Chapter – 4

Hybrid Pulse Width Modulation Algorithm for Vector Controlled Induction Motor Drive

4.1 Introduction:

The induction motors have been widely used in the industrial applications to convert the electric power to mechanical power. Hence, these are also known as workhorse of the industry. With the advent of Field Oriented Control (FOC) algorithm, the induction motor drives offer fast transient response as well as independent control on flux and torque, which is similar to that of the separately excited DC motor drive. The FOC is also known as vector control. The vector control is referred for controlling both amplitude and phase angle of the stator current. In the case of vector controlled induction motor drives, the rotor flux oriented control is usually employed although it is possible to implement stator flux oriented control. The stator or rotor flux linkages of the induction motors are necessary for the vector control. In terms of the methods of obtaining the unit vectors, the FOC drive systems are classified into two categories, the direct field oriented control (DFOC) and the indirect field oriented control (IFOC) algorithms. In DFOC, the unit vectors are generated from the flux quantities, which are directly
measured by using Hall-effect sensors, etc. Whereas in the IFOC algorithm, the space angle of the flux linkage is obtained as the sum of the monitored rotor angle (corresponding to the rotor speed) and the computed reference value of the slip angle (corresponding to the slip frequency). The IFOC algorithms are receiving wide attention in many applications. But, the indirect vector control algorithm uses hysteresis band type current controllers for the generation of gating signals. This causes increased ripple in steady state currents and variable switching frequency operation. To overcome the above said problems, space vector pulse width modulation (SVPWM) algorithm has been developed for vector controlled induction motor drive. However, the complexity involved in conventional SVPWM algorithm is more due to the calculation of angle, sector and reference voltage vector. Moreover, the conventional SVPWM algorithm gives inferior performance at higher modulation indices. As the CSVPWM is a continuous PWM (CPWM), it gives more switching losses.

This chapter presents a novel hybrid PWM (HPWM) algorithm that uses the SVPWM and various discontinuous PWM (DPWM) algorithms. The proposed HPWM algorithm uses the concept of imaginary switching times which are derived from instantaneous phase voltages. This algorithm does not require angle and sector information and hence reduces the complexity of the algorithm. Also, the proposed HPWM algorithm decreases total harmonic distortion (THD in the stator current)
and switching losses of the inverter at all modulation indices. Moreover, this chapter presents a detailed switching loss analysis of various PWM algorithms and the minimum switching loss PWM algorithms for induction motor drives.

4.2 Principle of Vector Control:

Traditionally, separately excited DC motors are popular in variable speed applications. Because, in the separately excited DC machine the armature and field currents can be controlled independently. The simplified DC machine model can be represented as shown in Fig. 4.1.

![Simplified DC machine model](image)

**Fig. 4.1 Simplified DC machine model**

Due to the electromagnetic interaction between the field flux and the armature MMF, the electromagnetic torque can be generated, which is proportional to the armature current. The expression for electromagnetic torque can be finally written as

\[ T_e = k_t \lambda_f I_a \]  \hspace{1cm} (4.1)

From the above discussion it can be concluded that to achieve fast torque control the armature current should be independently controlled, while keeping the field current as constant. But in induction motors, this
requirement has to be achieved through external control and the process is much more complex than that of DC machines.

The first major breakthrough in the area of high performance induction motors is the discovery of vector control (VC) (also known as field oriented control (FOC)) by F. Blaschke in 1972[2]. Blaschke examined how field orientation occurs naturally in a separately excited DC motor. In the DC motor, the armature and field current are always perpendicular to each other. Similar condition can be obtained in an induction motor by using vector control algorithm. Vector control algorithm provides a method of decoupling the two components of stator current, one producing the torque and the other producing flux. Hence, this algorithm gives independent control of torque and flux.

As the steady state operation is also the special case of the transient analysis by assuming the derivative terms to be zero, in variable speed drive applications, the study of transient analysis is most important. Hence, it is very useful to derive the vector control of induction motor using the transient model. The phasor diagram of the stator voltages, currents and rotor flux is shown in Fig. 4.2. The rotor flux linkage vector $\lambda_r$ is supposed to have q- and d-axis components as the voltages and currents have. But in vector control algorithm, it is possible to align the whole phasor with the synchronous reference frame. Hence, the analysis becomes simple and the field is represented by a
scalar instead of a phasor. The electromagnetic torque equation for the induction machine is given in (4.2).

\[ T_e = \frac{3}{2} P \frac{L_m}{L_r} \left( \lambda_{dr} i_{qs} - \lambda_{qr} i_{ds} \right) \]  

(4.2)

**Fig. 4.2 Phasor diagram of vector control of the induction motor**

According to the principle of vector control algorithm, if the rotor flux linkage is aligned with the synchronous reference frame then \( \lambda_{qr} = 0 \).

Hence, \( \lambda_r = \lambda_{dr} \) and the torque expression can be changed to

\[ T_e = \frac{3}{2} P \frac{L_m}{L_r} \lambda_r i_{qs} \]  

(4.3)

The above expression is similar to the torque expression of a separately excited DC motor. Hence, if the rotor flux linkage \( \lambda_r \) is kept constant, the
output torque will be proportional to the q-axis stator current in the synchronous reference frame. Apparently $\lambda_r$ and $i_{qs}$ are not independent. The stator and rotor flux linkages and the torque are coming from the AC voltages applied to the stator of the induction motor and the current generated. So instantaneous decoupling and independent control of the rotor flux linkages and stator currents are the two most important issues to deal with. Since the vector control is basically in the synchronous reference frame, the instant angle for the synchronous reference frame with respect to the stator reference frame needs to be determined first. According to Fig. 4.2, this angle is the position angle of the rotor flux linkage vector ($\theta_s$). If the rotor flux linkage vector position angle is known, all variables can be transformed into the synchronous reference frame, in which the quantities after transformation become DC quantities [2-3].

Based on the measure of rotor flux linkage position angle, the vector control schemes can be classified into two categories: direct vector control (or direct field oriented control (DFOC)) and indirect vector control (or indirect field oriented control (IFOC)). In the direct vector control scheme, the angle is determined from the flux measurements using Hall sensors or flux sensing windings. While in the indirect vector control scheme, the angle is computed from the measured rotor position angle and the slip angle. The relationship between these angles is given in (4.4).
$$\theta_s = \theta_r + \theta_{sl} = \int (\omega_r + \omega_{sl})dt = \int \omega_s dt$$

(4.4)

In order to get the $\theta_s$ for the DFOC scheme, the additional sensors or windings need to be installed inside the induction motor, which need special design for different type of induction motors and introduce a potential fault condition. So the more popular control scheme is the IFOC scheme, in which the rotor speed is measured using rotor position or speed sensors. The slip frequency is calculated and then the position angle of the rotor flux linkage vector is calculated using (4.4). In the normal operation, the rotor speed is always important and measured, which means the IFOC scheme will not increase the cost at all[2.-3]. The IFOC scheme for induction motor is derived in detail in the following section.

