CHAPTER 5

OBSERVABILITY ANALYSIS

5.1 Introduction

The analysis of observability is one of the main tasks that need to be addressed in a state estimator. The power system observability confirms that sufficient measurements exist which are distributed throughout the network such that there is a potential solution to the state estimation problem [77]. Thus, the derived observability criterion defines the solvability condition of the state estimation problem. Several researchers have proposed various forms of observability. These forms of observability are (i) Numerical

(ii) Topological (iii) Symbolic and (iv) Hybrid.

5.2 Types of Observability

(i) Numerical Observability: The numerical observability is based on determining the rank of the measurement matrix. Round off errors creep in due to the floating point operations carried out on ill conditioned equations [78].

(ii) Topological Observability: Topological observability involves determining the full rank of the spanning tree. The topological methods are concerned with the determination of such a spanning tree. However, a lot of computational effort is required in the formulation of the spanning tree [78].

(iii) Symbolic Observability: The various entries in the measurement matrix are either a 1 or 0 based on the presence of link between two nodes (1) or its absence (0). This method is fast and simple and usually takes care of redundant measurements [78, 79].

(iv) Hybrid: These methods blend the merits of topological and numerical methods [80]. Hence the inherent disadvantages of these two methods exist in the hybrid method also.
5.3 Determination of Symbolic Observability

When state estimation is implemented in the conventional manner, the value of the gain of measurement matrix is known. The expression is:

\[ \mathbf{I} = \mathbf{V} \mathbf{Y} \]  \hspace{1cm} (5.1)

In the above equation, \( \mathbf{I} \) is a vector representing the currents at the various buses, \( \mathbf{V} \) is a vector representing the voltages at the various buses and \( \mathbf{Y} \) is the admittance matrix of the power system at fundamental frequency. In a normal case the value of \( \mathbf{Y} \) is known. But when blind signal processing is employed, values of \( \mathbf{A} \) and \( \mathbf{s} \) are unknown (\( \mathbf{Y} \) and \( \mathbf{I} \) for harmonic state estimation). Hence the procedure to determine observability is to relax the assumption about the topology of the network. It is assumed that the topology of the network is known. Based on the topology, the symbolic observability method is applied to determine the observability of the power system network.

The sequence of steps for determining the observability of the system is as follows:

1. The KCL equation for \( n \) buses with all harmonics ignored is:

\[ \mathbf{V} = \mathbf{I} \mathbf{Z} \]  \hspace{1cm} (5.2)

On expanding 5.2, the equation is:

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_n
\end{bmatrix} =
\begin{bmatrix}
z_{11} & z_{12} & z_{1n} \\
z_{21} & z_{22} & z_{2n} \\
z_{n1} & z_{n2} & z_{nn}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_n
\end{bmatrix}
\]  \hspace{1cm} (5.3)

2. Consider a reference bus. The reference bus is assumed such that it has no harmonic current source and has minimum number of interconnections (Bus 1 for the SVC system).

3. Construct the symbolic impedance matrix for such a system. This is shown in step 5.
4. Build the admittance matrix for step 3, after accounting for the harmonic current sources only.

5. Eliminate the buses which do not comprise harmonic current sources. In the example, assume harmonic current sources are present at buses 3 to n.

\[
\begin{bmatrix}
V_3 \\
V_4 \\
V_n
\end{bmatrix} =
\begin{bmatrix}
z_{31} & z_{32} & z_{3n} \\
z_{41} & z_{42} & z_{4n} \\
z_{n1} & z_{n2} & z_{nn}
\end{bmatrix}
\begin{bmatrix}
I_3 \\
I_4 \\
I_n
\end{bmatrix}
\]

(5.4)

6. Substitute zero for columns which are devoid of the harmonic current source. The equation 5.3 is modified as:

\[
\begin{bmatrix}
V_3 \\
V_4 \\
V_n
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & z_{3n} \\
0 & 0 & z_{4n} \\
0 & 0 & z_{nn}
\end{bmatrix}
\begin{bmatrix}
I_3 \\
I_4 \\
I_n
\end{bmatrix}
\]

(5.5)

7. For the matrix in equation 5.5, determine the linear independence, based on the rank of the matrix.

### 5.3.1 Four Bus Model Implementation

The determination of symbolic observability is implemented in detail for system1 (Four bus System with SVC and HVDC).

i. Consider bus 1 as the reference bus as per step 2 of Section 5.3.

