CHAPTER TEN

SUMMARY AND CONCLUSIONS
10.1. INTRODUCTION

The knowledge of mathematics influences the life of man in all aspects and helps mankind in the creation of its history. The advent of automation and cybernetics marking the beginning of the new scientific industrial revolution in our country makes it all the more imperative to devote special attention to the study of mathematics.

Advances in psychological research, recognition of individual differences, changing socio-economic and political conditions, scientific and technological advancements, improved techniques of teaching, curriculum development and evaluation are some of the significant factors which have contributed to the revision of mathematics courses time and again. Recent trends in the field of curriculum, learning and teaching theories and evaluation and the objectives of teaching mathematics force on us the task of building up conceptual understanding of the discipline in the pupils instead of concentrating all efforts on the drilling of the computational processes alone.

10.2. THE PRESENT STUDY

The present investigation was carried out with a purpose of studying the development of algebraic concepts in pupils at the Junior Secondary stage in relation to the
differences due to grades, sex and levels of intelligence with implications for improving the syllabuses, for helping teachers to evaluate their own methods of teaching algebra, and for assisting counsellors in providing proper guidance to the pupil.

The choice of algebraic concepts at the Junior Secondary stage as the topic for investigation had special appeal primarily because algebra is introduced at this stage only; it forms an important branch of mathematical studies even at later stages and its teaching at present is in miserable condition, and secondly, because not much work of this nature on the subject is available not only in India but also in the world literature.

The study was advanced on the three hypotheses given below:

a. The levels of instruction, varying according to the content for each grade, are responsible for the variations in the understanding of algebraic concepts from one grade to another.

b. Sex-differences exist in relation to the development of algebraic concepts.

c. The development of algebraic concepts in pupils depends on their levels of intelligence, that is, the superior intelligence accounts for better understanding of algebraic concepts.
10.3. PROCEDURE AND TECHNIQUES

10.3.1. RESEARCH METHOD & TOOLS

The status-survey method was employed, using a cross-sectional approach and taking sections of pupils from each grade at the Junior Secondary stage with the help of Algebraic Concepts Test specially developed for tracing the development of algebraic concepts. Jalota's Group Test of General Mental Ability (1/60) was used for evaluating the intellectual status of pupils. Achievement scores in algebra used for validating the Algebraic Concept Test were provided by the teachers of mathematics from their respective institutions.

10.3.2. SAMPLE

The sampling method was resorted to at three stages; (i) for the try-out of the preliminary draft of the Algebraic Concepts Test; (ii) for finding out the reliability and validity of the Algebraic Concepts Test; and (iii) for completing the final phase of the study, that is, to collect data for studying the developmental trends of algebraic concepts from one grade to another in relation to differences due to sex and levels of intelligence. In the first two stages rigorous methods of randomization were not employed. For the try-out of the test five institutions in different parts of the Panjab,
Haryana and the Union Territory of Chandigarh were taken while reliability studies were conducted on the pupils drawn from two institutions only, such sample being considered enough for the purpose. The method of sampling for final data collection was based on the process of multi-stage randomization of clusters and one classroom was taken as the unit of sampling in order to collect data for completing the final phase of the study. A total sample of 2000 was taken at the three stages, out of which some cases had to be omitted due to incomplete information and finally 1863 cases were involved in the study. The split of the sub-samples at the three stages was 280, 283 and 1300 including girls and boys both.

10.3.3. DEVELOPING THE ALGEBRAIC CONCEPTS TEST

In order to evaluate the understanding of algebraic concepts in pupils relevant to the syllabuses at the Junior Secondary stage in the Panjab, Haryana and Chandigarh, the Algebraic Concepts Test was locally developed in the absence of any other standardized test of this nature. The preliminary draft of the test consisted of 150 items split into seven broad categories of concepts corresponding to generalized numbers, directed numbers, equations, parentheses, substitution, exponents and graphs. The try-out of the preliminary test was conducted on a sample of 280 representing the three grades,
namely, sixth, seventh and eighth. Item-difficulty and item-discrimination were used for analysing the items. Minimum proportion of responses for each item was found out for each grade which indicated knowledge of items beyond chance success by the groups. Only those items were selected for the final test which discriminated between the upper and lower 27 percent cases and which had item-difficulty values exceeding the minimum proportion indicating knowledge beyond chance success. Finally, 70 items were included in the final form of the Algebraic Concepts Test. The test was prepared in Hindi and it had all multiple-choice type items. The time-limit for completing the test was found out to be 55 minutes.

