PREFACE

Most of the problems in Applied Mathematics lead to the solution of differential equations which satisfy prescribed conditions. Frequently, these solutions turn out to be new functions called Special functions with interesting properties, the word special is used in the sense that they arise in the solutions of special problems.

The Special functions include functions as well as orthogonal and non-orthogonal polynomials, such as, Hypergeometric functions, Meijer’s G-functions, Lame functions, Cylindrical functions, Matheiu functions, Wave functions, Legendre polynomials, Hermite polynomials, Appell’s polynomials, Bernoulli and Euler’s polynomials etc..

In 1961, Charles Fox defined a more general function called the H-function in terms of a Mellin-Barnes type integral, in an attempt to unify existing results on Fourier kernels. Here I am giving the definitions of H-function of one variable, H-function of two variables and H-function of ‘r’ variables, which are useful in our study.

(i). According to Fox, C. the definition of H-function of one variable as follows

\[
H(x) = \frac{\Gamma(b_j - B_j s)}{\Gamma(1-a_j - A_j s)} \prod_{j=1}^{p} \Gamma(1-a_j - A_j s) \prod_{j=1}^{q} \Gamma(b_j - B_j s)
\]

\[
F(s) = \frac{\prod_{j=1}^{p} \Gamma(1-a_j - A_j s)}{\prod_{j=1}^{q} \Gamma(b_j - B_j s)} \prod_{j=n+1}^{p} \Gamma(a_j - A_j s)
\]

An empty product is interpreted as unity; m, n, p and q are integers satisfying 0 ≤ m ≤ q, 0 ≤ n ≤ p; A_j (j = 1, 2, . . . , p), B_j (j = 1, 2, . . . , q) are positive numbers and a_j (j = 1, 2, . . . , p), b_j (j = 1, 2, . . . , q) are complex numbers such that no poles of Π(β_j - β_j s), (j = 1, 2, . . . , m) coincide with any pole of Π(α_j - A_j s), (j = 1, 2, . . . , n), i.e A_j(b_k + N) ≠ B_k(a_j - M -1),

(k = 1, 2, . . . , m; j = 1, 2, . . . , n; M = 0, 1, 2, . . . ).
The contour \( L \) runs from \( \sigma - i \infty \) to \( \sigma + i \infty \), \( \sigma \) be a positive constant such that the points \( s = \frac{b_h + N}{B_h} \), \( (h = 1, 2, \ldots, m) \); \( N = (0, 1, 2, \ldots) \); which are the poles of \( \Gamma(b_j - B_js) \), \( (j = 1, 2, \ldots, m) \) lie to the right ; and the points \( s = \frac{a_j - M - 1}{A_j} \) \( (j = 1, 2, \ldots, n; M = 0, 1, 2, \ldots) \) which are the poles of \( \Gamma(1 - a_j + A_js) \), \( (j = 1, 2, \ldots, n) \) lie to the left of \( L \).

(ii) The H-function of two variables given by Prasad and Gupta is

\[
H[x,y] = H_{M,N;m,h,n;m_2,n_2}^{P,Q;p_1,q_1:p_2,q_2} \left[ \prod_{j=1}^{m_1} \Gamma(d_j - D_j s) \prod_{j=1}^{m_1} \Gamma(1-c_j + C_j s) \prod_{j=m+1}^{q_1} \Gamma(1-d_j + D_j s) \prod_{j=n+1}^{p_1} \Gamma(c_j - C_j s) \right] \frac{1}{(2\pi i)^2} \int L_1 L_2 |\phi_1(s)\phi_2(t)\psi(s,t)|e^{s'y'dt}, \quad i = \sqrt{-1}
\]

where \( x , y > 0 \),

\[
\phi_1(s) = \frac{\prod_{j=1}^{m_1} \Gamma(d_j - D_j s) \prod_{j=1}^{m_1} \Gamma(1-c_j + C_j s)}{\prod_{j=m+1}^{q_1} \Gamma(1-d_j + D_j s) \prod_{j=n+1}^{p_1} \Gamma(c_j - C_j s)}
\]