**4.3 Indirect Vector Control of Induction Motor**:

In the indirect vector control of induction motor, a VSI is supposed to drive the motor so that the slip frequency can be changed according to the particular requirement. Assuming the rotor speed is measured, and then the slip speed is derived in the feed-forward manner. The rotor voltage equations of the induction motor can be given as:

$$\frac{d\lambda_{dr}}{dt} + R_f i_{dr} - \omega_{sl} \lambda_{qr} = 0$$

$$\frac{d\lambda_{qr}}{dt} + R_f i_{qr} + \omega_{sl} \lambda_{dr} = 0$$

(4.5)

The rotor flux linkage expressions can be written as
\[ \lambda_{dr} = L_r i_{dr} + L_m i_{ds} \]
\[ \lambda_{qr} = L_r i_{qr} + L_m i_{qs} \]  \hspace{1cm} (4.6)

From (4.6), the rotor currents can be written as
\[ i_{dr} = \frac{1}{L_r} \lambda_{dr} - \frac{L_m}{L_r} i_{ds} \]
\[ i_{qr} = \frac{1}{L_r} \lambda_{qr} - \frac{L_m}{L_r} i_{qs} \]  \hspace{1cm} (4.7)

By substituting (4.7) in (4.5), (4.5) can be modified as
\[ \frac{d\lambda_{dr}}{dt} + \frac{R_r}{L_r} \lambda_{dr} - \frac{L_m R_r}{L_r} i_{ds} - \omega_{sl} \lambda_{qr} = 0 \]
\[ \frac{d\lambda_{qr}}{dt} + \frac{R_r}{L_r} \lambda_{qr} - \frac{L_m R_r}{L_r} i_{qs} + \omega_{sl} \lambda_{dr} = 0 \]  \hspace{1cm} (4.8)

For decoupling control, it is desirable that the rotor flux is aligned onto the d-axis of the synchronously rotating reference frame, then
\[ \dot{\lambda}_{qr} = 0 \Rightarrow \frac{d\lambda_{qr}}{dt} = 0 \text{ and } \lambda_{dr} = \lambda_r \]  \hspace{1cm} (4.9)

By substituting (4.9) in (4.8), (4.8) can be modified as
\[ \frac{L_r}{R_r} \frac{d\lambda_r}{dt} + \lambda_r = L_m i_{ds} \]
\[ \omega_{sl} = \frac{L_m R_r}{L_r \lambda_r} i_{qs} \]  \hspace{1cm} (4.10)

If rotor flux is constant then the rotor flux linkage expression can be written from (4.10) as
\[ \lambda_r = L_m i_{ds} \]  \hspace{1cm} (4.11)

The block diagram of indirect vector controlled induction motor drive can be shown as in Fig. 4.3. This shows how the rotor flux linkage position
can be obtained by integrating the sum of rotor speed and actual speed.

In the indirect vector control scheme, to regulate rotor flux linkage ($\lambda_r$) and rotor speed to desired values are the two objectives. Apparently the stator voltages that are required to generate the desired rotor flux linkage and rotor speed are not directly related to these variables. So the alternative way is to regulate the rotor flux linkage and rotor speed through PI controllers and the outputs of these two controllers give out the reference values for the $q$- and $d$-axis stator currents in synchronous reference frame. Then the actual $q$- and $d$-axis stator currents are regulated to these two reference currents to get the stator voltages [2-3].

**Fig. 4.3 Block diagram of indirect vector controlled induction motor**

### 4.4 Conventional SVPWM Algorithm:

In recent years VSI is widely used to generate a 3-phase variable frequency and variable voltage ac supply required for variable speed AC
drives. The ac voltage is defined by two characteristics, namely amplitude and frequency. Hence, it is essential to work out an algorithm that permits control over both of these quantities. PWM controls the average output voltage over a sufficiently small period called sampling period or subcycle, by producing pulses of variable duty-cycle. The 3-phase VSI can be represented as shown in Fig. 4.4. Every terminal of the induction motor will be connected to the pole of one of the inverter legs. Thus every one of the three pole voltages ($V_{ao}$, $V_{bo}$ and $V_{co}$), measured with respect to the dc bus centre (o), is either $+0.5V_{dc}$ or $-0.5V_{dc}$ at any instant.

![Fig. 4.4 Three-phase voltage source inverter](image)

With a 3-phase VSI there are eight possible switching states. For instance, if the upper switch of the inverter’s pole ‘a’ is on, whereas the other legs both have the lower switch turned on. Then the pole voltages are $(+0.5V_{dc}, -0.5V_{dc}, -0.5V_{dc})$ for poles a, b, c respectively. This
switching state can be designated as +-- or 100. To designate the all possible switching states code numbers 0 to 7 are used and are as shown in Fig. 4.5. In case of the switching states --- (V₀) and +++ (V₇), all the three poles are connected to the same dc bus, effectively shorting the induction motor and resulting in no transfer of power between the source and induction motor. These two states are called ‘zero voltage vectors’ or ‘zero states’. In case of the other switching states, power gets transferred between the source and induction motor. These states (1, 2…6) are called ‘active voltage vectors’ or ‘active states’ [41-43].

![Possible switching states of the 3-phase two-level VSI](image_url)

**Fig. 4.5 Possible switching states of the 3-phase two-level VSI**
For a given set of inverter pole voltages, the vector components \((V_{ds}, V_{qs})\) in the stationary reference frame are found by the forward Clarke transform as

\[
V_s = V_{ds} + jV_{qs} = \frac{2}{3} \left( V_{ao} + V_{bo} e^{j\frac{2\pi}{3}} + V_{co} e^{j\frac{4\pi}{3}} \right)
\]

The relationship between the phase voltages \(V_{an}, V_{bn}, V_{cn}\) and the pole voltages \(V_{ao}, V_{bo}\) and \(V_{co}\) is given by:

\[
V_{ao} = V_{an} + V_{no}; \quad V_{bo} = V_{bn} + V_{no}; \quad V_{co} = V_{cn} + V_{no}
\]

where, \(V_{no}\) is the common mode voltage. Since \(V_{an} + V_{bn} + V_{cn} = 0\),

\[
V_{no} = \frac{(V_{ao} + V_{bo} + V_{co})}{3}
\]

From (4.13) and (4.14) it is evident that the phase voltages \(V_{an}, V_{bn}, V_{cn}\) also result in the same space vector \(V_s\). The space vector \(V_s\) can also be resolved into two rectangular components namely \(V_{ds}\) and \(V_{qs}\). It is customary to place the q-axis along the a-phase axis of the induction motor. The relationship between \(V_{ds}, V_{qs}\) and \(V_{an}, V_{bn}, V_{cn}\) can be given by the 3-phase to 2-phase transformation as follows:

\[
\begin{bmatrix}
V_{qs} \\
V_{ds}
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
1 & -1 & -1 \\
\frac{-1}{2} & \frac{2}{2} & \frac{-\sqrt{3}}{2} \\
\frac{2}{2} & \frac{-\sqrt{3}}{2} & \frac{2}{2}
\end{bmatrix}\begin{bmatrix}
V_{an} \\
V_{bn} \\
V_{cn}
\end{bmatrix}
\]

The space vector \(V_s\) has a constant magnitude and rotating with angular speed \(\omega = 2\pi f\). The space vector locations for switching states may be
evaluated using (4.12) and depicted as voltage space vectors as shown in Fig. 4.6. The space vector locations form the vertices of a regular hexagon, forming six symmetrical sectors as shown in Fig. 4.6.

\[ V_k = \frac{2}{3} V_{dc} e^{j(k-1)\frac{\pi}{3}} \quad \text{where } k = 1, 2, \ldots, 6 \] (4.16)

\[ \text{Fig. 4.6 Voltage space vectors produced by an inverter} \]

From (4.12), it is shown that the active states can be represented as

To generate a given reference voltage vector in an average sense, a combination of switching states can be utilized by maintaining the volt-second balance. This combination of switching states, which generate a reference voltage vector in the given sampling time, is termed as a ‘switching sequence’. This does not mean that any arbitrary set of active vectors and zero vectors can be applied for maintaining the volt-second balance to generate a PWM waveform. There are certain restrictions, which need to be imposed so as to generate a PWM waveform, which results in minimum harmonic distortion. A well-designed PWM algorithm
is one in which there are no pulses of opposite polarity in the line-to-line voltage waveforms, that is, a line-to-line voltage must be either $+V_{dc}$ or 0 and must not be $-V_{dc}$ at any instant in the positive half-cycle, the existence of which would lead to large ripple currents. Further, the simultaneous switching of two phases must be avoided to utilize the available switching frequency of the inverter efficiently. Hence, when the reference voltage vector is within a given sector, the active states that can be applied are only those two vectors, which form the boundaries of that sector. The application of any other active states, results in a pulse of opposite polarity.

