave alike and it is established that the relaxation zones of the velocity and temperatures are almost equal. The velocity profiles for the dusty phase are drawn for various pressure gradients. Further, the graphs for the temperature profiles are drawn by considering the temperature function for the dust particles to be constant, linear, exponential and periodic functions. The flow discussions are made based on graphs and the numerical values calculated for the temperature functions under suitable assumptions. In addition, the discussion is made on thermal equilibrium time, $\tau$, of the dusty gas.
In chapter 5, the effects of applied external magnetic field on the dusty fluid flows are studied and the expressions for the velocity of the dust particles and fluid particles are obtained. The flow discussion is made by considering different pressure gradients for the fluid flows. The flow discussion is made in osculating and rectifying planes.