CHAPTER 5

CONCENTRIC CIRCLE CONSTELLATION MAPPING FOR PTS BASED PAPR REDUCTION SCHEMES WITHOUT SI

In this chapter, our aim is to propose a constellation mapping scheme for PTS based OFDM systems to eliminate the requirement of side information (SI). As discussed in chapter 3, the PTS is one of the popular non distortion PAPR reduction scheme for OFDM system. In this scheme, a block of modulated data symbols is partitioned into multiple sub-blocks or partitions, using one of the partitioning schemes discussed in [62]. The obtained data sub-blocks are multiplied with the phase rotation factor to avoid the peak formation. In this scheme a phase optimization technique can be used to reduce the computational complexity. But, at the receiver, the information about phase rotation factors is required to recover the original data block. Therefore, SI should be transmitted with each OFDM symbol for data recovery. The SI has prime importance for data recovery of original data block because any error in SI detection may cause the corruption of complete data block. Hence, in case of SI corruption the error performance of the OFDM system may degrade severely. The transmission of SI along with each OFDM symbol results in data rate loss in the OFDM system. The subcarriers used for transmitting the SI may be lost in frequency selective fading channel. In order to protect the SI, a good error control mechanism can be employed, but it increases the system complexity and processing delay in the system. Any error control scheme with a code rate less than one further reduces the data rate loss in the OFDM system.

In [84], an SI embedding scheme has been proposed by Cimini and Sollenberger based on a marking algorithm and decision statistic at transmitting and receiving ends, respectively. The scheme in [84] may not be reliable for large constellation size and applicable only for M-ary PSK modulation. Jayalath and Tellambura proposed a maximum-likelihood decoding [85] for eliminating the requirement of side information. In this scheme [85], the modulation symbols of given constellation and multiple signals generated by multiplication of phase factors have sufficient Hamming distance to decode the original signal, but the SI detection capability of this scheme degrades at lower SNR values. The methods proposed in [84], [85] embed SI but
suffer from the problem of peak re-growth [84], increased decoding complexity [84], [85] and incapable of general search algorithm [84].

In [86] Nguyen and Lampe proposed a combinatorial optimization based search algorithm, to find the optimal phase factors and precoding of data stream with small redundancy is used to embed SI, but it [86] requires one bit per sub block and hence transmission of SI is not completely eliminated. In [87], Zhou et al. proposed multi point square mapping scheme for PTS-OFDM signal, which maps quaternary data points over 16-QAM constellation points using (1, j, -1, -j) as a phase rotation factors, to eliminate the requirement of side information. In [88], Yang et al. proposed an SI embedding scheme, in which the candidate signals are obtained by cyclically shifting each of the sub-blocks in time domain and combining them in recursive order, but the receiver design of such a system is complex.

The schemes proposed in [84]-[86], [88] embed SI in the transmitted signal and extract SI from the received signal at the receiver, whereas the scheme proposed in [87] is completely free from SI, i.e. extraction of SI from the received signal is not required.

In [84]-[86], [88], the information about the phase factors used at the transmitter for minimizing the PAPR is recovered from the extracted SI. The reciprocal of recovered phase factors are further used to multiply the demodulated signal at the receiver to recover the original data signal, but this operation increases the computational complexity at the receiving end, whereas the scheme proposed in [87] does not require SI and therefore, no such multiplication operation needs to be performed at the receiver, hence the receiver structure of the scheme proposed in [87] is computationally less complex. In many of the SI embedding schemes [84]-[86], [88] the SI detection at low SNR is very poor, and due to which error performance of the OFDM system degrades severely.

In wireless standards like LTE, OFDM is used in downlink, where mobile station acts as receiver. The mobile stations have limited computational resources; therefore, a PAPR reduction scheme with less computational complexity at receiving end will be beneficial. As discussed above, the schemes proposed in [84]-[86], [88] have computationally complex receiver in comparison to the schemes proposed in [87], [89]. Hence, MPSM-PTS scheme is a viable choice for PTS-OFDM system.