The voltage vector $V_{ref}$ in Fig. 4.6 represents the reference voltage space vector or sample, corresponding to the desired value of the fundamental components for the output phase voltages. It is obtained by substituting the instantaneous values of the reference phase voltages, sampled at regular time intervals in (4.12). The $V_{ref}$ is sampled at equal intervals of time, $T_s$ referred to as subcycle or sampling time period. Different voltage vectors that can be produced by the inverter are applied over different durations within a subcycle such that the average vector produced over the subcycle is equal to $V_{ref}$, both in terms of magnitude and angle. It has been established that the vectors to be used to generate any sample are the zero vectors and the two active vectors forming the boundary of the sector in which the sample lies.
As all the six sectors are symmetrical, the discussion is limited to sector-I only. Let $T_1$ and $T_2$ be the durations for which the active states 1 and 2 are to be applied respectively in a given sampling time period $T_s$. Let $T_z$ be the total duration for which the zero states are to be applied. From the principle of volt-time balance, $T_1$, $T_2$ and $T_z$ can be calculated as:

$$V_{ref} \angle \alpha^\circ \times T_s = \frac{2}{3} V_{dc} \angle 0^\circ \times T_1 + \frac{2}{3} V_{dc} \angle 60^\circ \times T_2 + 0 \times T_z$$

(4.17)

Splitting the (4.17) into its real and imaginary components and solving, the switching times can be obtained as

$$T_1 = \frac{2\sqrt{3}}{\pi} M_i \left(\sin(60^\circ - \alpha)\right) T_s$$

(4.18)

$$T_2 = \frac{2\sqrt{3}}{\pi} M_i \left(\sin \alpha\right) T_s$$

(4.19)

where ‘$M_i$’ is the modulation index, given by $M_i = \frac{\pi V_{ref}}{2V_{dc}}$.

The reference voltage space vector shown in Fig. 4.6 describes a circular trajectory of radius $V_{ref}$ at an angular velocity $\omega$ in the complex plane. Clearly, the largest possible voltage magnitude that may be achieved using the space vector pulse width modulation strategy corresponds to the radius of the largest circle that can be inscribed within the hexagon of Fig. 4.6. This circle is tangential to the midpoints
of the lines connecting the ends of the active state vectors. The maximum fundamental phase voltage that may be achieved is:

\[ |V_{ref}|_{\text{max}} = \frac{2}{3} V_{dc} \frac{\sqrt{3}}{2} \]  \hspace{1cm} (4.20)

Then the corresponding maximum modulation index is given by

\[ M_{i,\text{max}} = \frac{\pi \times \frac{2}{3} V_{dc} \frac{\sqrt{3}}{2}}{2V_{dc}} = 0.906 \]  \hspace{1cm} (4.21)

As seen, the maximum peak fundamental magnitude that may be obtained with the SVPWM algorithm is about 90.6% of the inverter capacity. This represents a 15% increase in the maximum voltage compared with sinusoidal PWM.

The total zero state time \( T_z \) may be divided in an arbitrary fashion between the two zero states. A common solution is to divide \( T_z \) equally between the two zero states \( V_0 \) and \( V_7 \), that is

\[ T_Z = T_s - T_1 - T_2 \]  \hspace{1cm} (4.22)

Further, in the SVPWM method, the zero voltage vector time is distributed symmetrically at the start and end of the subcycle in a symmetrical manner. Moreover, to minimize the switching frequency of the inverter, it is desirable that switching should take place in one phase of the inverter only for a transition from one state to another. Thus, SVPWM uses 0127-7210 in sector-I, 0327-7230 in sector-II and so on. Fig. 4.7 shows a typical switching sequence when the reference voltage vector is situated in first sector.
In Fig. 4.7, the symbols $T_{ga}$, $T_{gb}$ and $T_{gc}$ respectively denote the time duration for which the top switch in each phase leg is turned on, from which, it can be seen that the chopping frequency of each phase of the inverter is equal to half of the sampling frequency [41-46].

**4.5 Hybrid PWM Algorithm for Reduced Harmonic Distortion:**

In the previous section, conventional space vector PWM (CSVPWM) algorithm has been developed. However, there is one disadvantage in the CSVPWM algorithm, that is, it requires the angle and sector information to generate the actual switching times. Therefore, for the practical implementation, the CSVPWM technique is very complex and it needs longer calculation time. To reduce the complexity involved in the CSVPWM, in this section, first, few PWM algorithms have been developed using the concept of imaginary switching times and then the hybrid PWM algorithm is developed by using the developed PWM sequences. In this
approach, the actual switching times for each inverter leg are deduced directly from the instantaneous phase voltages as a simple form.

4.5.1 Proposed Switching Sequences:
The proposed approach uses only the instantaneous reference phase voltages to calculate switching times. This method does not depend on the magnitude of the reference voltage space vector and its relative angle with respect to the reference axis. By the d-q transformation theory, the transformation from two-phase voltages to three-phase voltages can be obtained from the stationary frame reference voltages as given in (4.23).

\[
\begin{bmatrix}
V_{an} \\
V_{bn} \\
V_{cn}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
-1/2 & -\sqrt{3}/2 \\
-1/2 & +\sqrt{3}/2
\end{bmatrix}
\begin{bmatrix}
V_{qs} \\
V_{ds}
\end{bmatrix}
\]

(4.23)

If the reference voltage vector lies in the first sector as shown in Fig. 4.6, then the actual switching times can be deduced as follows [52, 73]:

\[
T_1 = \frac{T_s}{V_{dc}} V_{an} - \frac{T_s}{V_{dc}} V_{bn} = T_{an} - T_{bn}
\]

(4.23)

And

\[
T_2 = \frac{T_s}{V_{dc}} V_{bn} - \frac{T_s}{V_{dc}} V_{cn} = T_{bn} - T_{cn}
\]

(4.24)

From the above expressions, the imaginary switching time periods proportional to the instantaneous values of the reference phase voltages are defined as

\[
T_{an} = \left( \frac{T_s}{V_{dc}} \right) V_{an} ;
T_{bn} = \left( \frac{T_s}{V_{dc}} \right) V_{bn} ;
T_{cn} = \left( \frac{T_s}{V_{dc}} \right) V_{cn}
\]

(4.25)
Thus, the active voltage vector switching times can be represented by the
time difference between the imaginary switching time periods. The
switching times $T_{an}, T_{bn}$ and $T_{cn}$ could be negative when the
instantaneous reference voltages are negative. Hence, these times are
called as imaginary switching times. The active vector switching times
can be calculated in each sampling interval as follows:

$$T_{Max} = Max(T_{an}, T_{bn}, T_{cn})$$
Let $T_{Min} = Min(T_{an}, T_{bn}, T_{cn})$
$$T_{Mid} = Mid(T_{an}, T_{bn}, T_{cn})$$

where Max, Min and Mid are the nominal values used during the
sampling interval. The function $Max(T_{an}, T_{bn}, T_{cn})$ selects the maximum
value among $T_{an}, T_{bn}$ and $T_{cn}$. Similarly $Min(T_{an}, T_{bn}, T_{cn})$ selects the
minimum value and $Mid(T_{an}, T_{bn}, T_{cn})$ selects the middle value. Therefore
the active state times $T_1$ and $T_2$ may be expressed as [73]

$$T_1 = T_{Max} - T_{Mid}$$
$$T_2 = T_{Mid} - T_{Min}$$

(4.27)

The effective time ($T_{eff}$) can be defined as the time between $T_{max}$ and
$T_{min}$ and is given by sum of two active vector switching times. The
effective time means the time duration in which the effective voltage is
supplied to the motor terminals. The zero voltage vectors switching time
is calculated using (4.22). The CSVPWM algorithm employs equal
division of zero voltage vector time within a sampling time period.
However, by utilizing the freedom of zero state time division, various
discontinuous PWM (DPWM) algorithms can be generated. In the proposed PWM sequences the zero state time will be shared between two zero states as $T_0$ for $V_0$ and $T_7$ for $V_7$ respectively, and can be expressed as [6]

$$T_0 = k_o T_z$$
$$T_7 = (1 - k_o) T_z$$

(4.28)

If $k_o = 0.5$, 0 and 1, then CSVPWM, DPWMMAX and DPWMMIN can be obtained respectively. When $k_o = 0$, any one of the phases is clamped to positive DC bus for 120 degrees over a fundamental interval and when $k_o = 1$, any one of the phases is clamped to negative DC bus for 120 degrees over a fundamental interval. Thus, in the first sector, CSVPWM uses 0127-7210 sequence, DPWMMAX uses 721-127 sequence and DPWMMIN uses 012-210 sequence. The switching sequences pertaining to all six sectors for the above PWM algorithms are listed in Table 4.1.

