CHAPTER-IV

PARALLEL REDUCT AND ITS PROPERTIES IN

OBJECT-ORIENTED ROUGH SET MODEL
4.1. Introduction

This chapter deals with a new type of Reduct in the Object-Oriented Rough Set Model which is called Parallel Reduct. In the Object–Oriented Rough Set Models, objects are treated as instances of classes, and structural hierarchies among objects are illustrated based on ‘is-a’ relationship and ‘has-a’ relationship [30]. In this chapter, two definitions of Parallel Reducts were given, and the properties were introduced. Algorithms are developed and presented to get Parallel Reduct and Core of Parallel Reduct in the Object-Oriented Rough set model.

The concepts of Dynamic Reducts delas to solve the problem of large amount of classes or incremental classes were presented. Parts of data are selected to process reduction, and then intersection is opted for all Reducts as stable Reducts in the Object-Oriented Rough Set Model. The method must count all of the Reducts in these Object-Oriented Rough Set Models. Its time complexity is NP-Complete. Object-Oriented Rough Set Models is later extended and a new method to compute the stable Reducts in a series of Sub-Object-Oriented Rough Set Models called Parallel Reduct is introduced. These Parallel Reducts have all of advantages of Dynamic
Reducts but avoid the intersection of all of Reducts in a series Sub-Object-Oriented Rough Set Models may be empty. The time complexity of Dynamic Reducts in the Object-Oriented Rough Set Model is NP-complete and the way to get Dynamic Reducts is not complete because the intersection of all of Reducts in a series Sub-Object-Oriented Rough Set Models may be empty. The time complexity of Parallel Reducts could be the same as that of the algorithm of Reducts in the Object-Oriented Rough Set Model. It is shown that one can generate always the Parallel Reducts with any cases. Just like Dynamic Reducts, Parallel Reducts could deal with increased number of classes, dynamic data and tremendously large number of classes, and one can simultaneously compute to get Parallel Reducts in the Object-Oriented Rough Set Model.

The Parallel Reducts and the properties from the dependence of classes or classes with names along with the significance were proposed along with the algorithms to count Parallel Reducts and the Core of Parallel Reducts in the Object-Oriented Rough Set Model.

**Related Definitions**

The following definition gives positive region

**Definition 4.1:** Let $OORS(C, N, O)$ be the Object-Oriented Rough Set Model

Where $C = (C, \supseteq_C, \subseteq_C)$, $N=(N, \supseteq_N, \subseteq_N)$, $O=(O, \supseteq_O, \subseteq_O)$ be the well-defined class, name and object structures, respectively, $OORS/N_{DECI} \{Y1, Y2, ..., Yp\}$, where
$Y_j$ is an equivalence class. The positive region is defined as

$$\text{POS}_B(N_{\text{DEC}}) = \bigcup_{Y_i \in \text{OORS}/N_{\text{DEC}}} \text{POS}_B(Y_i)$$

(4.1)

Positive region is union of all lower approximations with respect to definition of lower approximation defined in (1.6) of chapter -1.

**Example 10:** This example is in continuation of Example-3. The Positive Region for this Object-Oriented Rough Set Model is as follows:

$$\text{POS}_{\text{college}}(\text{courses}) = \{\text{university1, university2, university3, university4, university5, university6, university7, university8, university9, university10, university11, university12, university13, university14}\}$$

$\text{Eeq}(1)$, where $\text{Eeq}(i)$ Represents Example Equation $i$

The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.

$$R = \{\text{Government University, Central University, Deemed University, Foreign University, Government University.department, Central University.department, Deemed University.department, Foreign University.department}\}$$

$\text{Eeq}(2)$

$$\text{POS}_{\text{college}}(\text{department}) = \{\text{university1, university2, university3, university4, university5, university6, university7, university8, university9, university10, university11, university12, university13, university14}\}$$

$\text{Eeq}(3)$
The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.

\[
R = \left\{ \text{GovernmentUniversity, CentralUniversity, DeemedUniversity, } \right. \\
\left. \text{ForeignUniversity, GovernmentUniversity.department, } \right. \\
\left. \text{CentralUniversity.department, DeemedUniversity.department, } \right. \\
\left. \text{ForeignUniversity.department} \right\}
\]

\[ Eeq(4) \]

\[ POS_{department}(courses) = \left\{ \text{university1, university2, university3, university4, university5, } \right. \\
\left. \text{university6, university7, university8, university9, university10, } \right. \\
\left. \text{university11, university12, university13, university14} \right\} \]

\[ Eeq(5) \]

The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.

\[
R = \left\{ \text{GovernmentUniversity, CentralUniversity, DeemedUniversity, } \right. \\
\left. \text{ForeignUniversity, GovernmentUniversity.department, } \right. \\
\left. \text{CentralUniversity.department, DeemedUniversity.department, } \right. \\
\left. \text{ForeignUniversity.department} \right\}
\]

\[ Eeq(6) \]

\[ POS_{department}(college) = \left\{ \text{university1, university2, university3, university4, university5, } \right. \\
\left. \text{university6, university7, university8, university9, university10, } \right. \\
\left. \text{university11, university12, university13, university14} \right\} \]

\[ Eeq(7) \]

The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.
\( R = \{ \text{Government University, Central University, Deemed University,} \) \\
\( \quad \text{Foreign University, Government University.department,} \) \\
\( \quad \text{Central University.department, Deemed University.department,} \) \\
\( \quad \text{Foreign University.department} \) \\
\( \text{Eeq(8)} \)

\( POS_{\text{course}} (\text{college}) = \{ \text{university1, university2, university3, university4, university5, university6, university7, university8, university9, university10, university11, university12, university13, university14} \} \)

\( \text{Eeq(9)} \)

The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.