Motivated by MPSM mapping, in this chapter we propose a quaternary to concentric circle constellation mapping scheme to completely eliminate the requirement of side information.
Theoretical results for SER performance of CCM-PTS-OFDM over AWGN channel are derived using minimum distance decoding and circular boundary decoding algorithms. The SER analysis of MPSM-PTS scheme over AWGN channel is also carried out by minimum distance decoding and verified by simulation results. The simulation results for SER performance of CCM-PTS-OFDM over fading channel using minimum distance decoding and circular boundary decoding have been found out and compared with MPSM-PTS OFDM system. The PAPR and SER performances of the CCM-PTS method [89] are compared with the existing MPSM-PTS method of [87]. The advantages of CCM-PTS over MPSM-PTS in terms of SER performance and computational complexity are shown. Since the methods [87], [89] are SI-free, the drawbacks associated with SI transmission, as discussed above, are automatically removed.

This chapter is organised as follows: In section 5.1, we describe the existing SI embedding schemes. The OFDM system model using concentric circle constellation mapping scheme is discussed in section 5.2. A SI free scheme i.e. multipoint square mapping [87] for PTS based OFDM systems to eliminate the requirement of side information is described in section 5.3. The proposed scheme for mapping the quaternary data to concentric circle constellation using (1, j,-1,-j) as phase rotation factors is presented in section 5.4. SER analysis of concentric circle constellation and MPSM schemes are given in section 5.5. In section 5.6 we discuss the computational complexity analysis of concentric circle constellation and MPSM schemes. In section 5.7, we present the mathematical and simulation results of the PTS based OFDM system utilizing concentric circle constellation or MPSM scheme. Finally, we conclude this chapter in section 5.8.

5.1 EXISTING SI EMBEDDING SCHEMES

As mentioned earlier, conventional PTS schemes require the transmission of SI with the transmitted OFDM signal, which not only results in loss of data rate but also results degradation in error performance, in case of erroneous detection of SI. Therefore, many SI embedding schemes [84]-[86], [88] have been proposed in literature.

In [84], Cimini and Sollenberger embedded a marker on the transmitted data sequence and at the receiver SI is extracted from embedded marking sequence. The scheme proposed in [84] uses 16 partitions (M) and Walsh Hadamard codes of length 16 are used as phase factors to multiply with the sub blocks. In this scheme, if any of the sub block needs to multiply by a
phase factor 1, then we keep that sub block as it is, otherwise we rotate the data on every other subcarrier in that sub block by $\pi/4$. At the receiver, after performing the subcarrier demodulation, raise the fourth power of demodulated data symbols (for QPSK modulation) and detect them differentially for extracting the side information without using quantization. The Euclidean distance of the detected SI is calculated with each of the Walsh Hadamard code words and one of the codes with minimum distance is chosen as the recovered phase factors. In this decoding algorithm, the SI detection capability depends on the Hamming distance of the code words. In [84], the length of Walsh Hadamard code words is 16, which has a minimum Hamming distance of 8. In [84], the error in SI detection has been evaluated using Walsh Hadamard code, over AWGN and fading channel. It has been shown in [84] that over AWGN channel, the probability of error in SI detection at 3.2dB SNR, is $10^{-3}$, whereas, over fading channel it requires about 8dB SNR for the same probability of error in SI detection.

It is noteworthy that when a PTS-OFDM system uses 4 sub blocks then to multiply these sub blocks with phase factors, we require Walsh Hadamard codes of length 4, which have a minimum Hamming distance of 2, and due this smaller value of Hamming distance there are more chances of performance loss in SI detection. The lesser number of code words in Walsh Hadamard matrix also has significant effect on PAPR performance because for $M=4$, we can have only 4 candidates for searching the OFDM signal with lowest PAPR, due to which the PAPR reduction capability of the scheme proposed in [84] is very limited. Hence, for smaller number of sub blocks, it is not considered as a good PAPR reduction scheme.