**Table 4.1 Switching sequences in all six sectors:**

<table>
<thead>
<tr>
<th>Sector</th>
<th>CSVPWM</th>
<th>DPWMMIN</th>
<th>DPWMMAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0127-7210</td>
<td>012-210</td>
<td>721-127</td>
</tr>
<tr>
<td>II</td>
<td>0327-7230</td>
<td>032-230</td>
<td>723-327</td>
</tr>
<tr>
<td>III</td>
<td>0347-7430</td>
<td>034-430</td>
<td>743-347</td>
</tr>
<tr>
<td>IV</td>
<td>0547-7450</td>
<td>054-450</td>
<td>745-547</td>
</tr>
<tr>
<td>V</td>
<td>0567-7650</td>
<td>056-650</td>
<td>765-567</td>
</tr>
<tr>
<td>VI</td>
<td>0167-7610</td>
<td>016-610</td>
<td>761-167</td>
</tr>
</tbody>
</table>
Various DPWM algorithms can be generated with step change of $k_o$ between zero and one.

**Fig. 4.8** Generation of DPWM0 method by varying the $k_o$

**Fig. 4.9** Generation of DPWM2 method by varying the $k_o$
By changing the value of $k_o$ at the boundary of sectors as shown in Fig. 4.8 and Fig. 4.9, DPWM0 and DPWM2 can be obtained. Similarly by changing the value of $k_o$, at the middle of the sector as shown in Fig.4.10 and Fig.4.11, DPWM1 and DPWM3 can be obtained.

\[ V_1 \]
\[ V_2 \]
\[ V_3 \]
\[ V_4 \]
\[ V_5 \]
\[ V_6 \]

**Fig. 4.10 Generation of DPWM1 method by varying the $k_o$**

\[ V_1 \]
\[ V_2 \]
\[ V_3 \]
\[ V_4 \]
\[ V_5 \]
\[ V_6 \]

**Fig. 4.11 Generation of DPWM3 method by varying the $k_o$**
The modulating waveforms of sinusoidal PWM (SPWM), CSVPWM and all possible DPWM algorithms are given in Fig. 4.12.

![Diagrams of various PWM methods](image)

**Fig. 4.12 Modulating waveforms of various PWM methods at \( M_i = 0.7 \)**

Thus, by varying \( k_o \), the switching time periods for zero voltage vectors can be changed, accordingly different DPWM methods can be obtained. In the DPWM methods, any one of the phases is clamped to the
positive or negative DC bus for utmost a total of $120^\circ$ over a fundamental cycle. Hence, the switching losses of the associated inverter leg are eliminated. Moreover, within a sampling time period three switchings will occur in CSVPWM algorithm whereas two switchings in all the above DPWM algorithms. Hence, the switching frequency of the above DPWM algorithms is reduced by 33% compared with CSVPWM. Hence a switching frequency coefficient is introduced as defined in (4.29).

$$k_{sw} = \frac{f_{swCSVPWM}}{f_{swDPWM}}$$

(4.29)

4.5.2 Analysis of Harmonic Distortion Using Stator Flux Ripple:

In the space vector approach, the applied voltage vector equals the reference voltage vector only in an average sense over the given sampling interval, and not in an instantaneous fashion. The difference between applied voltage vector and reference voltage vector is the ripple voltage vector, which depends on space and modulation index. The ripple voltage vectors and trajectory of the stator flux ripple can be represented in a complex plane as shown in Fig. 4.13. The corresponding d-axis and q-axis components of the stator flux ripple vector are as shown in Fig. 4.14, from which it is observed that the application of a zero voltage vector results in a variation of the q-axis component of the flux ripple and the application of any active voltage vector results in variation of the both the d-axis and q-axis components. The error volt-seconds corresponding to the voltage ripple vectors are given by
\[ V_{r1}T_1 = \left( \frac{2}{3} V_{dc} \sin \alpha \right) T_1 + j \left( \frac{2}{3} V_{dc} \cos \alpha - V_{ref} \right) T_1 \]  \hspace{1cm} (4.30)

\[ V_{r2}T_2 = -\left( \frac{2}{3} V_{dc} \sin(60^\circ - \alpha) \right) T_2 + j \left( \frac{2}{3} V_{dc} \cos(60^\circ - \alpha) - V_{ref} \right) T_2 \]  \hspace{1cm} (4.31)

\[ V_{r0}T_0 = -jV_{ref} T_0 = -j \frac{2M_i V_{dc}}{\pi} T_0 = j\lambda_{q0} \]  \hspace{1cm} (4.32)

\[ V_{r7}T_7 = -jV_{ref} T_7 = -j \frac{2M_i V_{dc}}{\pi} T_7 = j\lambda_{q7} \]  \hspace{1cm} (4.33)

Fig. 4.13 Voltage ripple vectors and trajectory of the flux ripple

Fig. 4.14 q-axis and d-axis components of the flux ripple vectors
From (4.18) and (4.19)

\[
\sin \alpha = \frac{\pi T_2}{2\sqrt{3}M_i T_s}
\]

(4.34)

\[
\cos \alpha = \frac{\pi (T_1 + 0.5T_2)}{3M_i T_s}
\]

(4.35)

\[
\cos (60^\circ - \alpha) = \frac{\pi (0.5T_1 + T_2)}{3M_i T_s}
\]

(4.36)

and \(\left(\frac{2}{3} V_{dc} \sin \alpha\right)T_1 = \left(\frac{2}{3} V_{dc} \sin(60^\circ - \alpha)\right)T_2\) (4.37)

By substituting (4.34) - (4.37) in (4.30) and (4.31),

\[
V_{r1T1} = \frac{\pi V_{dc}}{3\sqrt{3}M_i} \frac{T_1 T_2}{T_s} + j\left(\frac{2V_{dc} \pi (T_1 + 0.5T_2)}{9M_i T_s} - \frac{2V_{dc} M_i}{\pi}\right)T_1
\]

(4.38)

\[
= \lambda_d + j\lambda_q
\]

\[
V_{r2T2} = -\frac{\pi V_{dc}}{3\sqrt{3}M_i} \frac{T_1 T_2}{T_s} + j\left(\frac{2V_{dc} \pi (0.5T_1 + T_2)}{9M_i T_s} - \frac{2V_{dc} M_i}{\pi}\right)T_2
\]

(4.39)

\[
= -\lambda_d + j\lambda_q
\]

The q-axis ripple and d-axis ripple are given in (4.40) and (4.41).

\[
\lambda_q = \frac{\lambda_q}{T_0}, \quad \text{if } 0 \leq t \leq T_0
\]

\[
= \lambda_q + \frac{\lambda_q}{T_1} t_1, \quad \text{if } T_0 \leq t \leq (T_0 + T_1)
\]