\( R = \{ \text{Government University, Central University, Deemed University,} \) \\
\( \quad \text{Foreign University, Government University.department,} \) \\
\( \quad \text{Central University.department, Deemed University.department,} \) \\
\( \quad \text{Foreign University.department} \) \\
\( \text{Eeq(10)} \)

\( POS_{\text{course}} (\text{department}) = \{ \text{university1, university2, university3, university4, university5, university6, university7, university8, university9, university10, university11, university12, university13, university14} \} \)

\( \text{Eeq(11)} \)

The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.

\( R = \{ \text{Government University, Central University, Deemed University,} \) \\
\( \quad \text{Foreign University, Government University.department,} \) \\
\( \quad \text{Central University.department, Deemed University.department,} \) \\
\( \quad \text{Foreign University.department} \) \\
\( \text{Eeq(12)} \)
The following definition gives ratio of positive region to total number of objects.

**Definition 4.2:** Let $OORS(C, N, O)$ be the Object-Oriented Rough Set Model where $C = (C, C, ⊇_C)$, $N = (N, N, ⊇_N)$, $O = (O, O, ⊇_O)$ well-defined class, name and object structures, respectively. It can be said then, $N_{CON}$ depends on $N_{DEC}$ to a degree $k (0 ≤ k ≤ 1)$, if

$$K = \gamma (N_{CON}, N_{DEC}) = \frac{\text{POS}_{N_{CON}} (N_{DEC})}{|OORS|} \quad (4.2).$$

$\gamma(department, college) = 14/14 = 1$

$\gamma(department, courses) = 14/14 = 1$

$\gamma(college, courses) = 14/14 = 1$

**4.2. Parallel Reducts in the Object-Oriented Rough Set Model**

**Definition 4.3:** Let $OORS(C, N, O)$ be the Object-Oriented Rough Set Model where $C = (C, C, ⊇_C)$, $N = (N, N, ⊇_N)$, $O = (O, O, ⊇_O)$ be the well defined class, name, object structures, respectively. Let $OORS'(C', N', O')$ be the Sub-Object-Oriented Rough Set Model, where $C' = (C', C', ⊇_{C'})$, $N' = (N', N, ⊇_{N'})$, $O' = (O', O, ⊇_{O})$ be the well defined class, name, object structures respectively. Let $C' \subseteq C$, $N' \subseteq N$, $O' \subseteq O$, $OORS' \subseteq OORS$, $OORS'$ is called Sub-Object-Oriented Rough Set model of $OORS$. 

109
Let \( \rho(OORS) \) be the set of all Sub-Object-Oriented Rough Set model of \( OORS \). The intersection of all Reducts in a series Sub-Object-Oriented Rough Set Models may be empty. These empty reducts are excluded by the type of reducts are called Parallel Reducts. The symbol \( F \) denotes a non-empty subset of \( \rho(OORS) \) which excludes the empty element \( \emptyset \) i.e \( \emptyset \not\in F \).

The following definition gives Parallel Reducts

**Definition 4.4:** Let \( OORS(C, N, O) \) be the Object-Oriented Rough Set Model where \( C = (C, \mathcal{C}, \supseteq) \), \( N = (N, \mathcal{N}, \supseteq) \), \( O = (O, \mathcal{O}, \supseteq) \) be the well defined class, name, object structures, respectively. Let \( OORS'(C', N', O') \) be the Sub-Object-Oriented Rough Set Model, where \( C' = (C', \mathcal{C}, \supseteq) \), \( N' = (N', \mathcal{N}, \supseteq) \), \( O' = (O', \mathcal{O}, \supseteq) \) be the well defined class, name, object structures respectively. Let \( C' \subseteq C, N' \subseteq N, O' \subseteq O, OORS' \subseteq OORS \), \( OORS' \) is called Sub-Object-Oriented Rough Set model of \( OORS \).

Let \( \rho(OORS) \) be the set of all Sub-Object-Oriented Rough Set model of \( OORS \). Let \( P \subseteq N_{CON} \) is called a Parallel Reduct of \( F \) if \( P \) satisfies the following two conditions.

1) For any sub-object-oriented rough set model \( OORS' \in F \) it satisfies

\[
POS_{\rho}(OORS', d) = POS_{\emptyset}(OORS', d).
\]

2) There exist at least a Sub-Object-Oriented Rough Set Model \( OORS' \in F \) such that

\[
POS_{\rho}(OORS', d) \neq POS_{\emptyset}(OORS', d). \tag{4.3}
\]

Where \( P = n_i \in H_N(n), n_i \in N_{CON} \).
Q = n_i \in H_N(n), n_i \in N_{CON},

d = n \in H_N(n), n \in N_{DEC}.

The Parallel Reduct is a set of the least classes or classes with names which keep the positive regions of all sub-object-oriented rough set models in F \( \subseteq \rho(OORS) \). There may be more than one Parallel Reduct in the Rough Set Model of F.

**Example 11**: This example is in continuation of example 10. The Parallel Reduct in this Model is as follows:

If university1 object is deleted, then object set for this Sub-Object-Oriented Rough Set Model is as follows.

\[
\{ \text{university2, university3, university4, university5, university6, university7, university8, university9, university10, university11, university12, university13, university14} \}
\]

Eq(13)

\[\text{POS}_{\text{college}}(\text{courses}) = \{ \text{university2, university3, university4, university5, university6, university7, university8, university9, university10, university11, university12, university13, university14} \}\]

Eq(14)

The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.
\[ R' = \{ \text{GovernmentUniversity, CentralUniversity, DeemedUniversity}, \]
\[ \text{ForeignUniversity, GovernmentUniversity.department,} \]
\[ \text{CentralUniversity.department, DeemedUniversity.department,} \]
\[ \text{ForeignUniversity.department} \] \]

Eq(15)

\[ \text{POS}_{\text{college}} \left( \text{department} \right) = \{ \text{university2, university3, university4, university5, university6,} \]
\[ \text{university7, university8, university9, university10, university11,} \]
\[ \text{university12, university13, university14} \} \]

Eq(16)

The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.