![Figure 5.1 Bus 1 as Reference for forming the Symbolic Matrix (Four Bus Model)](image-url)
ii. For the four bus model, the symbolic impedance matrix is:

\[
\begin{bmatrix}
V_2 \\
V_3 \\
V_4
\end{bmatrix} =
\begin{bmatrix}
2 & -1 & 0 \\
-1 & 2 & -1 \\
0 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
\]

(5.6)

iii. Applying Step 6 to System 1. The symbolic impedance matrix is:

\[
\begin{bmatrix}
V_2 \\
V_3 \\
V_4
\end{bmatrix} =
\begin{bmatrix}
0 & -1 & 0 \\
0 & 2 & -1 \\
0 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
\]

(5.7)

iv. For the above reduced matrix, the Y matrix is:

\[
\begin{bmatrix}
0.4286 & 0.2143 \\
0.1429 & 0.5714
\end{bmatrix}
\]

(5.8)

From the Y matrix it is found that rows 2 and 3 are linearly independent. Their rank is equal to two = Number of Harmonic Current Sources (two in system 1). Thus, for the four bus system (System1) if measurements are made at bus 2 and bus 3, the system is observable.

5.3.2 IEEE 14 Bus Implementation

The various steps for implementing the observability of the IEEE 14 bus are as follows:

i. Based on step.2 of Section 5.3, assume bus 1 as the reference bus. The IEEE 14 bus system is symbolically represented as:
ii. The symbolic impedance matrix for the IEEE 14 bus with bus 1 as reference is obtained as:

\[
\begin{bmatrix}
V_2 \\
V_3 \\
V_4 \\
V_5 \\
V_6 \\
V_7 \\
V_8 \\
V_9 \\
V_{10} \\
V_{11} \\
V_{12} \\
V_{13} \\
V_{14}
\end{bmatrix} = \begin{bmatrix} 3 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & -1 & 5 & -1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & -1 & 3 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 3 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & -1 & 0 & 4 & -1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 2 & 0
\end{bmatrix} \begin{bmatrix} I_2 \\
I_3 \\
I_4 \\
I_5 \\
I_6 \\
I_7 \\
I_8 \\
I_9 \\
I_{10} \\
I_{11} \\
I_{12} \\
I_{13} \\
I_{14}\end{bmatrix}
\]

(5.9)

iii. Applying step 6 of Section 5.3, the reduced symbolic impedance matrix is:
iv. For this matrix the reduced symbolic admittance matrix, ignoring the other zero elements is:

\[
\begin{bmatrix}
V_2 & V_3 & V_4 & V_5 & V_6 & V_7 & V_8 & V_9 & V_{10} & V_{11} & V_{12} & V_{13} & V_{14} \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
I_2 \\
I_3 \\
I_4 \\
I_5 \\
I_6 \\
I_7 \\
I_8 \\
I_9 \\
I_{10} \\
I_{11} \\
I_{12} \\
I_{13} \\
I_{14} \\
\end{bmatrix}
\]

Next, from the reduced admittance matrix, buses 3, 6 and 8 are linearly independent. Also buses 3, 5, 6 (i); 10, 11, 12 (i); 12, 13, 14 (i); 11, 12, 13 (i) and 3, 13, 14 (ii) are independent. Their rank is three, so they are all linearly independent. Hence, when (i) only linear loads or (ii) linear loads and harmonic current sources are present, the system is observable. Thus, for the system 2 if measurements are made at buses 3, 6 and 8 the system is observable, where only harmonic current sources are present.

5.4 Meter Placement Algorithm

The cost of the harmonic meters is high. Hence, it is difficult to employ harmonic meters at all the buses of a power system. It thus becomes mandatory to install harmonic meters in those buses which generate appreciable harmonic content. Various meter placement algorithms are suggested which help to identify the placement of these harmonic meters. In this work, the linear independence and minimum variance technique is applied to deploy the meters.
It is assumed in this algorithm that all the harmonic sources exist for all \( h \). (i.e.) \( h=5, 7, 11, 13, 17, 19, \) etc.

The meter placement algorithm is formulated as follows:

1. The symbolic impedance matrix is made for the specific power system network.

2. From this matrix a reduced matrix consisting of only the columns of the (assumed) harmonic sources is made.

3. If the resultant matrix has zeros in the entire row, they are eliminated.

4. The admittance matrix of the above matrix is formed.

5. The rows of the admittance matrix are tested for linear independence.

6. The variance of such a combination of rows is determined. The value of minimum variance, in cases where large number of linearly independent rows exists determines the rows where the meters are to be placed on the buses.