(4.40)

\[
= \lambda_q + \lambda_q + \frac{\lambda_q}{T_2} t_2, \quad \text{if } (T_0 + T_1) \leq t \leq (T_0 + T_1 + T_2)
\]

\[
= -\lambda_q + \frac{\lambda_q}{T_7} t_3, \quad \text{if } (T_s - T_7) \leq t \leq T_s
\]
\[ \lambda_d = 0, \quad \text{if } 0 \leq t \leq T_0 \]
\[ = \frac{\lambda_d}{T_1} t_1, \quad \text{if } T_0 \leq t \leq (T_0 + T_1) \]
\[ = \frac{\lambda_d}{T_2} t_2, \quad \text{if } (T_0 + T_1) \leq t \leq (T_0 + T_1 + T_2) \]
\[ = 0, \quad \text{if } (T_s - T_7) \leq t \leq T_s \]

where, \[ t_1 = t - T_0; \quad t_2 = t - T_0 - T_1; \quad t_3 = t - T_0 - T_1 - T_2 \]

The mean square stator flux ripple over a sampling interval can be calculated as

\[
\lambda^2_{(\text{rms})} = \frac{1}{T_s} \int_0^{T_s} \lambda^2_{q} \, dt + \frac{1}{T_s} \int_0^{T_s} \lambda^2_{d} \, dt
\]
\[
= \frac{1}{3} \left[ \frac{\lambda_{q0}^2 T_0}{T_s} + \left[ \frac{\lambda_{q0}^2 + (\lambda_{q0} + \lambda_{q1})^2 + \lambda_{q0}(\lambda_{q0} + \lambda_{q1})}{T_s} \right] \frac{T_1}{T_s} + \left[ (\lambda_{q0} + \lambda_{q1})^2 - \lambda_{q7}(\lambda_{q0} + \lambda_{q1}) + \frac{\lambda_{q7}^2}{T_s} \right] \frac{T_2}{T_s} \right]
\]
\[
+ \frac{\lambda_{q7}^2}{T_s} + \frac{\lambda_d^2}{T_s} \left( T_1 + T_2 \right) \]

By using the above formula, the mean square flux ripple can be easily computed and graphically represented for CSVPWM, DPWMMAX, DPWMIN, DPWM0, DPWM1, DPWM2 and DPWM3 methods. The mean square stator flux ripple characteristics obtained from (4.43) for various PWM algorithms and for different modulation indices are shown in Fig.4.15 – Fig.4.20.
Fig. 4.15 Variation of stator flux ripple over the first sector for $M_i = 0.4$ and $k_{sw}=1$ (E: CSVPWM, AB: DPWM1, CD: DPWM3, AD: DPWMMAX, DPWM2 and CB: DPWMMIN, DPWM0)

Fig. 4.16 Variation of stator flux ripple over the first sector for $M_i = 0.8$ and $k_{sw}=1$ (E: CSVPWM, AB: DPWM1, CD: DPWM3, AD: DPWMMAX, DPWM2 and CB: DPWMMIN, DPWM0)
Fig. 4.17 Variation of stator flux ripple over the first sector for $M_i = 0.906$ and $k_{sw}=1$ (E: CSVPWM, AB: DPWM1, CD: DPWM3, AD: DPWMMAX, DPWM2 and CB: DPWMMIN, DPWM0)

Fig. 4.18 Variation of stator flux ripple over the first sector for $M_i = 0.4$ and $k_{sw}=2/3$ (E: CSVPWM, AB: DPWM1, CD: DPWM3, AD: DPWMMAX, DPWM2 and CB: DPWMMIN, DPWM0)
Fig. 4.19 Variation of stator flux ripple over the first sector for $M_i = 0.8$ and $k_{sw}=2/3$ (E: CSVPWM, AB: DPWM1, CD: DPWM3, AD: DPWMMAX, DPWM2 and CB: DPWMMIN, DPWM0)

Fig. 4.20 Variation of stator flux ripple over the first sector for $M_i = 0.906$ and $k_{sw}=2/3$ (E: CSVPWM, AB: DPWM1, CD: DPWM3, AD: DPWMMAX, DPWM2 and CB: DPWMMIN, DPWM0)
From Fig. 4.15 – Fig. 4.20, it can be observed that replacing $\alpha$ by $(60^\circ - \alpha)$ in the rms stator flux ripple expressions of CSVPWM does not change its value. It is equivalent to interchanging of $T_1$ and $T_2$ and that of $\lambda_{q1}$ and $\lambda_{q2}$. It can be seen that such a swapping does not change rms stator flux ripple expressions of CSVPWM. Thus for a given sampling time and $V_{\text{ref}}$, 0127 sequence produces equal mean square stator flux ripple at spatial angles $\alpha$ and $(60^\circ - \alpha)$. However, the rms ripple over a subcycle is not symmetric about the center of the sector for the other PWM sequences. Swapping of $T_1$ and $T_2$ and that of $\lambda_{q1}$ and $\lambda_{q2}$, in the rms stator flux ripple expression of DPWMMIN lead to the rms stator flux ripple expression of DPWMMAX. Thus, for a given $V_{\text{ref}}$ and $T_s$, these can be shown as follows:

$$
\lambda_{\text{rms}}^2_{\text{CSVPWM}}(\alpha) = \lambda_{\text{rms}}^2_{\text{CSVPWM}}(60^\circ - \alpha)
$$

$$
\lambda_{\text{rms}}^2_{\text{DPWMIN}}(\alpha) = \lambda_{\text{rms}}^2_{\text{DPWMMAX}}(60^\circ - \alpha)
$$

(4.44)

Also, from Fig. 4.15 – Fig. 4.20, it can be observed that, DPWMMIN algorithm leads to less rms ripple than DPWMMAX algorithm in the first half of sector-I. Conversely, DPWMMAX algorithm leads to less rms ripple than DPWMMIN algorithm in the second half of sector-I as given in (4.45).

$$
\lambda_{\text{rms}}^2_{\text{DPWMIN}}(\alpha) < \lambda_{\text{rms}}^2_{\text{DPWMMAX}}(\alpha) \text{ if } 0^\circ < \alpha < 30^\circ
$$

$$
\lambda_{\text{rms}}^2_{\text{DPWMIN}}(\alpha) > \lambda_{\text{rms}}^2_{\text{DPWMMAX}}(\alpha) \text{ if } 30^\circ < \alpha < 60^\circ
$$

(4.45)
The CSVPWM algorithm (0127-7210 sequence) has three switchings per sampling time period, whereas all the other DPWM algorithms (DPWMMIN, DPWMMAX, DPWM0, DPWM1, DPWM2 and DPWM3) have only two switchings per sampling time period. Hence, subcycle duration $T_s = (2T/3)$ is considered for these DPWM algorithms, while $T_s = T$ for CSVPWM algorithm. This ensures comparison of PWM algorithms at the same average switching frequency, $f_{sw}$. Hence the switching frequency coefficient is taken as $k_{sw} = 2/3$ in this thesis for the simulation studies. If $k_{sw} = 2/3$, from Fig. 4.15 – Fig. 4.20, it can be observed that at lower modulation indices, the CSVPWM algorithm gives superior performance whereas at higher modulation indices the DPWM algorithms give superior performance. Out of all possible DPWM algorithms, DPWM3 algorithm gives less THD at all modulation indices.

4.5.3 Proposed Hybrid PWM Algorithms:

To minimize the harmonic distortion in the line current, the rms current ripple or rms stator flux ripple over every sampling time period should be reduced. The proposed hybrid PWM (HPWM) algorithm employ the best sequence out of the above possible sequences to minimize the rms current ripple in every sampling time period. The development of HPWM technique for reduced current ripple involves determination of superior performance for every sequence. The zone of superior performance for a given sequence is the spatial zone within a sector
where the given sequence results in less mean square stator flux ripple than the other sequences considered. By taking the combination of different sequences various HPWM algorithms can be generated. But, the proposed HPWM algorithm consists of all the above possible sequences. In the proposed HPWM algorithm, in every sampling time period the rms stator flux ripples are compared with each other and the sequence, which has less rms stator flux ripple is applied to minimize the THD. Thus, the proposed HPWM algorithm uses the DPWM algorithms in conjunction with CSVPWM algorithm. Flow chart of the proposed Hybrid PWM algorithm is shown appendix-III.