\[ R' = \{ \text{GovernmentUniversity, CentralUniversity, DeemedUniversity}, \]
\[ \text{ForeignUniversity, GovernmentUniversity.department,} \]
\[ \text{CentralUniversity.department, DeemedUniversity.department,} \]
\[ \text{ForeignUniversity.department} \] \]

Eq(17)

\[ \text{POS}_{\text{department}} \left( \text{courses} \right) = \{ \text{university2, university3, university4, university5, university6,} \]
\[ \text{university7, university8, university9, university10, university11,} \]
\[ \text{university12, university13, university14} \} \]

Eq(18)

The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.
The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.

\[ R' = \{ \text{GovernmentUniversity}, \text{CentralUniversity}, \text{DeemedUniversity}, \text{ForeignUniversity}, \text{GovernmentUniversity.department}, \text{CentralUniversity.department}, \text{DeemedUniversity.department}, \text{ForeignUniversity.department} \} \]  

Eeq(19)

\[ \text{POS}_{\text{department}}(\text{college}) = \{ \text{university2}, \text{university3}, \text{university4}, \text{university5}, \text{university6}, \text{university7}, \text{university8}, \text{university9}, \text{university10}, \text{university11}, \text{university12}, \text{university13}, \text{university14} \} \]  

Eeq(20)

The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.

\[ R' = \{ \text{GovernmentUniversity}, \text{CentralUniversity}, \text{DeemedUniversity}, \text{ForeignUniversity}, \text{GovernmentUniversity.department}, \text{CentralUniversity.department}, \text{DeemedUniversity.department}, \text{ForeignUniversity.department} \} \]  

Eeq(21)

\[ \text{POS}_{\text{courses}}(\text{college}) = \{ \text{university2}, \text{university3}, \text{university4}, \text{university5}, \text{university6}, \text{university7}, \text{university8}, \text{university9}, \text{university10}, \text{university11}, \text{university12}, \text{university13}, \text{university14} \} \]  

Eeq(22)

The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.

\[ R' = \{ \text{GovernmentUniversity}, \text{CentralUniversity}, \text{DeemedUniversity}, \text{ForeignUniversity}, \text{GovernmentUniversity.department}, \text{CentralUniversity.department}, \text{DeemedUniversity.department}, \text{ForeignUniversity.department} \} \]  

Eeq(23)
\( \text{POS}_{\text{courses}}(\text{department}) = \{ \text{university } 2, \text{university } 3, \text{university } 4, \text{university } 5, \text{university } 6, \text{university } 7, \text{university } 8, \text{university } 9, \text{university } 10, \text{university } 11, \text{university } 12, \text{university } 13, \text{university } 14 \} \)

Eeq(24)

The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.

\[ R' = \{ \text{Government University, Central University, Deemed University,} \]
\[ \text{Foreign University, Government University.department,} \]
\[ \text{Central University.department, Deemed University.department,} \]
\[ \text{Foreign University.department} \] \]

Eeq(25)

If single object is deleted from the object set, say university1 or university2…………………university14, then one of the Reduct is same as that we have obtained earlier in Example 4 of Chapter-1.

The one of the Reduct of this Sub-Object-Oriented Rough Set model is as follows.

\[ R' = \{ \text{Government University, Central University, Deemed University,} \]
\[ \text{Foreign University, Government University.department,} \]
\[ \text{Central University.department, Deemed University.department,} \]
\[ \text{Foreign University.department} \] \]

Eeq(26)

If any two objects are deleted, the Reduct is same as that was obtained earlier in Example-4 of Chapter-1 except for deletion of two objects university9, university10.
If university9, university10 objects are deleted, then object set for this Sub-Object-Oriented Rough Set Model is as follows.

\[
\{ \text{university} 1, \text{university} 2, \text{university} 3, \text{university} 4, \text{university} 5, \text{university} 6, \\
\text{university} 7, \text{university} 8, \text{university} 11, \text{university} 12, \text{university} 13, \text{university} 14 \}
\]

\text{Eeq}(27)

\[
\text{POS}_{\text{college}}(\text{courses}) = \{ \text{university} 1, \text{university} 2, \text{university} 3, \text{university} 4, \text{university} 5, \\
\text{university} 6, \text{university} 7, \text{university} 8, \text{university} 11, \text{university} 12, \\
\text{university} 13, \text{university} 14 \}
\]

\text{Eeq}(28)

The one of the Reduct of this Sub-Object-Oriented Rough Set model is as follows.

\[
R' = \{ \text{Government University}, \text{Central University}, \text{Foreign University}, \\
\text{Government University.department}, \text{Central University.department}, \\
\text{ForeignUniversity.department} \}
\]

\text{Eeq}(29)

The Reduct obtained in the \text{Eeq}(29), is called Parallel Reduct of \text{Eeq}(15) since positive region while considering \text{Eeq}(27) is different from \text{Eeq}(14) where as the positive region considering \text{Eeq}(14) is equal to the one obtained through \text{Eeq}(16).

If objects university1, university2, university3, university4 are deleted, then the resulting object set for the Sub-Object-Oriented Rough Set Model is
The one of the Reduct of this Sub-Object-Oriented Rough Set model is as follows:

\[
\{ \text{university } 5, \text{university } 6, \text{university } 7, \text{university } 8, \text{university } 9, \text{university } 10, \text{university } 11, \text{university } 12, \text{university } 13, \text{university } 14 \}
\]

Eeq(30)

\[
POS_{\text{college}}(\text{courses}) = \{ \text{university } 5, \text{university } 6, \text{university } 7, \text{university } 8, \text{university } 9, \text{university } 10, \text{university } 11, \text{university } 12, \text{university } 13, \text{university } 14 \}
\]

Eeq(31)

The Reduct obtained in the Eeq(32), is called Parallel Reduct of Eeq(29) since positive region while considering Eeq(30) is different from Eeq(28) where as the positive region considering Eeq(14) is equal to the one obtained through Eeq(16).