10.3.1. RELIABILITY OF A.C. TEST

The re-test reliability of the test was worked out. Internal consistency in terms of inter-concept correlations was also found out. The reliability studies were based on a sample of 283 (133 girls and 150 boys) representing three grades, namely, six, seven and eight (actual sample was taken from the seventh, eighth and ninth grades in the beginning of the session). The modal chronological ages of the pupils at these grades were 12+, 13+ and 14+ respectively. The following conclusions were drawn for the reliability results of the test.
1) The coefficients of reliability for different groups, namely, (i) girls in each grade, (ii) boys in each grade, (iii) girls and boys in each grade and (iv) total sample ranged between .77 and .92 which speak of the soundness of the Algebraic Concepts Test from the reliability point of view.

ii) The coefficients of reliability were comparatively lower for the sixth grade pupils and were found in the increasing order from sixth onwards to the eighth grade. The increasing trend of the coefficients of reliability was perhaps due to the reason that the understanding of algebraic concepts was better with higher grade pupils and the degree of fluctuation of scores decreased in the two administrations of the test with increased amount of knowledge.

iii) The reliability coefficients were comparatively lower for the girls than for boys in the same grade except in the case of the eighth grade pupils. The reason seems to be closely connected with aptitude for algebra. Higher coefficient of reliability in case of the eighth grade girls (pupils taken from the ninth grade) was probably due to the selective nature of the group of girls opting for mathematics willingly unlike other groups. In the ninth grade girls may opt for mathematics or leave that altogether in lieu of some other elective, so only those girls take up
mathematics who have an aptitude for it. Algebraic aptitude, therefore, seems to have influenced the reliability coefficient.

iv) The coefficients of reliability were .86 for all girls, .89 for all boys and .92 for the total sample. The coefficients were high enough to justify the sensitivity of the test for variability as it was likely to be greater when all the three grades were thrown together than what it could be with any one group (due to wider range of age and increase in the range of scores).

v) The standard error of estimates was least for the seventh grade girls and maximum for the sixth grade girls and boys both.

vi) The mean scores on the two administrations of the A.C. Test were, however, significantly different for the sixth and seventh grade pupils. Two reasons were apparently responsible for disturbing stability associated with retest reliability, namely, (i) perhaps practice given to the pupils in revising their previous grade-work during the period of two administrations helped them to score higher on the second administration, and (ii) the familiarity with the test boosted up their scores on the second administration.

vii) The mean scores of pupils taken for reliability studies were higher than for the sample taken for locating developmental trends of algebraic concepts. The explanation here is very simple. The reliability studies were based on
the pupils taken from two institutions alone but in the latter case, the sample was broad-based. It may, however, be pointed out at this stage that the two institutions taken for reliability studies were the ones which showed consistently good results at the university examinations during the previous years.

vii) The internal consistency taken in the sense of inter-concept correlations was low varying between 0.22 and .65. It speaks of heterogeneity of the test, partly due to the different mental-functions being measured by the different sub-tests (concepts) and partly due to uneven stress on various concepts in teaching at different grade levels.