4.5.4 Results and Discussion:

To validate the proposed HPWM based vector controlled induction motor drive, numerical simulation studies have been carried out by using Matlab/Simulink. The simulation diagrams, parameters and specifications of induction motor used in this thesis are given in Appendix – I&III. Various conditions of the drive such as starting, steady state, step change in load and speed reversal are simulated.

The results for CSVPWM based vector controlled induction motor drive are given in Fig. 4.21 – Fig 4.25. Fig.4.21 shows starting transients of speed, stator currents, torque and d, q axes currents. Fig.4.22 presents the plots under steady state operating conditions. The transient during step change in load (a load torque of 8 N-m is applied at 0.6 sec and removed at 0.8 sec) is shown in Fig.4.23. The transients of speed,
stator currents and torque during speed reversal from +1200 to -1200 rpm and -1200 to +1200 rpm are shown in Fig.4.24 and Fig.4.25 respectively. The results for proposed HPWM based vector controlled induction motor drive are given in Fig. 4.26 – Fig. 4.30. Fig.4.26 shows starting transients of speed, stator currents, torque and d, q axes currents. Fig.4.27 presents the plots under steady state operating conditions. The transients of speed, stator current and torque during step change in load (a load torque of 8 N-m is applied at 0.6 sec and removed at 0.8 sec) is shown in Fig.4.28. The transients during speed reversal from +1200 to -1200 R.P.M. and -1200 to +1200 R.P.M. are shown in Fig.4.29 and Fig.4.30 respectively. The harmonic spectra of stator currents for CSVPWM and proposed HPWM based vector control of induction motor along with their total harmonic distortion (THD) values are shown in Fig. 4.31 and Fig. 4.32 respectively.

By observing the steady state results of CSVPWM and HPWM based vector control of induction motor, it can be observed that the proposed HPWM algorithm gives less ripples in current and torque. Also, HPWM gives less THD compared with CSVPWM algorithm. Hence, with the proposed HPWM algorithm the harmonic distortion in the line current and hence steady state ripples in the torque and current can be reduced. From the q-axis and d-axis currents, it can be observed that the q-axis current is changing according to the torque requirement and d-axis current is almost constant at all operating conditions.
Fig. 4.21 (a) Transients in speed, stator currents and torque during the starting for CSVPWM algorithm based vector controlled induction motor drive (b) Transients during starting in actual q-axis and d-axis currents (synchronous reference frame)
Fig. 4.22 (a) Steady state plots of speed, stator currents and torque for CSVPWM algorithm based vector controlled induction motor drive (b) Steady state plots of actual q-axis and d-axis currents (synchronous reference frame)
Fig. 4.23 (a) Transients in speed, stator currents and torque during the step change in load (a load torque of 8 N-m is applied at 0.6 sec and removed at 0.8 sec) for CSVPWM algorithm based vector controlled induction motor drive (b) Transients during the step change in load in actual q-axis and d-axis currents (synchronous reference frame)
Fig. 4.24 (a) Transients in speed, stator currents and torque during the speed reversal (+1200 to -1200 R.P.M) for CSVPWM algorithm based vector controlled induction motor drive (b) Transients during the speed reversal in actual q-axis and d-axis currents (synchronous reference frame)
Fig. 4.25 (a) Transients in speed, stator currents and torque during the speed reversal(-1200 to +1200 R.P.M) for CSVPWM algorithm based vector controlled induction motor drive (b) Transients during the speed reversal in actual q-axis and d-axis currents (synchronous reference frame)
Fig. 4.26 (a) Transients in speed, stator currents and torque during the starting for proposed HPWM algorithm based vector controlled induction motor drive (b) Transients during starting in actual q-axis and d-axis currents (synchronous reference frame)
Fig. 4.27 (a) Steady state plots of speed, stator currents and torque for proposed HPWM algorithm based vector controlled induction motor drive (b) Steady state plots of actual q-axis and d-axis currents (synchronous reference frame)
Fig. 4.28 (a) Transients in speed, stator currents and torque during the step change in load (a load torque of 8 N-m is applied at 0.6 sec and removed at 0.8 sec) for proposed HPWM algorithm based vector controlled induction motor drive (b) Transients during the step change in load in actual q-axis and d-axis currents (synchronous reference frame)
Fig. 4.29 (a) Transients in speed, stator currents and torque during the speed reversal (+1200 to -1200 R.P.M) for proposed HPWM algorithm based vector controlled induction motor drive (b) Transients during the speed reversal in actual q-axis and d-axis currents (synchronous reference frame)
Fig. 4.30 (a) Transients in speed, stator currents and torque during the speed reversal(-1200 to +1200 R.P.M) for proposed HPWM algorithm based vector controlled induction motor drive (b) Transients during the speed reversal in actual q-axis and d-axis currents (synchronous reference frame)
Moreover, as the proposed HPWM algorithm uses the DPWM algorithms in conjunction with the CSVPWM algorithm, it reduces the switching losses of the inverter also. Thus, the harmonic distortion in the line current, steady state ripples in the torque and switching losses of the inverter can be reduced with the proposed HPWM algorithm. Also,
the proposed HPWM algorithm is developed based on the imaginary switching times, it can be decreased the complexity involved in the existing HPWM algorithms. However, the detailed analysis of the switching loss characteristics is given in the next section.

**4.6 MSLPWM Algorithm for Reduced Switching Loss of the Inverter:**

The switching losses of a pulse width modulated VSI fed induction motor drive are load dependent and increase with the current magnitude. According to the switching devices manufacturers’, the switching losses are approximately proportional to the current magnitude. The switching losses of the inverter also depend on the type of PWM method used [54]. With continuous PWM methods, all the three phase currents are commutated within each carrier cycle of a full fundamental cycle. Therefore, for all continuous PWM methods the switching losses are the same and independent of the load power factor angle. However, with the discontinuous PWM methods, the switching losses are significantly influenced by the type of modulation method and load power factor angle. In the DPWM methods, the devices are clamped to either negative bus or positive bus for a total of 120° and hence reduce the switching losses of inverter over continuous PWM methods. Hence, in DPWM algorithms, the load power factor and the modulation method together determine the time interval that the load current is not commutated. Since the switching losses of the inverter are strongly dependent on and linearly increase with the magnitude of the commutating phase current,
selecting a DPWM method with reduced switching losses can significantly contribute to the performance of an induction motor drive. Therefore, it is necessary to derive the switching loss characteristics to compare the switching losses of various DPWM algorithms. This section presents a comparison of inverter switching losses due to conventional SVPWM and existing DPWM methods.

The switching loss in an IGBT depends mainly on the dc link voltage ($V_{dc}$), instantaneous line current and turn-on and turn-off times. However, the $V_{dc}$ and times are assumed to be constant for different instantaneous line currents. Hence, to study the switching losses of the inverter, it is sufficient to consider the product of instantaneous line current magnitude of a particular phase and the number of switchings per sampling time period in that phase ($n_a$), corresponding to the PWM sequence considered. This product is referred to as the switching loss factor (SLF). Since the three phases are symmetric, it is enough to analyze one phase only. The switching losses of a PWM-VSI induction motor drive can be modeled analytically by assuming linear current turn-on and turn-off characteristics with respect to time for the inverter switching devices and considering only fundamental component of the load current. Let the phase current be

$$i_a = I_{\text{max}} \sin(\omega t - \phi)$$

(4.46)

where $i_a$ is the instantaneous fundamental phase current, $I_{\text{max}}$ is the maximum value of the fundamental phase current and $\phi$ is the line side
power factor angle. The average switching energy loss per subcycle \((E_{\text{sub}})\) in an inverter leg is as given by [76]

\[
E_{\text{sub}}(\text{avg}) = \frac{1}{\pi} \int_{0}^{\pi} n_a |I_{\text{max}}| \sin(\omega t - \phi) d(\omega t) = \frac{1}{\pi} \int_{0}^{\pi} n_a \sin(\omega t - \phi) d(\omega t) \quad (4.47)
\]