The Reduct obtained in the Eeq(32), is called Parallel Reduct of Eeq(15) since positive region while considering Eeq(30) is different from Eeq(14) where as the positive region considering Eeq(14) is equal to the one obtained through Eeq(16).
If university5, university6, university7, university8 objects are deleted, then the resulting object set for the Sub-Object-Oriented Rough Set Model is

\[
\{ \text{university 1, university 2, university 3, university 4, university 9, university 10, university 11, university 12, university 13, university 14} \}
\]

Eq(33)

\[
POS_{\text{college}}(\text{courses}) = \{ \text{university 1, university 2, university 3, university 4, university 9, university 10, university 11, university 12, university 13, university 14} \}
\]

Eq(34)

The one of the Reduct of this Sub-Object-Oriented Rough Set model is as follows:

\[
R' = \{ \text{GovernmentUniversity, DeemedUniversity, ForeignUniversity, GovernmentUniversity.department, DeemedUniversity.department, ForeignUniversity.department} \}
\]

Eq(35)

The Reduct obtained in the Eq(35), is called Parallel Reduct of Eq(32) since positive region while considering Eq(33) is different from Eq(31) where as the positive region considering Eq(14) is equal to the one obtained through Eq(16).

The Reduct obtained in the Eq(35), is called Parallel Reduct of Eq(29) since positive region while considering Eq(33) is different from Eq(28)
where as the positive region considering Eq(14) is equal to the one obtained through Eq(16).

The Reduct obtained in the Eq(35), is called Parallel Reduct of Eq(26) since positive region while considering Eq(33) is different from Eq(24) where as the positive region considering Eq(14) is equal to the one obtained through Eq(16).

If university11, university12, university13, university14 objects are deleted, then the resulting object set for the Sub-Object-Oriented Rough Set Model is

$$\begin{align*}
\{ & \text{university1, university2, university3, university4, university5, university6, university7, university8, university9, university10} \\
& \}
\end{align*}$$

Eq(36)

$$POS_{\text{college}}(\text{courses}) = \begin{cases} 
\text{university1, university2, university3, university4, university5, university6, university7, university8, university9, university10} \\
\end{cases}$$

Eq(37)

The one of the Reduct of this Sub-Object-Oriented Rough Set model is as follows:

$$R' = \begin{cases} 
\text{GovernmentUniversity, CentralUniversity, DeemedUniversity, GovernmentUniversity.department, CentralUniversity.department, DeemedUniversity.department} \\
\end{cases}$$

Eq(38)
The Reduct obtained in the Eq(38), is called Parallel Reduct of Eq(35) since positive region while considering Eq(36) is different from Eq(34) whereas the positive region considering Eq(14) is equal to the one obtained through Eq(16).

The Reduct obtained in the Eq(38), is called Parallel Reduct of Eq(32) since positive region while considering Eq(36) is different from Eq(31) whereas the positive region considering Eq(14) is equal to the one obtained through Eq(16).

The Reduct obtained in the Eq(38), is called Parallel Reduct of Eq(29) since positive region while considering Eq(36) is different from Eq(25) whereas the positive region considering Eq(14) is equal to the one obtained through Eq(16).

The Reduct obtained in the Eq(38), is called Parallel Reduct of Eq(26) since positive region while considering Eq(36) is different from Eq(25) whereas the positive region considering Eq(14) is equal to the one obtained through Eq(16).

Some of the Parallel Reducts are from the Eq(28), Eq(32), Eq(35), Eq(38) are
Another approach for Calculating Positive Region

Given \( P \subseteq N_{CON}, P \in H_N(n), \)

\( Q \in N_{DEC}, Q \in H_N(n), \)

\[
POS_P(Q) = \bigcup_{P=1}^{m} \left\{ x \in OORS : [x]_Q \subseteq [x]_P \right\}
\] (4.4)

The result representing corresponding class and its class names which are other than decision names and condition names belongs to \( H_N(n) \).
Example 12: This example is in continuation of example 11. The Parallel Reduct in the Object-Oriented Rough Set Model is as follows:

Positive region is considered with respect to 4.4.

If university1 object is deleted, then object set for this Sub-Object-Oriented Rough Set Model is as follows.

\[
\text{POS}_{\text{college}}(\text{courses}) = \{\text{university} 2, \text{university} 3, \text{university} 4, \text{university} 5, \text{university} 6, \text{university} 7, \text{university} 8, \text{university} 9, \text{university} 10, \text{university} 11, \text{university} 12, \text{university} 13, \text{university} 14\}
\]

Eeq(40)

\[
\text{POS}_{\text{college}}(\text{department}) = \{\text{university} 2, \text{university} 3, \text{university} 4, \text{university} 5, \text{university} 6, \text{university} 7, \text{university} 8, \text{university} 9, \text{university} 10, \text{university} 11, \text{university} 12, \text{university} 13, \text{university} 14\}
\]

Eeq(41)

The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.

\[
R' = \{\text{Government University}, \text{Central University}, \text{Deemed University}, \text{Foreign University}, \text{Government University.department}, \text{Central University.department}, \text{Deemed University.department}, \text{Foreign University.department}\}
\]

Eeq(42)
The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.

\[ R' = \begin{cases} 
\text{GovernmentUniversity, CentralUniversity, DeemedUniversity,} \\
\text{ForeignUniversity, GovernmentUniversity.department,} \\
\text{CentralUniversity.department, DeemedUniversity.department,} \\
\text{ForeignUniversity.department} 
\end{cases} \]

Eq(44)

\[ POS\text{\_department}(\text{courses}) = \begin{cases} 
\text{university2, university3, university4, university5, university6,} \\
\text{university7, university8, university9, university10, university11,} \\
\text{university12, university13, university14} 
\end{cases} \]

Eq(45)

The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.

\[ R' = \begin{cases} 
\text{GovernmentUniversity, CentralUniversity, DeemedUniversity,} \\
\text{ForeignUniversity, GovernmentUniversity.department,} \\
\text{CentralUniversity.department, DeemedUniversity.department,} \\
\text{ForeignUniversity.department} 
\end{cases} \]