As mentioned earlier, in [85], Jayalath and Tellembura proposed a receiver structure based on ML decoding algorithm for eliminating the requirement of SI. In this type of PTS based system with 4 partitions and 4 phase factors, 64 searches are required to found out the optimal set of phase factors, which requires a receiver structure with 64 branches to decode the side information. The computational complexity of such a receiver is very high and is found to be unsuitable for practical implementation. As seen from Fig.1 shown in [85], Jayalath and Tellembura proposed, to use a receiver structure with less number of branches, to limit the computational complexity of the receiver, but by limiting the number of branches in the receiver structure, the numbers of candidate OFDM signals for PAPR reduction also get reduced. In [85] only six signals for $M=4$ have been used for searching an OFDM signal with lowest PAPR. Therefore, by having very limited number of candidate signals, the PAPR reduction capability of the scheme proposed in [85] is very limited in comparison to
conventional PTS scheme. Hence, the scheme proposed in [85] is not a good choice from PAPR reduction and computational complexity point of view.

Nguyen and Lampe proposed a trellis shaping based SI embedding scheme [86], which requires pre-processing of data stream before PAPR reduction. As mentioned in [86], it requires few redundant bits per sub block to embed the SI. As seen from Fig. 6 shown in [86], the PAPR reduction capability of this scheme is very close to the conventional PTS scheme, but it has been found that its SI detection capability is not significant. Based on the results given in [86], over AWGN and fading channel it requires 7.8dB and 11.7 dB SNR, respectively, for achieving $10^{-3}$ probability of error in SI detection, which seems to degrade the error performance of the overall OFDM system at low SNR. Hence, the requirement of redundant bits for embedding the SI and its poor SI detection capability at low SNR makes it unattractive for PTS-OFDM system.

In [88], Yang et al. proposed a SI embedding scheme, which generates the candidate OFDM signals after cyclically shifting the sub blocks and combining them in recursive order. As seen from the Fig. 2 shown in [88], at the receiving end, we require $M$ detectors to retrieve the original data signal. As mentioned in [88], this type of the detector requires $K$ (number of cyclic shifts) times more additions and multiplications in comparison to conventional PTS receiver, which makes this type of receiver computationally complex. Hence, this type of receiver is not found suitable for wireless standard like LTE in downlink. Also, based on the results given in [88], over AWGN and fading channel, it requires 5dB and 9dB SNR respectively, for achieving $10^{-3}$ probability of error in SI detection. Hence, receiver computational complexity and poor SI detection performance at low SNR makes this scheme unsuitable for wireless standard like LTE.

Based on the above discussion, we can now conclude that SI embedding schemes [84]-[86], [88] are not suitable because of their poor SI detection capability at low SNR and increased computational complexity of the receiver. Therefore, we focus on SI free schemes.

5.2 SYSTEM MODEL

We have considered an OFDM system with PTS based PAPR reduction scheme. The block diagram of PTS based OFDM system utilizing concentric circle constellation is shown in Fig 5.1. In this system model, an OFDM system with $N=256$ subcarriers and a PTS based PAPR reduction scheme with $S=4$ partitions is used. The partitioning of the data block is performed
by using adjacent partitioning scheme and $W=4$ pure rotational phase factors $B=\{1,j,-1,-j\}$ are utilized in PTS-OFDM system. The proposed scheme can be applied for any number of subcarriers in OFDM system.

As shown in Fig. 5.1 the binary input signal is first converted to quaternary data signal and then quaternary to concentric circle mapping scheme is performed by using Table 5.1. After that, these obtained modulated data symbols are converted into $N$ parallel substreams. The obtained data block of $N$ modulated data symbols $\{X_k\}_{k=0}^{N-1}$ is partitioned into $S=4$ sub-blocks using adjacent partitioning scheme. The IDFT or IFFT of each of the sub-blocks is performed to obtain $S$ partial transmitted sequences. Each of these partial transmit sequences are multiplied by the phase rotation factors and then combined to avoid the peak formation. A phase optimization is required for achieving better PAPR reduction with low computational complexity.

Figure 5.1: CCM-PTS OFDM system transceiver
The obtained discrete time domain OFDM signal \(x')\) is given by (3.29). After that the parallel OFDM signal is converted into serial and cyclic prefix of 1/16\(^{th}\) OFDM symbol duration is inserted to eliminate the effect of ISI. The discrete time OFDM signal is passed through digital to analog (D/A) converter to obtain the analog signal. Finally, the analog signal is amplified by HPA to achieve the desired signal power. At the receiver, the received signal is converted into digital by using A/D converter and then cyclic prefix is removed. The obtained serial OFDM signal is converted into parallel using S/P converter. After that, subcarrier demodulation is performed by taking the FFT of OFDM signal obtained from S/P converter. In order to retrieve the original quaternary data signal, quaternary to concentric circle constellation de-mapping is performed, using Table 5.2. Finally, to obtain binary input, quaternary to binary data conversion is performed.