### 5.4.1 Four Bus Implementation

The meter placement algorithm is implemented for the Four buses System with SVC and HVDC as harmonic current source at buses 3 and 2 as given below:

i. For the four bus model, the linear independence testing is given in detail in Section 5.3.1.

ii. For rows 2 and 3 the minimum variance is obtained. Hence the harmonic meters are placed at buses 2 and 3.

iii. This agrees well with the assumed location of harmonic current sources at buses two and three.

### 5.4.2 IEEE 14 Bus Implementation

The meter placement algorithm is implemented for IEEE 14 bus system with ASD, SVC and HVDC as harmonic current sources at buses 3, 6 and 8 respectively. The implementation details are as follows:
i. For IEEE 14 bus, the linear independence testing is given in detail in Section 5.3.2.

ii. For rows 3, 6 and 8 the minimum variance is obtained. Hence the harmonic meters are placed at buses 3, 6 and 8.

iii. This agrees well with the assumed location of harmonic current sources at buses three, six and eight.

5.5 Harmonic Source Identification Algorithm

Several techniques of harmonic source identification algorithms are discussed in chapter 2. Majority of these algorithms require measurement values of active and reactive power or the actual impedance matrix [19, 21] for harmonic source identification. The mutual information theory for harmonic source identification in the power system is employed [81] which calls for prior information on the number of harmonic sources present in the power system. The researcher proposes a modification of this method by incorporating a threshold based on the IEEE-512 standards.

The harmonic source identification algorithm is implemented for the harmonic current identification of four bus system and for the IEEE 14 Bus system.

5.5.1 Four Bus Model Implementation

The four bus system used in the work is described in Section 4.5.1. The system has two harmonic sources HVDC at Bus 2 and the SVC at Bus 3.

The details of the method are given below:

Step 1: Obtain the voltage and current values at all buses. Determine the mutual information.

Step 2: Compute the descriptor from the mutual information. The descriptor of the mutual information is defined as the permissible upper and lower limits of mutual information. From the value of the descriptor, estimate the range of the descriptor.

Step 3: Based on the IEEE 512 standards and the range of the descriptor, a constant called as harmonic source threshold (HST) is formulated at the various harmonic frequencies,
(h = 5, 7, 11, 13 and 17) at all the buses of the four bus system and for all the four ICA algorithms. The HST is defined as the limiting value of the specific current value of the harmonic frequency in terms of the fundamental current determined at the point of coupling as per IEEE 519 standards. For the harmonic frequency h = 5 and h = 7, the minimum HST at the point of common coupling is 0.401. Similarly, for the harmonic frequency h = 11 and h = 13, the minimum HST value is 0.211 and for h = 13, the minimum HST value is 0.153. The bus with these HST values is defined as the harmonic injection bus (HIB) or the suspicious bus.

5.5.1.1 Implementation Results

The harmonic source identification algorithm is applied as explained in Section 5.5.1. The results of the implementation for all the four ICA algorithms are given in Table 5.1.

The results of all the ICA algorithms indicate that HIBs exist at Buses 2 and 3. This is true to the actual system considered in Section 4.5.1 where the harmonic sources are considered at Buses 2 and 3.

5.5.2 IEEE 14 Bus Model Implementation

The IEEE 14 bus system employed for harmonic source identification is described in Section 4.5.2. The system has three harmonic current sources – ASD at Bus 3, HVDC at Bus 6 and the SVC at Bus 8. The method of implementation is similar to the four bus model explained in Section 5.5.1. However, since the system is different, the HST values at the various harmonic frequencies vary slightly. The minimum HST values at the point of coupling for h = 5 and h = 7 are 0.932 and the minimum values of HST for h = 11 and h = 13 are 0.875. The minimum HST value of h = 17 is 0.451.

5.5.2.1 Implementation Results

The harmonic source identification algorithm is applied to the IEEE 14 Bus System as explained in Section 5.5.1. The results of the implementation for all the four ICA algorithms are given in Table 5.2.

From the table, it is evident that for all the ICA algorithms HIBs exist at Buses 3, 6 and 8. This is in accordance with the actual system considered in Section 4.5.2 where the harmonic sources are considered at Buses 3, 6 and 8.
5.6 Conclusions

The drawbacks of various types of observability are described. The symbolic observability of the four bus system and the IEEE 14 bus system is determined. A simple meter placement algorithm for these two systems is also suggested. A harmonic source identification algorithm based on the threshold of the IEEE 512 standard depending on the mutual information of the harmonic current and harmonic voltage is developed.