Eq(46)

\[ POS\text{\_department}(\text{college}) = \begin{cases} 
\text{university2, university3, university4, university5, university6,} \\
\text{university7, university8, university9, university10, university11,} \\
\text{university12, university13, university14} 
\end{cases} \]

Eq(47)

The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.
$$R' = \left\{ \text{Government University, CentralUniversity, DeemedUniversity, } \right. $$
$$\left. \text{ForeignUniversity, Government University.department, } \right. $$
$$\left. \text{CentralUniversity.department, DeemedUniversity.department, } \right. $$
$$\left. \text{ForeignUniversity.department} \right\}$$

Eeq (48)

$$POS_{\text{courses}}(\text{college}) = \left\{ \text{university2, university3, university4, university5, university6, } \right. $$
$$\left. \text{university7, university8, university9, university10, university11, } \right. $$
$$\left. \text{university12, university13, university14} \right\}$$

Eeq (49)

The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.

$$R' = \left\{ \text{Government University, CentralUniversity, DeemedUniversity, } \right. $$
$$\left. \text{ForeignUniversity, Government University.department, } \right. $$
$$\left. \text{CentralUniversity.department, DeemedUniversity.department, } \right. $$
$$\left. \text{ForeignUniversity.department} \right\}$$

Eeq (50)

$$POS_{\text{courses}}(\text{department}) = \left\{ \text{university 2, university 3, university 4, university 5, university 6, } \right. $$
$$\left. \text{university 7, university 8, university 9, university 10, university 11, } \right. $$
$$\left. \text{university 12, university 13, university 14} \right\}$$

Eeq (51)

The objects representing corresponding class and its class names which are other than decision names will form one of the Reduct as shown below.
if single object is deleted from the object set, say university1 or university2…………….university14, then one of the Reduct is same as that we have obtained earlier in Example 4 of Chapter-1.

The one of the Reduct of this Sub-Object-Oriented Rough Set model is as follows.

\[
R' = \begin{cases} 
\{Government\ University, \ CentralUniversity, \ DeemedUniversity, \\
ForeignUniversity, Government University.department, \\
CentralUniversity.department, DeemedUniversity.department, \\
ForeignUniversity.department \end{cases} 
\]

Eq(52)

If any two objects are deleted, the Reduct is same as that was obtained earlier in Example 4 of Chapter-1 except for deletion of two objects university9, university10.

If university9 and university10 objects are deleted, then object set for this Sub-Object-Oriented Rough Set Model is as follows.

\[
\{university\ 1, university\ 2, university\ 3, university\ 4, university\ 5, university\ 6, \\
university\ 7, university\ 8, university\ 11, university\ 12, university\ 13, university\ 14\} 
\]

Eq(54)
The one of the Reduct of this Sub-Object-Oriented Rough Set model is as follows.

\[ R' = \{ Government\ University, CentralUniversity, ForeignUniversity, \\
Government\ University.department, CentralUniversity.department, \\
ForeignUniversity.department \} \]

Eeq(56)

The Reduct obtained in the Eeq(56), is called Parallel Reduct of Eeq(42) since positive region while considering Eeq(54) is different from Eeq(41) where as the positive region considering Eeq(41) is equal to the one obtained through Eeq(43).

If university1, university2, university3, university4 objects are deleted, then the object set for the Sub-Object-Oriented Rough Set Model is

\[ \{ university\ 5, university\ 6, university\ 7, university\ 8, university\ 9, \\
university\ 10, university\ 11, university\ 12, university\ 13, university\ 14 \} \]

Eeq(57)

\[ POS_{college}(courses) = \{ university\ 5, university\ 6, university\ 7, university\ 8, \\
university\ 9, university\ 10, university\ 11, university\ 12, \\
university\ 13, university\ 14 \} \]

Eeq(58)
The one of the Reduct of this Sub-Object-Oriented Rough Set model is as follows:

\[
\{ \text{CentralUniversity, DeemedUniversity, ForeignUniversity,} \\
\text{CentralUniversity.department, DeemedUniversity.department,} \\
\text{ForeignUniversity.department} \}
\]

Eeq(59)

The Reduct obtained in the Eeq(59), is called Parallel Reduct of Eeq(56) since positive region while considering Eeq(57) is different from Eeq(55) where as the positive region considering Eeq(41) is equal to the one obtained through Eeq(43).

The Reduct obtained in the Eeq(59), is called Parallel Reduct of Eeq(42) since positive region while considering Eeq(57) is different from Eeq(41) where as the positive region considering Eeq(41) is equal to the one obtained through Eeq(43).

If university5, university6, university7, university8 objects are deleted, then the resulting object set for the Sub-Object-Oriented Rough Set Model is

\[
\{ \text{university1, university2, university3, university4, university9, university10,} \\
\text{university11, university12, university13, university14} \}
\]

Eeq(60)
The one of the Reduct of this Sub-Object-Oriented Rough Set model is as follows:

\[
POS_{college}(\text{courses}) = \left\{ \text{university}1, \text{university}2, \text{university}3, \text{university}4, \right.
\]
\[
\text{university}9, \text{university}10, \text{university}11, \text{university}12,
\]
\[
\text{university}13, \text{university}14
\]

Eeq(61)

The Reduct obtained in the Eeq(62),is called Parallel Reduct of Eeq(59) since positive region while considering Eeq(60) is different from Eeq(58) where as the positive region considering Eeq(41) is equal to the one obtained through Eeq(43).

The Reduct obtained in the Eeq(62), is called Parallel Reduct of Eeq(56) since positive region while considering Eeq(60) is different from Eeq(55) where as the positive region considering Eeq(41) is equal to the one obtained through Eeq(43).