**Table 5.1:** Quaternary to concentric circle constellation mapping using phase factors \((1, j, -1, -j)\) for \(d=1\)

<table>
<thead>
<tr>
<th>Quaternary Symbol</th>
<th>Initially Mapped Quaternary data points to concentric circle constellation</th>
<th>Constellation Points After Multiplication with the Phase Factors in (S)</th>
<th>1</th>
<th>j</th>
<th>-1</th>
<th>-j</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 + j0</td>
<td>0 + j0</td>
<td>0 + j0</td>
<td>0 + j0</td>
<td>0 + j0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0 + j2</td>
<td>-2 + j0</td>
<td>0 - j2</td>
<td>2 + j0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-4 + j0</td>
<td>0 - j4</td>
<td>4 + j0</td>
<td>0 + j4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2\sqrt{2}(1-j)</td>
<td>2\sqrt{2}(1-j)</td>
<td>2\sqrt{2}(-1+j)</td>
<td>2\sqrt{2}(-1-j)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 5.2:** De- Mapping of Concentric Circle Constellation Symbols To Quaternary Data Points

<table>
<thead>
<tr>
<th>Demodulated Constellation symbols</th>
<th>De-mapped Constellation Point</th>
<th>Recovered Quaternary data</th>
</tr>
</thead>
<tbody>
<tr>
<td>{0 + j0}</td>
<td>0 + j0</td>
<td>0</td>
</tr>
<tr>
<td>{j2, -2, -j2, 2}</td>
<td>0 + j2</td>
<td>1</td>
</tr>
<tr>
<td>{-4, -j4, 4, j4}</td>
<td>-4 + j0</td>
<td>2</td>
</tr>
<tr>
<td>{2\sqrt{2}(1-j), 2\sqrt{2}(1+j), 2\sqrt{2}(-1+j), 2\sqrt{2}(-1-j)}</td>
<td>2\sqrt{2}(1-j)</td>
<td>3</td>
</tr>
</tbody>
</table>
5.3 MULTIPOINT SQUARE MAPPING

In this scheme, the quaternary data points are initially mapped to four different constellation points of 16-QAM using Table 5.3. It can be seen from Fig.5.2, that quaternary data points (0, 1, 2 and 3) are initially mapped to four different constellation points located at \{3+3j, -3+j, -1-j, 1-3j\} and are denoted by square, diamond, circle and hexagon respectively. It is noteworthy that initially mapped constellation points are lying in four different quadrants. The constellation points \{3+3j, -3+j, -1-j, 1-3j\} after multiplication with phase rotation factor \(B=\{1, j, -1, -j\}\), are rotated by \(\left\{0, \frac{\pi}{2}, \frac{3\pi}{2}\right\}\), as shown in Fig. 5.3 and covers all 16 points of 16-QAM constellation. Any initially mapped quaternary data point after multiplication with phase factor \{1, j, -1, -j\} lies on the vertices of a square, hence the name “Multipoint Square Mapping (MPSM)”. The constellation points are unique and can be de-mapped to obtain the quaternary data signal using Table 5.4. Hence, as per Table 5.4, if any of the data point is received as \{3+3j, -3+j, -3-3j or 3-3j\}, \{-3+j, -1-3j, 3-j or 1+3j\}, \{-1-j, 1-j, 1+j or -1+j\} or \{1-3j, 3+j, -1+3j or -3-j\} then it will be de-mapped to constellation the constellation points \{3+3j, -3+j, -1-j, 1-3j\} respectively, and these are nothing but the four initially mapped quaternary data points 0, 1, 2 or 3 respectively.