To obtain the measure of the inverter switching losses, the average switching energy loss per subcycle must be multiplied by the number of subcycles per second, i.e., the sampling frequency \((f_s)\). The sampling frequency of the CSVPWM is two times the switching frequency \((f_{sw})\), while it is three times the switching frequency \((f_{sw})\) for the above DPWM methods. The average switching energy loss over a fundamental cycle for CSVPWM equals \((2/\pi)\). The normalized switching loss due to given PWM algorithm can be obtained as given in (4.48) [76].

\[
P_{\text{sw}} = \frac{E_{\text{sub}}(\text{avg}) \ast f_s}{(2/\pi) \ast 2f_{sw}} \quad (4.48)
\]

From (4.48), the normalized switching loss due to existing DPWM methods can be obtained and given in (4.49).

\[
P_{\text{sw}} = \frac{3\pi}{4} E_{\text{sub}}(\text{avg}) \quad (4.49)
\]

By observing the DPWM0, DPWM1 and DPWM2 modulating waveforms, it can be given that there is a 30° phase-angle \((\delta)\) distance between their dc-link clamped 60° segments. Hence, a new PWM algorithm can be introduced as generalized DPWM (GDPWM) algorithm which covers the DPWM0, DPWM1 and DPWM2 algorithms. The \(\delta\) and \(\phi\)
dependent switching phase current and normalized switching energy loss per subcycle waveforms of GDPWM algorithm is shown in Fig. 4.33.

Fig. 4.33 The average switching loss of GDPWM

Fig. 4.34 space vector illustration of GDPWM algorithm
The GDPWM algorithm can be represented by using space vector as shown in Fig. 4.34. For $\delta=0^\circ$, $30^\circ$ and $60^\circ$, DPWM0, DPWM1 and
DPWM2 algorithms can be obtained respectively. The variation of average switching energy loss per subcycle ($E_{\text{sub}}$) over a fundamental cycle for various DPWM algorithms for different values of $\delta$ and $\phi$ is shown in Fig. 4.35- Fig. 4.38. From these, it can be observed that at unity power factor DPWM1 clamps a phase around its current peak, which leads to a significant reduction in $E_{\text{sub}}(\text{avg})$ over the remaining DPWM and conventional SVPWM algorithms. Therefore, the switching loss mainly depends on the power factor angle by which the line current lags/leads the line voltage. Moreover, it may be observed that the effect of different PWM sequences on the switching loss does not depend on $V_{\text{ref}}$ or the fundamental frequency.

![Diagram](image)

(a) $\delta = 0^\circ$ and $\phi = 0^\circ$
(b) $\delta = 0^\circ$ and $\phi = 30^\circ$

(c) $\delta = 0^\circ$ and $\phi = 60^\circ$
Fig. 4.35 Variation of normalized switching energy loss in an inverter leg over a fundamental cycle for DPWM0 method: (a) $\delta = 0^\circ$ and $\phi = 0^\circ$ (b) $\delta = 0^\circ$ and $\phi = 30^\circ$ (c) $\delta = 0^\circ$ and $\phi = 60^\circ$ (d) $\delta = 0^\circ$ and $\phi = 90^\circ$

(a) $\delta = 30^\circ$ and $\phi = 0^\circ$
(b) $\delta = 30^\circ$ and $\phi = 30^\circ$

(c) $\delta = 30^\circ$ and $\phi = 60^\circ$
(e) $\delta = 30^\circ$ and $\phi = 90^\circ$

Fig. 4.36 Variation of normalized switching energy loss in an inverter leg over a fundamental cycle for DPWM1 method: (a) $\delta = 30^\circ$ and $\phi = 0^\circ$ (b) $\delta = 30^\circ$ and $\phi = 30^\circ$ (c) $\delta = 30^\circ$ and $\phi = 60^\circ$ (d) $\delta = 30^\circ$ and $\phi = 90^\circ$

(a) $\delta = 60^\circ$ and $\phi = 0^\circ$
(b) $\delta = 60^\circ$ and $\phi = 30^\circ$

(c) $\delta = 60^\circ$ and $\phi = 60^\circ$
Fig. 4.37 Variation of normalized switching energy loss in an inverter leg over a fundamental cycle for DPWM2 method: (a) $\delta = 60^\circ$ and $\phi = 0^\circ$ (b) $\delta = 60^\circ$ and $\phi = 30^\circ$ (c) $\delta = 60^\circ$ and $\phi = 60^\circ$ (d) $\delta = 60^\circ$ and $\phi = 90^\circ$
(b) $\phi = 30^\circ$

(c) $\phi = 60^\circ$
Fig. 4.38 Variation of normalized switching energy loss in an inverter leg over a fundamental cycle for DPWM3 method:
(a) $\phi = 0^\circ$ (b) $\phi = 30^\circ$ (c) $\phi = 60^\circ$ (d) $\phi = 90^\circ$

From Fig. 4.35 – Fig. 4.38, the expressions for normalized inverter switching loss corresponding to different DPWM algorithms are given in (4.50) and (4.51) [54], [65].

\[ P_{sw,GDPWM} = \frac{3}{4} \left( 2 - \sin(60^\circ + \delta - \phi) \right) \quad \text{for } 0^\circ \leq \phi \leq (30^\circ + \delta) \]
\[ = \frac{3}{4} \left( \sqrt{3} \sin(150^\circ + \delta - \phi) \right) \quad \text{for } (30^\circ + \delta) \leq \phi \leq 90^\circ \]  

(4.50)

\[ P_{sw,DPWM3} = \frac{3}{4} \left( 2 - \sqrt{3} \cos \phi + \sin(60^\circ + \delta - \phi) \right) \quad \text{for } 0^\circ \leq \phi \leq 30^\circ \]
\[ = \frac{3}{4} \left( \sin \phi + \sin(60^\circ + \delta - \phi) \right) \quad \text{for } 30^\circ \leq \phi \leq (30^\circ + \delta) \]
\[ = \frac{3}{4} \left( 2 + \sin \phi - \sqrt{3} \sin(150^\circ + \delta - \phi) \right) \quad \text{for } (30^\circ + \delta) \leq \phi \leq 90^\circ \]  

(4.51)
The normalized switching loss expressions for DPWM0, DPWM1 and DPWM2 can be obtained from (4.50) by substituting $\delta=0^\circ$, $30^\circ$ and $60^\circ$ respectively. From (4.50) and (4.51), it can be observed that the normalized switching loss expressions of existing DPWM methods are the function of both $\delta$ and $\phi$. The variation of normalized switching loss of different DPWM algorithms with power factor angle (both at lagging and leading) is shown in Fig. 4.39.

![Fig. 4.39 The variation of normalized switching loss of different DPWM algorithms with power factor angle (A: DPWM0; B: DPWM1; C: DPWM2 and D: DPWM3)](image)

From the above figure, it can be found that the DPWM1 gives minimum switching loss at unity power factor, because DPWM1 clamps at peak phase current at unity power factor. Similarly, DPWM0 and DPWM2 gives minimum switching losses at $30^\circ$ leading and lagging power factors respectively. But, DPWM3 gives minimum switching loss...
near the zero power factor. The value of $\delta$ can be varied in accordance with $\phi$ to achieve reduction in switching losses. The minimum switching power loss solution of the DPWM0, DPWM1 and DPWM2 yields to GDPWM algorithm. The optimal solution of GDPWM is obtained by selecting $\delta = (\pi/6) + \phi$ for $-(\pi/6) \leq \delta \leq (\pi/6)$. Outside this range $\delta = 60^\circ$ for lagging power factor angle and $\delta = 0^\circ$ for leading power factor angle. The normalized switching loss variations for different DPWM algorithms and GDPWM algorithm are shown in Fig. 4.40. This shows that the GDPWM algorithm combines the DPWM0 and DPWM1 and DPWM2 algorithms and also gives less switching loss.