\[
\phi_2(t) = \frac{\prod_{j=1}^{m_2} \Gamma(1 - f_j t) \prod_{j=1}^{n_2} \Gamma(1 - e_j + E_j t) \prod_{j=g+1}^{p_2} \Gamma(-f_j + F_j t)}{\prod_{j=h+1}^{q_2} \Gamma(-e_j - E_j t)}
\]

\[
\psi(s,t) = \frac{\prod_{j=1}^{M} \Gamma(b_j - \beta_j s - B_j t) \prod_{j=1}^{N} \Gamma(1-a_j + \alpha_j s + A_j t)}{\prod_{j=M+1}^{Q} \Gamma(1-b_j + \beta_j s + B_j t) \prod_{j=N+1}^{P} \Gamma(a_j - \alpha_j s - A_j t)}
\]

\( H[x,y] \) is an analytic function of \( x \) and \( y \) if \( V_1 < 0, V_2 < 0 \) where \( V_1 \) and \( V_2 \) are defined by

\[
V_1 = \sum_{j=1}^{p} \alpha_j - \sum_{j=1}^{q_1} \beta_j - \sum_{j=1}^{q_1} D_j
\]

\[
V_2 = \sum_{j=1}^{Q} C_j
\]
and

\[ V_2 = \sum_{j=1}^{p} A_j + \sum_{j=1}^{p} E_j - \sum_{j=1}^{Q} B_j - \sum_{j=1}^{q} F_j \]

The double integral converges absolutely; if \(|\arg x| < (1/2) \pi \Delta_1\) and \(|\arg y| < \pi \Delta_2\) where

\[ \Delta_1 = \sum_{j=1}^{N} \alpha_j - \sum_{j=N+1}^{P} \alpha_j + \sum_{j=1}^{M} \beta_j - \sum_{j=M+1}^{Q} \beta_j + \sum_{j=1}^{m} D_j - \sum_{j=m+1}^{q} D_j + \sum_{j=1}^{n} C_j - \sum_{j=n+1}^{p} C_j \]

and

\[ \Delta_2 = \sum_{j=1}^{N} A_j - \sum_{j=N+1}^{P} A_j + \sum_{j=1}^{M} B_j - \sum_{j=M+1}^{Q} B_j + \sum_{j=1}^{m} F_j - \sum_{j=m+1}^{q} F_j + \sum_{j=1}^{n} E_j - \sum_{j=n+1}^{p} E_j \]

(iii) The H-function of several complex variables (or the multivariable H-function) \(x_1, x_2, \ldots, x_r\) given by Srivastava and Panda interms of the multiple contours integral as:

\[
H[x_1, \ldots, x_r] = H_{\mathbf{0}, \mathbf{N}}: \mathbf{p}, \mathbf{q}, \mathbf{c}, \mathbf{D}, \mathbf{C}, \mathbf{P}, \mathbf{E} : \mathbf{p}, \mathbf{q}, \mathbf{c}, \mathbf{D}, \mathbf{C}, \mathbf{P}, \mathbf{E} \\
= \frac{1}{(2\pi i)^{r-1}} \int \cdots \int \theta(s_1, \ldots, s_r) \prod_{k=1}^{r} \phi_k(s_k) s_k^{x_k} ds_1 \cdots ds_r ,
\]

where \(i = \sqrt{-1}\)

\[
\theta(s_1, \ldots, s_r) = \prod_{j=1}^{p} \frac{\Gamma(1-a_j + \sum_{k=1}^{r} \alpha_j^{(k)} s_k)}{\Gamma(a_j - \sum_{k=1}^{r} \alpha_j^{(k)} s_k) \Gamma(1-b_j + \sum_{k=1}^{r} \beta_j^{(k)} s_k)}
\]
\[
\varphi_k(s_k) = \frac{\prod_{j=1}^{m_k} \Gamma\left(\frac{d_j^{(k)} - D_j^{(k)}}{s_k}\right) \prod_{j=1}^{n_k} \Gamma\left(1 - c_j^{(k)} + C_j^{(k)} s_k\right)}{\prod_{j=m_k+1}^{d_k} \Gamma\left(1 - d_j^{(k)} + D_j^{(k)} s_k\right) \prod_{j=n_k+1}^{p_k} \Gamma\left(c_j^{(k)} - C_j^{(k)} s_k\right)}, \quad k = 1, \ldots, r.
\]