10.3.3.2 VALIDITY OF THE A.C. TEST

Concurrent and factorial validities were established for the A.C. Test. Content validation was employed in the process of test-development. The following conclusions were drawn:

1) The mean-scores on the A.C. Test differed significantly between sixth and seventh, sixth and eighth, and seventh and eighth grade pupils and were found in the increasing order from sixth onwards to the eighth grade, thereby establishing the sensitivity of the test for discriminations which is one of the essential conditions for a test to be valid.
ii) The results of cluster-analysis showed that all the sub-tests (concepts) appeared on only one common factor.

iii) The centroid method of factor-analysis, however, showed the existence of two common factors. The first factor identified with 'Algebraic Aptitude' contributed to 34.82 percent of the common factor variance. The second factor identified with 'symbolic substitution' accounted for 14.84 percent of the common factor variance. The results established the univocalness of the test to be very high which accounted for nearly half of the variance in most of the sub-tests (concepts) excepting concepts B, E and D.

iv) The concurrent validity coefficients against teachers' evaluations varied from .496 to .708. These values were deemed to be quite satisfactory, specially when the Algebraic Concepts Test was validated against the external criterion of teachers' evaluation which, it was feared, did not measure conceptual understanding of algebra exclusively, but tested the manipulative skills also in algebra.

10.4. DEVELOPMENT OF ALGEBRAIC CONCEPTS IN PUPILS AT THE JUNIOR SECONDARY STAGE

After having finalized the A.C. Test, it was administered along with the General Intelligence Test to a sample of 1400 pupils randomized in clusters from the institutions in the Panjab, Haryana and Chandigarh but
finally only 1300 cases were effectively involved in the study and others had to be deleted due to incomplete information or due to wide variations from the modal ages of 12+\ , 13+\ and 14+\ for the pupils of grades six, seven and eight respectively.

The data were analysed firstly to test the three hypotheses together by making use of the three-way analysis of variance and four-way analysis of variance. The three variables involved in the former were as follows:

Grades varied in three ways, sex varied in two ways and levels of intelligence varied in two ways the high and the low groups. The criterion variable was the total algebraic concepts scores which were mutually independent of one another in different cells. In the latter, the variable of concepts which was varied in seven ways was also included besides the three variables taken in the three-way analysis of variance. The criterion variables in the four-way analysis were the scores earned by pupils on individual concepts. The distribution of cases was again done mutually exclusive of one another in different cells.

Each of the three hypotheses was further tested by employing different techniques. The grade-differences were examined by using significance of difference between mean scores on the total algebraic concepts and on individual
concepts. Ogives were also drawn. The sex differences were tested by employing the technique of significance of difference between the mean scores earned on total test as well as on each concept separately by girls and boys in different grades and in the total sample. The effect of intelligence on the understanding of algebraic concepts was evaluated with the help of correlational technique. Item-difficulty values were found out for each group in order to evaluate the knowledge of items by the groups beyond chance success.

The analyses of data led to draw the following conclusions:

a. The contribution to variance due to grades was found to be significant at .01 level in the three-way and four-way classifications of the analysis of variance and the mean scores on the Algebraic Concepts Test showed an increase from the sixth to the eighth grade. Detailed item-wise analysis also revealed that the algebraic concepts tend to develop from lower to the higher grades. The understanding of most of the items was found to be better in the next higher grades than the previous ones which was supported by an upward increase in the item-difficulty values from grade six onwards to grade eight. The developmental trends based on the mean scores of each grade
for each concept and the total concepts indicated incline from lower to the higher grades in each concept except for concept B on directed numbers in which case the sixth grade pupils showed better understanding of it than the seventh grade pupils. It does not, however, seem to be a major factor in upsetting the first hypothesis, namely, grade to grade differences exist in the understanding of algebraic concepts due to the variations in the subject-matter and the instruction programme. In the case of the concepts where the differences did not appear significant between the mean scores of sixth and seventh, sixth and eighth, and seventh and eighth grade pupils, perhaps either the concepts were not at all introduced to the pupils through the instruction programme or they were inadequately treated in the class rooms.