The Reduct obtained in the Eeq(62), is called Parallel Reduct of Eeq(53) since positive region while considering Eeq(60) is different from Eeq(51) where as the positive region considering Eeq(41) is equal to the one obtained through Eeq(43).
If university11, university12, university13, university14 objects are deleted, then the resulting object set for the Sub-Object-Oriented Rough Set Model is

\[
\{\text{university1, university2, university3, university4, university5, university6, university7, university8, university9, university10}\}
\]

Eq(63)

POS\_college (courses) = \{university1, university2, university3, university4, university5, university6, university7, university8, university9, university10\}

Eq(64)

The one of the Reduct of this Sub-Object-Oriented Rough Set model is as follows:

\[
R' = \{\text{GovernmentUniversity, CentralUniversity, DeemedUniversity, GovernmentUniversity.department, CentralUniversity.department, DeemedUniversity.department}\}
\]

Eq(65)

The Reduct obtained in the Eq(65), is called Parallel Reduct of Eq(62) since positive region while considering Eq(63) is different from Eq(61) where as the positive region considering Eq(41) is equal to the one obtained through Eq(43).

The Reduct obtained in the Eq(65), is called Parallel Reduct of Eq(59) since positive region while considering Eq(63) is different from Eq(58)
where as the positive region considering Eq(41) is equal to the one obtained through Eq(43).

The Reduct obtained in the Eq(65), is called Parallel Reduct of Eq(56) since positive region while considering Eq(63) is different from Eq(52) where as the positive region considering Eq(41) is equal to the one obtained through Eq(43).

The Reduct obtained in the Eq(65), is called Parallel Reduct of Eq(52) since positive region while considering Eq(63) is different from Eq(52) where as the positive region considering Eq(41) is equal to the one obtained through Eq(43).

Some of the Parallel Reducts are from the Eq(55), Eq(59), Eq(62), Eq(65) are exactly same that obtained in Eq(39).

The following definition represents all Reducts

**Definition 4.5:** Let $OORS(C, N, O)$ be the Object-Oriented Rough Set Model where $C = (C, \supseteq_c, \subseteq_c), N = (N, \supseteq_n, \subseteq_n), O = (O, \supseteq_o, \subseteq_o)$ be the well-defined class, name and object structures, respectively. Let $OORS'(C', N', O')$ be the Sub-Object-Oriented Rough Set Model, where $C' = (C', \supseteq_c, \subseteq_c), N' = (N', \supseteq_n, \subseteq_n), O' = (O', \supseteq_o, \subseteq_o)$ be the well defined class, name, object structures respectively. Let $C' \subseteq C, N' \subseteq N, O' \subseteq O, OORS' \subseteq OORS, OORS'$
is called Sub-Object-Oriented Rough Set Model of $OORS$, Let be the $\rho(OORS)$ set of all Sub-Object-Oriented Rough Set Model of $OORS$, All Reducts of a Object-Oriented Rough Set Model of OORS is denoted by $RED(OORS)$. The Reduct in the Object-Oriented Rough Set Model is a set of the least classes or classes with names which could keep the positive regions of all subsystems $F \subseteq \rho(OORS)$. There may be more than one Parallel Reduct in the Object-Oriented Rough Set Model of $F$. Such several Parallel Reducts are possible in the Object-Oriented Rough Set Model.

This definition represents $F$ – Parallel Core.

**Definition 4.6:** Let $PR$ be the set of Parallel Reducts in the Object-Oriented Rough Set Model of $F$, then the intersection of elements of $PR$ is called the core of $F$–Parallel Reduct in the Object-Oriented Rough Set Model denoted by $PCORE = \cap PR$

Just like the classes or classes with names, and Core of Reducts, the Core of $F$-Parallel Reduct in the Object-Oriented Rough Set Model is a set of classes or classes with names which can not be reduced in all of $F$-Parallel Reducts. If one of elements in the Core of $F$ - Parallel Reducts in the Model is reduced, all of Sub-Object-Oriented Rough Set Model in $F$ could not keep their positive regions.
Example 13: This example is in continuation of Example-12. We have the Parallel Core in the Object-Oriented Rough Set Model

\[ PCORE = \]

\[
\left\{ \begin{array}{l}
\{ \text{CentralUniversity, DeemedUniversity, ForeignUniversity,} \\
\{ \text{CentralUniversity.department,} \\
\{ \text{DeemedUniversity.department, ForeignUniversity.department} \\
\{ \text{GovernmentUniversity, DeemedUniversity, ForeignUniversity,} \\
\{ \text{GovernmentUniversity.department,} \\
\{ \text{DeemedUniversity.department, ForeignUniversity.department} \\
\{ \text{GovernmentUniversity, CentralUniversity, ForeignUniversity,} \\
\{ \text{GovernmentUniversity.department,} \\
\{ \text{CentralUniversity.department, ForeignUniversity.department} \\
\{ \text{GovernmentUniversity, CentralUniversity, DeemedUniversity,} \\
\{ \text{GovernmentUniversity.department,} \\
\{ \text{CentralUniversity.department, DeemedUniversity.department} \\
\end{array} \right. \]

[Eq (50)]

Proposition 1: Let \( OORS (C, N, O) \) be the Object-Oriented Rough Set Model where \( C = (C, \mathcal{C}_c, \subseteq_c) \), \( N = (N, \mathcal{C}_N, \supseteq_N) \), \( O = (O, \mathcal{C}_O, \supseteq_O) \) be the well-defined class, name and object structures, respectively. Let \( OORS' (C', N', O') \) be the Sub-Object-Oriented Rough Set Model, where \( C' = (C', \mathcal{C}_c, \subseteq_c) \), \( N' = (N', \mathcal{C}_N, \supseteq_N) \), \( O' = (O', \mathcal{C}_O, \supseteq_O) \) be the well-defined class, name, object structures respectively. Let \( C' \subseteq C, N' \subseteq N, O' \subseteq O, OORS' \subseteq OORS \), \( OORS' \) is called Sub-Object-Oriented Rough Set Model of \( OORS \), \( R \) is Reduct of
Sub-Object-Oriented Rough Set Model, there exist Parallel Reduct \( PR \) in the Object-Oriented Rough Set Model of \( F \) such that \( R \subseteq PR \).