![Graph showing switching loss variations](image)

**Fig. 4.40** The variation of normalized switching loss of different DPWM algorithms with power factor angle (A: DPWM0; B: DPWM1; C: DPWM2; D: DPWM3 and E=GDPWM)

The GDPWM does not consist of DPWM3 algorithm. But from Fig.4.39, it can be observed that the DPWM3 gives minimum switching losses outside the $-(5\pi/12) \leq \phi \leq (5\pi/12)$ range. In this thesis, by taking
DPWM3 also into account minimum switching loss PWM (MSLPWM) algorithm with existing DPWM algorithms is proposed. The variations of normalized switching losses of various DPWM algorithms and proposed minimum switching loss PWM (MSLPWM) algorithm are shown in Fig. 4.41. The comparison between GDPWM and proposed MSLPWM is shown in Fig. 4.42.

**Fig. 4.41** The variation of normalized switching loss of different DPWM algorithms with power factor angle (A: DPWM0; B: DPWM1; C: DPWM2; D: DPWM3 and E= MSLPWM)

**Fig. 4.42** The variation of normalized switching loss of GDPWM (A) and MSLPWM (B) algorithms
To minimize the normalized switching losses of inverter with existing DPWM algorithm, the type of clamping and the value of $\delta$ must be as shown in Table 4.2.

**Table 4.2 Minimum switching loss PWM (MSLPWM)**

<table>
<thead>
<tr>
<th>PWM algorithm</th>
<th>Leading power factor</th>
<th>Lagging power factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-90^0 \leq \delta \leq -60^0$</td>
<td>$-60^0 \leq \delta \leq -30^0$</td>
</tr>
<tr>
<td>GDPWM</td>
<td>$\delta = 0^0$</td>
<td>$\delta = 0^0$</td>
</tr>
<tr>
<td>MSLPWM</td>
<td>$\delta = 120^0 -</td>
<td>\phi</td>
</tr>
</tbody>
</table>

By changing the value of $\delta$ in accordance with the power factor angle ($\phi$), the normalized switching losses of an inverter can be reduced. Thus, the proposed MSLPWM algorithm gives minimum switching losses over the other existing DPWM algorithms.

**4.6.1 Results and Discussion:**

The simulation results for the proposed MSLPWM algorithm based vector controlled induction motor are given in Fig 4.43 – Fig. 4.52 at unity power factor and $30^0$ lagging power factors. As the proposed MSLPWM algorithm uses the only existing DPWM algorithms, it gives more steady state ripples when compared with the CSVPWM algorithm at lower modulation indices. Here, the simulation results are given at lower modulation indices. Fig. 4.43 shows the starting transients of MSLPWM based vector controlled induction motor drive at unity power factor.
conditions. The steady state plots of proposed MSLPWM based vector controlled induction motor drive are shown in Fig 4.44 at UPF condition.

![Graphs showing speed, stator currents, and torque](image)

**Fig. 4.43** (a) Transients in speed, stator currents and torque during the starting for MSLPWM algorithm based vector controlled induction motor at Unity power factor (when DPWM1 is selected) (b) Transients during starting in actual q-axis and d-axis currents
Fig. 4.44 (a) Steady state plots of speed, stator currents and torque for proposed MSLPWM algorithm based vector controlled induction motor at Unity power factor (when DPWM1 is selected)  (b) Steady state plots of actual q-axis and d-axis currents.
Fig. 4.45 (a) Transients in speed, stator currents and torque during the step change in load (a load torque of 8 N-m is applied at 0.6 sec and removed at 0.8 sec) for proposed MSLPWM algorithm based vector controlled induction motor at Unity power factor (when DPWM1 is selected) (b) Transients during the step change in load in actual q-axis and d-axis currents
Fig. 4.46 (a) Transients in speed, stator currents and torque during the speed reversal(+1200 to -1200 R.P.M) for proposed MSLPWM algorithm based vector controlled induction motor at Unity power factor (when DPWM1 is selected) (b) Transients during the speed reversal in actual q-axis and d-axis currents
Fig. 4.47 (a) Transients in speed, stator currents and torque during the speed reversal (-1200 to +1200 R.P.M) for proposed MSLPWM algorithm based vector controlled induction motor at Unity power factor (when DPWM1 is selected) (b) Transients during the speed reversal in actual q-axis and d-axis currents
Fig. 4.48 (a) Starting transients of MSLPWM based Vector controlled induction motor at 30° lagging power factor (when DPWM2 is selected) (b) Transients during the speed reversal in actual q-axis and d-axis currents
Fig. 4.49 (a) Steady state plots of MSLPWM based Vector controlled induction motor at 30° lagging power factor (when DPWM2 is selected) (b) Transients during the speed reversal in actual q-axis and d-axis currents
Fig. 4.50 (a) Transients during step change in load (a load torque of 8 N-m is applied at 0.6 sec and removed at 0.8 sec) for MSLPWM based Vector controlled induction motor at 30° lagging power factor (when DPWM2 is selected) (b) Transients during the speed reversal in actual q-axis and d-axis currents
Fig. 4.51 (a) Transients during speed reversal (+1200 to -1200 R.P.M) for MSLPWM based Vector controlled induction motor at 30° lagging power factor (when DPWM2 is selected) (b) Transients during the speed reversal in actual q-axis and d-axis currents
Fig. 4.52 (a) Transients during speed reversal (-1200 to +1200 R.P.M) for MSLPWM based Vector controlled induction motor at 30° lagging power factor (when DPWM2 is selected) (b) Transients during the speed reversal in actual q-axis and d-axis currents
Fig 4.45 shows the transients of MSLPWM based drive during the step change in load condition. The transients during the speed reversal for proposed MSLPWM based drive at UPF are shown in Fig. 4.46 and Fig. 4.47. Fig. 4.48 shows the starting transients of MSLPWM based vector controlled induction motor drive at 30° lagging power factor conditions. The steady state plots of proposed MSLPWM based vector controlled induction motor drive are shown in Fig 4.49 at 30° lagging power factor condition. Fig 4.50 shows the transients of MSLPWM based drive during the step change in load condition. The transients during the speed reversal for proposed MSLPWM based drive at 30° lagging power factor are shown in Fig. 4.51 and Fig. 4.52. From the above results it can be observed that there is a change in stator current by applying a load torque of 8 N-m. Also there is a change in stator current as well as in torque during speed reversal case. From the q-axis and d-axis currents, it can be observed that the q-axis current is changing according to the torque requirement and d-axis current is almost constant at all operating conditions. The THD of line currents of the proposed algorithms are shown in Fig. 4.53.

4.7 Summary:

The indirect vector control algorithm with hysteresis type current controllers gives more steady state ripple and variable switching frequency operation. To overcome this problem, the CVSPWM algorithm has been used traditionally, which needs angle and sector information.
**Fig. 4.53** Harmonic spectra of proposed MSLPWM based vector controlled induction motor drive at (a) Unity power factor (when DPWM1 is selected)  (b) 30° lagging power factor (when DPWM2 is selected)

Hence, to reduce the complexity involved in the CSVPWM algorithm, this chapter presented novel and simplified PWM sequences based on the imaginary switching times. The proposed approaches do not require angle information. Moreover, to reduce the steady state ripples in torque and current, further HPWM algorithm is presented for vector controlled
induction motor drive. The proposed HPWM algorithm uses the existing DPWM algorithms in conjunction with the CSVPWM algorithm. Finally, the switching loss analysis for the various PWM algorithms is carried out and MSLPWM algorithm is developed for the vector controlled induction motor drive. The harmonic spectra of stator currents for MSLVPWM based vector control of induction motor along with their total harmonic distortion (THD) values are shown in Fig. 4.53. Thus, the proposed PWM algorithms reduce the harmonic distortion and switching losses of the inverter.