b) The contribution to variance due to sex was again found to be significant at .01 level in the three-way and four-way classifications of analysis of variance which indicated significant sex-differences in the understanding of algebraic concepts. Girls and boys differed significantly at each grade-level as also in the total sample with regard to total algebraic concepts scores and in each case the mean scores of boys was higher than that of girls in the corresponding group. Taking the concepts one at a time,
it was observed that boys excelled girls in the understanding of all concepts except in concept E in which case eighth-grade girls had a better understanding of it than boys, and in most cases these differences were statistically significant. Item-wise analysis showed that boys in general had knowledge of greater number of items at each grade-level with minor exceptions. They had high item-difficulty values for most of the items than the girls had. In the context of these findings, it can be safely inferred that the boys had a tendency to excel girls in the understanding of algebraic concepts.

c) The third hypothesis was tested to trace the development of algebraic concepts in girls and boys at each grade level in relation to intelligence. It was established that the three-way and four-way classification of analysis of variance yielded significant F-ratios due to intelligence. Grade-wise correlations between total algebraic concepts scores and intelligence scores were all significant at .01 level. Most of the correlations worked out concept-wise were also significant. The results led to infer that superior intelligence accounted for higher scores on the algebraic concepts. Developmental trends on algebraic concepts for girls and boys of lower and upper 27 percent cases as classified according to intelligence test scores showed that girls and boys of higher intelligence in each grade,
as well as in the total sample, invariably had better mean scores than the low intelligence group of pupils corresponding to particular sex and grades. In the seventh grade, there seemed to be maximum difference between the upper and lower intelligence groups which explained the significant interaction due to grades and intelligence in the three-way analysis of variance as also in the four-way analysis of variance.

10.5. IMPLICATIONS AND APPLICATIONS OF THE PRESENT STUDY

The findings of the present investigation have very important implications for improving the quality of instruction of algebra at the Junior Secondary stage which may be summarized as below:

a) The algebraic concepts tend to develop from lower to the higher grades and all concepts are not understood simultaneously. It would be appropriate to include only those operations relevant to the concepts emerging at various grade levels in the syllabuses. The concepts which are not understood at lower levels may be deferred to the higher levels but the concepts which are understood earlier need not be postponed till the higher grade.

b) Sex-differences exist in the acquisition of algebraic concepts. There seems to be a possibility of having different syllabuses for girls and boys at the
junior secondary stage. They may be made more varied for boys than for girls. This suggestion, however, does not deprive those girls from taking higher courses who show an aptitude for algebra.

c) Levels of intelligence account for significant differences in the understanding of algebraic concepts and therefore, there should be more challenging syllabuses for the gifted than for averages or below averages.

d) The research findings revealed that the mean scores of pupils at each grade-level were not very high even though most of the items had emerged by the eighth grade. It is a reflection on poor teaching in schools; probably because teachers themselves are not aware of these concepts or do not stress the acquisition of concepts at this stage. In our schools, there is more emphasis on skills in algebra than on concepts. Programme-teaching may be organised around the concepts which are understood by different grade pupils beyond chance success. Adequate emphasis on the teaching of concepts may be laid in the class rooms. There is no harm if teachers themselves are acquainted with algebraic concepts which they are supposed to transmit to the pupils at the junior secondary stage and the training programme of prospective junior secondary school teachers may be re-organised in this light.

e) The Algebraic Concepts Test may be administered to the inservice and prospective junior secondary school teachers in order to evaluate their algebraic competence.
10.3. SUMMARY AND FURTHER RESEARCH

The present investigation was a very much needed survey of the existing status in the teaching of algebra, but its scope was limited because of the necessity of the initial work of developing the Algebra Concepts Test. The following possibilities for further research seem to be of considerable importance:

d) Organized teaching of algebra with a view to emphasize conceptual understanding besides acquiring skills in certain operations should be done in one or two experimental schools and then the results should be evaluated in order to have a better picture of the capabilities of pupils to understand algebraic concepts.

e) Sex-differences may be worked out after making adjustments for intelligence and controlling most of the environmental factors.

f) A study of understanding of algebraic concepts in relation to socio-economic status of pupils may also be conducted.

g) Effort may be made to find out if algebraic concepts can be mastered at a level lower than where algebra is started routinely, that is, the readiness to acquire such concepts should be tested at early levels to reorganize the introduction of algebra in the courses.