**Example 14:** This example is in continuation of example 12. For example, we have one Reduct of Sub-Object-Oriented Rough Set Model

\[
R = \left\{ \begin{array}{l}
Government University, Central University, Deemed University, \\
Foreign University, Government University.\text{department}, \\
Central University.\text{department}, Deemed University.\text{department}, \\
Foreign University.\text{department}
\end{array} \right\}
\]

Eq(51)

We have the one of the Parallel Reduct of Object-Oriented Rough Set Model.

\[
PR = \left\{ \begin{array}{l}
Government University, Deemed University, Foreign University, \\
Government University.\text{department}, Deemed University.\text{department}, \\
Foreign University.\text{department}
\end{array} \right\}
\]

Eq(52)

From the \( R \) and \( PR \) we can have \( R \subseteq PR \).

**Proposition 2:** Let \( GDR \) be a Generalized Dynamic Reduct of \( F \) in the Object-Oriented Rough Set Model, then it is also subset of one of Parallel Reduct of \( F \) in the Object-Oriented Rough Set Model.

**Example 15:** This example is in continuation of example 13. This example give Generalized Dynamic Reduct in the Object-Oriented Rough Set Model.
Consider Sub-Object-Oriented Rough Set Model with the following objects

\[
\{\text{university 5, university 6, university 7, university 8, university 9, university 10, university 11, university 12, university 13, university 14}\}
\]

Eq(53)

The Generalized Dynamic Reduct in the Object-Oriented Rough Set Model with respect to the above object set is as follows.

\[
\text{GDR} = \{\text{Central University, Deemed University, Foreign University, Central University\, college, Deemed University\, college, Foreign University\, college, Central University\, department, Deemed University\, department, Foreign University\, department, Central University\, courses, Deemed University\, courses, Foreign University\, courses}\}
\]

Eq(54)

The one of the Parallel Reduct of Object-Oriented Rough Set Model is as follows.

\[
\text{PR} = \{\text{Central University, Deemed University, Foreign University, Central University\, department, Deemed University\, department, Foreign University\, department}\}
\]

Eq(55)

From GDR and PR, we have $\text{GDR} \subseteq \text{PR}$.

**Proposition 3:** Let $OORS (C, N, O)$ be the Object-Oriented Rough Set Model where $C = (c, \varnothing, \supseteq_c)$, $N = (n, \varnothing, \supseteq_n)$, $O = (o, \varnothing, \supseteq_o)$ be the well-defined class, name and object structures, respectively. Let $OORS' (C', N', O')$ be the Sub-Object-Oriented Rough Set Model, where $C' = (c', \varnothing, \supseteq_c)$,
$N'=(N',\exists N,\supseteq_n), O'=(O',\exists o,\supseteq_o)$ be the well defined class, name, object structures respectively. Let $C' \subseteq C, N' \subseteq N, O' \subseteq O, OORS' \subseteq OORS$, $OORS'$ is called Sub-Object-Oriented Rough Set Model of $OORS$. Let $\rho(OORS)$ be the set of all Sub-Object-Oriented Rough Set Model of $OORS$, $B$ is Reduct of $OORS'$. If $\forall OORS \subseteq F$ then there is Parallel Reduct $PR$ of $F$ such that $PR \subseteq B$.

**Example 16:** This example is in continuation of Example-14. Considering one of the Parallel Reduct of Object-Oriented Rough Set Model.

$$PR = \begin{cases} 
\text{Central University, DeemedUniversity, ForeignUniversity,} \\
\text{Central University.department, DeemedUniversity.department,} \\
\text{ForeignUniversity.department} 
\end{cases} \quad \text{Eq}(56)$$

Consider Sub-Object-Oriented Rough Set Model with the following objects

$$\begin{cases} 
\text{university 2, university 3, university 4, university 5, university 6,} \\
\text{university 7, university 8, university 9, university 10, university 11,} \\
\text{university 12, university 13, university 14} 
\end{cases} \quad \text{Eq}(57)$$

The one of the Reduct of this Sub-Object-Oriented Rough Set model is as follows:

$$R = \begin{cases} 
\text{CentralUniversity, DeemedUniversity, ForeignUniversity,} \\
\text{CentralUniversity.department, DeemedUniversity.department,} \\
\text{ForeignUniversity.department} 
\end{cases} \quad \text{Eq}(58)$$

From PR and R, we have $PR \subseteq R$
**Corollary 1:** Let \( OORS (C, N, O) \) be the Object-Oriented Rough Set Model where \( C = (C, \preceq_C) \), \( N = (N, \preceq_N) \), \( O = (O, \preceq_O) \) be the well-defined class, name and object structures, respectively. Let \( OORS' (C', N', O') \) be the Sub-Object-Oriented Rough Set Model, where \( C' = (C', \preceq_C) \), \( N' = (N', \preceq_N) \), \( O' = (O', \preceq_O) \) be the well-defined class, name, object structures respectively. Let \( C' \subseteq C, N' \subseteq N, O' \subseteq O, OORS' \subseteq OORS, OORS' \) is called Sub-Object-Oriented Rough Set Model of \( OORS \). Let \( \rho(OORS) \) set of all Sub-Object-Oriented Rough Set Model of \( OORS \). If \( OORS' \in F \) such that any element of \( F \) is included \( OORS' \), then any Reduct of \( OORS' \) is Parallel Reduct of \( F \) in the Object-Oriented Rough Set Model.

