INTRODUCTION

Partially defined infinitary operations occur in the contexts ranging from integration theory to programming language semantics. The general cardinal algebras studied by Tarski[34] in 1949, Σ-structures studied by Higgs[14] in 1980, Housdorff topological commutative groups studied by Bourbaki[9] in 1966, sum-ordered partial monoids & sum-ordered partial semirings studied by Arbib, Manes, Benson[20] and Streenstrup[33] are some of the algebraic structures of the above type.

The study of $pfn(D, D)$ (the set of all partial functions of a set D to itself), $Mfn(D, D)$ (the set of all multi functions of a set D to itself) and $Mset(D, D)$ (the set of all total functions of a set D to the set of all finite multi sets of D) play an important role in the theory of computer science, and to abstract these structures Manes and Benson[20] introduced the notion of sum-ordered partial semiring (so-ring) as a system $(R, \Sigma, \cdot, \leq, 1)$ where R is a nonempty set, Σ is a partial addition defined on some (not necessarily all) families $(x_i : i \in I)$ in R, '$\cdot$' is a binary operation satisfying the following:

(1) *Unary Sum Axiom.* If $(x_i : i \in I)$ is a one element family in R and $I = \{j\}$, then $\Sigma(x_i : i \in I)$ is defined and equals x_j.

(2) *Partition-Associativity Axiom.* If $(x_i : i \in I)$ is a family in R and $(I_j : j \in J)$ is a partition of I, then $(x_i : i \in I)$ is summable if and only if $(x_i : i \in I_j)$ is summable for every j in J and $(\Sigma(x_i : i \in I_j) : j \in J)$ is summable. we write $\Sigma(x_i : i \in I) = \Sigma(\Sigma(x_i : i \in I_j) : j \in J)$.

(3) $(R, \cdot, 1)$ is a monoid with multiplicative operation '$\cdot$' with unit 1, and the additive and multiplicative structures obey the following distributive laws.
If $\Sigma(x_i : i \in I)$ is defined in R, then for all y in R, $\Sigma(y \cdot x_i : i \in I)$ and $\Sigma(x_i \cdot y : i \in I)$ are defined and $y \cdot [\Sigma_i x_i] = \Sigma_i(y \cdot x_i)$; $[\Sigma_i x_i] \cdot y = \Sigma_i(x_i \cdot y)$.

(4) the *sum ordering* $\leq$ defined on R by $x \leq y$ if and only if there exists a h in R such that $y = x + h$, for $x, y \in R$, is a partial order on R.

Manes and Benson[20] besides developing the algebra of so-rings have observed connections with the theory of inverse semigroups.

Motivated by the study of partially-additive semantics by Arbib, Manes[4] [1980, 1982] and the usage of matrices over semirings to determine the relationship between formal power series, automata, formal language theory by Schiitzenberger [1960,1961,1962], Nivat [1968], Salomaa and Soittola [1978], Martha E. Streenstrup[33] developed a matrix theory of so-rings in 1985.

Continuing this study, G.V.S. Acharyulu[3] in 1992 studied conditions under which an arbitrary so-ring becomes a $pfn(D, D)$, $Mfn(D, D)$ and $Mset(D, D)$ and investigated the condition under which the equation $F = AF + B$ has a nontrivial solution.

In view of ordering and partial addition in the so-ring, the Φ-representation of algebras studied by A. Walendziak[35] (which is true for semirings) and the ideal theory of semirings (Jonathan S. Golan[16]) are not applicable to so-rings and hence there is a need to study these for so-rings. The objectives of this thesis are: (1) to obtain a Φ-representation of so-rings, (2) to develop an ideal theory for so-rings and (3) to make a detailed study of partial semimodules over partial semirings.

This thesis is divided into four chapters. In chapter 0 we briefly survey the work done by Manes, Benson[18],[19],[20], Arbib[4],[5], Streenstrup[33], G.V.S. Acharyulu[3] on so-monoids and so-rings and collect all the necessary results relevant to the work of this thesis from the literature.

we now briefly state the main results of this thesis.

In chapter 1 we introduce the concepts of quotient structure and Φ-representation for so-rings and we obtain the following:

(1). Let θ be a partial semiring congruence on a so-ring R such that R/θ is a partial semiring. Then R/θ is a so-ring if and only if θ has the diagonal property. (Theorem 1.1.8)

(2). Any so-ring is a subdirect product of subdirectly irreducible so-rings. (Theorem 1.2.8)

(3). Let R be a so-ring, $\phi \in ConR$ and $\{\theta_i \mid i \in I\}$ be a system of congruences of $R \ni 0_R = \Pi_\phi(\theta_i : i \in I)$. If the mapping $f : R \to \prod R_i$ is defined by $f(x) = ([x]_{\theta_i} : i \in I)\ \forall x \in R$ where $R_i = R/\theta_i$, then $f(R)$ is a $f(\phi)$-product of so-rings $R_i, i \in I$. (Theorem 1.3.10)