An empty product is interpreted as unity; \( n, p, q, m_k, n_k, p_k, k = 1, 2, \ldots, r \) are non-negative integers such that: \( 0 \leq n \leq p, q \geq 0, 0 \leq m_k \leq qk \) and \( 0 \leq n_k \leq p_k; k = 1, 2, \ldots, r \) and \( \alpha_j^{(k)}, \beta_j^{(k)}, C_j^{(k)}, D_j^{(k)} \) are all positive.

The contour \( L_k \) in the complex plane \( s_k \) is of the Mellin-Barnes type which runs from \(-i\infty\) to \(i\infty\) with indentations, if necessary to ensure that all the poles of \( \Gamma(d_j^{(k)} - D_j^{(k)} s_k) \), \( j = 1, 2, \ldots, m_k \) are to the right path, and those of \( \Gamma(1 - c_j^{(k)} + C_j^{(k)} s_k) \), \( j = 1, 2, \ldots, n_k \) and \( \Gamma\left(1 - a_j + \sum_{j=1}^{r} \alpha_j^{(k)} s_k\right) \), \( j = 1, 2, \ldots, n \) are to the left of \( L_k \).

The study of integral transforms involving special functions also play an important role in finding the solution of differential and integral equations of Applied mathematics. If \( K(x, s) \) denote a definite function of \( x \) on interval \((a, b)\), then the transformation of the function \( F(x) \) with respect to \( K(x, s) \) called kernel, is defined by

\[
f(x) = \frac{b}{a} \int_{a}^{b} K(x, s) F(x) \, dx.
\]

In the present study, we have taken integral transforms like Mellin, Laplace of H-function of all kind, which are useful in various disciplines as Differential equations, Statistics, Pure and Applied mathematics.

The Main objective of the study is to establish integral transforms of H-function of one and several variables with general class of polynomials \( S_m^{m_{(x)}} \) and Struve’s function \( H_{\nu, \nu, \nu, \nu}^{\lambda} \).

The work is mainly divided into 6 Chapters.

**CHAPTER-I**, starts with the definition of special functions followed by a brief account of the historical survey of the work done by different researchers chronologically. Also it contains the various kinds of Integral transforms, in addition to that we given types of class of polynomials and generalized Struve’s function.
CHAPTER – II contains four sections. In first section, we have given brief summary. In second section we given some of the notations and results used in this Chapter. In the third section we established Mellin transform of product of class of polynomials with H-function of two variables. In fourth section, we established Laplace transform of product of class of polynomial with H-function of two variables.

CHAPTER – III contains three sections. In first and second sections a brief summary and notations are given respectively. In third section we established integral transform involving the product of Struve’s function and H-function of two variables.

CHAPTER – IV contains four sections. In first and second sections, a brief summary and notations, results are given respectively. In third section we established Mellin transform of product of class of polynomials, generalized Struve’s function with H-function of two variables. In fourth section, we established the Laplace transform of product of class of polynomials, generalized Struve’s function and H-function of two variables.

CHAPTER – V contains four sections. In first and second sections, we gave summary and notations, results are given respectively. In third section we established Mellin transform of the product of general class polynomials, H-function of one variable and H-function of ‘r’ variables. In fourth section, we established Mellin transform of product of general class of polynomials, Struve’s function, H-function of one variable and H-function of ‘r’ variables.

CHAPTER – VI contains three sections. In first and second sections we gave summary and Notations, results respectively. In third section, we established three integral formulae. First formula contains the integral involving the product of second class of multivariable polynomials with H-function of one variable. Second formula contains the integral involving the product of first class of multivariable polynomials with H-function of one variable. The third formula contains the integral involving the product of second class of multivariable polynomials, H-function of one variable and H-function of ‘r’ variables.

The thesis is concluded with a list of research papers arranged in an alphabetical order under the head: ‘References’.