**Corollary 2:** Let \( OORS (C, N, O) \) be the Object-Oriented Rough Set Model where \( C = (C, \preceq_C) \), \( N = (N, \preceq_N) \), \( O = (O, \preceq_O) \) be the well-defined class, name and object structures, respectively. Let \( OORS' (C', N', O') \) be the Sub-Object-Oriented Rough Set Model, where \( C' = (C', \preceq_C) \), \( N' = (N', \preceq_N) \), \( O' = (O', \preceq_O) \) be the well-defined class, name, object structures respectively. Let \( C' \subseteq C, N' \subseteq N, O' \subseteq O, OORS' \subseteq OORS, OORS' \) is called Sub-Object-Oriented Rough Set Model of \( OORS \). Let \( \rho(OORS) \) set of all Sub-Object-Oriented Rough Set Model of \( OORS \). If \( OORS' \in F \) such that

\[
\begin{align*}
1) \text{if } PR \text{ is one of } F \text{'s-Parallel Reducts, there exist a Parallel Reduct } PR' \text{ of } F \text{ such that } PR \subseteq PR'.
\end{align*}
\]
(2) if $PR'_i$ is one of $F'_i$’s Parallel Reducts, there exists a Parallel Reduct $PR$ of $F$ such that $PR \subseteq PR'_i$.

**Corollary 3:** Let $OORS(C, N, O)$ be the Object-Oriented Rough Set Model where $C = (C, \triangleright_C, \supseteq_C)$, $N = (N, \triangleright_N, \supseteq_N)$, $O = (O, \triangleright_O, \supseteq_O)$ be the well-defined class, name and object structures, respectively. Let $OORS'(C', N', O')$ be the Sub-Object-Oriented Rough Set Model, where $C' = (C', \triangleright_C, \supseteq_C)$, $N' = (N', \triangleright_N, \supseteq_N)$, $O' = (O', \triangleright_O, \supseteq_O)$ be the well-defined class, name, and object structures respectively. Let $C' \subseteq C, N' \subseteq N, O' \subseteq O, OORS' \subseteq OORS, OORS'$ is called Sub-Object-Oriented Rough Set Model of $OORS$. Let $\rho(OORS)$ set of all Sub-Object-Oriented Rough Set Model of $OORS$, if for any $OORS' \in F$, there exist $OORS'_i \in F'_i$ such that $OORS' \subseteq OORS'_i$. Then any Parallel Reduct $PR$ of $F$ there exist Parallel Reduct of $PR$ of $F'_i$ such that $PR \subseteq PR'_i$.

**Proposition 4:** The core of $OORS' \in F$ is included in the core of F-Parallel Reduct in the Object-Oriented Rough Set Model.

The core of $OORS'$ in the Example-7 of Chapter-3 is included in the core of F-Parallel Reduct in the Example-13 of this chapter.

### 4.3. F-Parallel Reducts in Object-Oriented Rough Set Model

**Definition 4.6:** The core of $F$-Parallel Reducts in the Object-Oriented
Rough Set Models, which eliminates intersection of all Sub-Object-Oriented Rough Set Model may be empty. From the above propositions and the corollaries, the Parallel Reduct is the extension of both Reduct and Generalized Dynamic Reduct in the Object-Oriented Rough Set Model.

If there is only one element in \( F \) then its Parallel Reduct and Generalized-Dynamic Reduct in the Object-Oriented Rough Set Model are the same. The Parallel Reducts of a Rough Set Models will not be empty in any cases while the \( F - \text{Generalized Dynamic} \) Reducts of a Object-Oriented Rough Set Models may be empty. The Parallel Reducts of a Rough Set Models are stable just like \( F - \text{Generalized Dynamic} \) Reducts in the Object-Oriented Rough Set Model.

### 4.4. Algorithm for \( F \) – Parallel Reducts

We will introduce two algorithms of \( F \) – Parallel Reducts in the Object-Oriented Rough Set Model.

**Algorithm 1:**

**Input:** \( F \subseteq \rho(OORS) \)

**Output:** one of \( F \) – Parallel Reducts in Object-Oriented Rough Set Model.

\( n = \text{number of classes}. \)

\( PR = \phi \)

\( P_i = POS_{\rho}(Q), \) Where \( P = n_i \in H_N(n), n_i \in N_{\text{CON}} \),

\( Q = n_i \in H_N(n), n_i \in N_{\text{DEC}} \)
$PR = PR \cup P_i$

$For(i=1;i<n;i++)$

Let $C_i \in C$

$C = C - C_j$

$P_i = POS_p(Q)$, Where $P = n_i \in H_n(n), n_j \in N_{CON}$,

$Q = n_i \in H_n(n), n_j \in N_{DEC}$,

$PR = PR \cup P_i$

$Output PR$.

**Algorithm 2:**

Input: $F \subseteq \rho(OORS)$

Output: the core of F-Parallel Reducts in the Object-Oriented Rough Set Model.

$PCORE = \phi$ ;

Consider all Parallel Reduct Set in the Object-Oriented Rough Set Model say

say $R_1, R_2, R_3, ... R_i$

$PRCORE = R_1 \cap R_2 \cap ... \cap R_i$

$Output PRCORE$
In the above algorithm, $PCORE$ stands for the Core of Parallel Reducts in the Object-Oriented Rough Set Model. From the algorithm 1, in order to get Parallel Reducts one need not count all Reducts of Object-Oriented Rough Set Models. But one must count all Reducts of Sub-Object-Oriented Rough Set Models for getting Generalized Dynamic Reducts in the Object-Oriented Rough Set Model. One can Parallel compute when the Parallel Reducts in the Object-Oriented Rough Set Model are counted.

It is a NP-hard problem to count all of Parallel Reducts, just like Dynamic Reducts, in any real application.

### 4.5. Conclusions

In the chapter a new notion of Reduct called Parallel Reduct is proposed, along with more of its properties. The Parallel Reduct in the Object-Oriented Rough Set Model is the extension of Reduct and Generalized Dynamic Reduct in the Object-Oriented Rough Set Model. The notions of Object-Oriented Rough Set Model such as positive region, Core and Reduct were extended. It is shown how to compute the Parallel Reduct which is as stable as Dynamic Reducts. The complexity of computation of Parallel Reduct in the Object-Oriented Rough Set Model may be same as that of the algorithm of Reduct, but it is lower compared to Generalized Dynamic Reducts in the Object-Oriented Rough Set Model.