(4). Let R be a so-ring, ϕ, $\theta_i(i \in I) \in ConR$. Define $f : R \to \prod R_i$ by $x \mapsto ([x]_{\theta_i} : i \in I)$ where $R_i = R/\theta_i$, $i \in I$. Then $< (R_i : i \in I), f >$ is a ϕ-representation of R if and only if $0_R = \prod_\phi(\theta_i : i \in I)$. (Theorem 1.3.13)

In chapter 2 we obtained the ideal theory of so-rings and we show the following:

(1). Let R be a complete so-ring, then for any S contained in R,
$< S >= \{x \in R \mid x \leq \Sigma r_i x_i r_i', x_i \in S, r_i, r_i' \in R\}$. (Theorem 2.1.5)

(2). Let I, J be ideals of a so-ring R satisfying the decomposition property. Then $I + J$ is an ideal of R. In fact $I \bigvee J = I + J$. (Theorem 2.1.32)

(3). If P is a proper ideal of a complete so-ring R then the following conditions are equivalent

(i). P is prime

(ii). $\{arb \mid r \in R\} \subseteq P \Leftrightarrow a \in P$ or $b \in P$. (Theorem 2.2.3)

(4). The ideal lattice of a complete so-ring R is isomorphic to the lattice of all open sets of the stone space of R. (Theorem 2.2.19)

(5). If I is an ideal of a complete so-ring R then the following are equivalent

(i). I is semiprime

(ii). $\{ara \mid r \in R\} \subseteq I \Leftrightarrow a \in I$. (Theorem 2.2.23)

(6). If I is an ideal of a commutative complete so-ring R then $\sqrt{I} = \{a \in R \mid a^n \in I$ for some positive integer $n\}$. (Theorem 2.2.30)

(7). For any partial semiring R, $SP(R) = \{x \in R \mid (G,P)$ is a super *sp*-system $\ni x \in G \Rightarrow 0 \in G\}$. (Theorem 2.3.33)

In chapter 3 we recall the notion of left partial semimodule over partial semirings studied by Streenstrup[33] and we show the following:

(1). If M is a left austere partial semimodule over R then $(0:M) = (0:m)$ for every nonzero m in M. (Theorem 3.1.15)

(2). Let M be a left partial semimodule over R and let $\{N_j \mid j \in J\}$ be a family of absorbing subsemimodules of M such that $\bigcap_{j \in J} N_j = N$ and $\bigcup_{j \in J} N_j = N'$. Then $N \sqsubseteq M$ and $N' \sqsubseteq M$. (Theorem 3.2.31)

(3). Let M be a multiplication partial semimodule over R and N be a subsemimodule of M. Then N is a prime subsemimodule of M if and only if $(N:M)$ is a prime partial ideal of R. (Theorem 3.3.7)

(4). Let M be a multiplication partial semimodule over R and N be a subsemimodule of M. Then the following conditions are equivalent

(1). N is a prime subsemimodule

(2). for any subsemimodules U, V of M, $UV \subseteq N$ implies $U \subseteq N$ or $V \subseteq N$

(3). for any $m_1, m_2 \in M$, $m_1 m_2 \subseteq N$ implies $m_1 \in N$ or $m_2 \in N$.

(Theorem 3.3.15)

(5). Let M be a multiplication partial semimodule over R and N be a subsemimodule of M. Then the following conditions are equivalent

(1). N is a semiprime subsemimodule

(2). for any subsemimodule U of M, $U^2 \subseteq N$ implies $U \subseteq N$

(3). for any $m \in M$, $m^2 \subseteq N$ implies $m \in N$. (Theorem 3.3.22)

(6). If M is a multiplication partial semimodule over R and N is a subsemimodule of M. Then $\bigcap V(N) = \{m \in M \mid m^n \subseteq N \text{ for some positive integer } n\}$.
(Theorem 3.3.28)

In chapter 4 we introduce the notion of left Euclidean and Dale norms on partial semirings and we obtain the following:

(1). The following conditions on a left Euclidean partial semiring are equivalent

(i). R is a PLIS-semiring

(ii). there exists a left Euclidean norm δ defined on R satisfying the condition that if $a = qb + r$ for $r \in R \setminus \{0\}$ and $\delta(r) < \delta(b)$ then $a \notin Rb$.

(Theorem 4.1.22)

(2). If R is a commutative antisimple partial semiring and δ is a Dale norm defined on R then

(i). $U(R) = \{a \in R \mid \delta(a) = 1\}$

(ii). R is a division partial semiring if and only if $\delta(R)$ is finite. (Theorem 4.1.29)