<table>
<thead>
<tr>
<th>Quaternary data</th>
<th>Initially Mapped Quaternary data points to 16 QAM constellation</th>
<th>Constellation points after multiplication with phase factors in (S)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1) (j) (-1) (-j)</td>
</tr>
<tr>
<td>0</td>
<td>3+3j</td>
<td>3+3j (-3+3j) (-3-3j) (3-3j)</td>
</tr>
<tr>
<td>1</td>
<td>-3+j</td>
<td>-3+j (-1-3j) (3-j) (1+3j)</td>
</tr>
<tr>
<td>2</td>
<td>-1-j</td>
<td>-1-j (1-j) (1+j) (-1+j)</td>
</tr>
<tr>
<td>3</td>
<td>1-3j</td>
<td>1-3j (3+j) (-1+3j) (-3-j)</td>
</tr>
</tbody>
</table>

The de-mapping scheme does not require any side information (SI) about the phase rotation factors at the receiver, thus, eliminating the major constraints of PTS technique. This approach extends the constellation size but do not results any data rate loss because
corresponding to each quaternary data point only one 16-QAM symbol is sent over each subcarrier, which keep bandwidth requirement unchanged. The only price paid for SI elimination capability achieved by CCM-PTS is the requirement of increased $E_b/N_0$. But, it provides consistent BER performance, whereas SI embedding schemes and the schemes transmitting SI with each of OFDM symbol yields very poor BER performance in case of erroneous SI detection, which is more likely to happen at low $E_b/N_0$.

Figure 5.2: Initial mapping of quaternary data points over 16-QAM constellation

Figure 5.3: Mapping of quaternary data points over 16-QAM constellation and effect of phase rotation factors $(1, j, -1, -j)$
### Table 5.4: De-mapping of 16-QAM constellation symbols to quaternary data points

<table>
<thead>
<tr>
<th>Demodulated Constellation symbols</th>
<th>De-mapped Constellation Point</th>
<th>Recovered Quaternary data</th>
</tr>
</thead>
<tbody>
<tr>
<td>{3+3j, -3+3j, -3-3j or 3-3j}</td>
<td>3+3j</td>
<td>0</td>
</tr>
<tr>
<td>{-3+j, -1-3j, 3-j or 1+3j}</td>
<td>-3+j</td>
<td>1</td>
</tr>
<tr>
<td>{-1-j, 1-j, 1+j or -1+j}</td>
<td>-1-j</td>
<td>2</td>
</tr>
<tr>
<td>{1-3j, 3+j, -1+3j or -3-j}</td>
<td>1-3j</td>
<td>3</td>
</tr>
</tbody>
</table>

### 5.4 CONCENTRIC CIRCLE CONSTELLATION MAPPING

In this mapping scheme, like MPSM the original bit stream is converted into quaternary data and then quaternary data points are mapped to concentric circle constellation (CCC), as shown in Fig. 5.4. The constellation points are located at origin and two concentric circles of radius $2d$ and $4d$. As shown in Fig. 5.4, quaternary data points 0, 1, 2 and 3 are mapped to four different points of CCC, located at $0$, $j2d$, $-4d$ and $2\sqrt{2}(1-j)d$ and are denoted by diamond, triangle, circle and square respectively as shown in Fig. 5.4. Concentric circles of radius $2d$ and $4d$ ensure a minimum Euclidean distance of $2d$ between any two constellation points. In QPSK constellation ($\pm 1 \pm j$), the minimum Euclidean distance between two constellation points is 2. If we take $d = 1$ then Euclidean distance between any two nearest constellation points of concentric circle constellation can be maintained same. The constellation points $0$, $j2d$, $-4d$ and $2\sqrt{2}(1-j)d$ after multiplication with phase rotation factors lie on the circle of same radius. As shown in Fig.5.4, the constellation points $0$, $j2d$, $-4d$ and $2\sqrt{2}(1-j)d$ , after multiplication with phase rotation factors \{1, j, -1, -j\} are mapped to \{0\}, \{j2d, -2d, -j2d, 2d\}, \{-4d, -j4d, 4d, j4d\} and \{2\sqrt{2}(1-j)d, 2\sqrt{2}(1+j)d, 2\sqrt{2}(-1+j)d, 2\sqrt{2}(-1-j)d\} respectively, therefore, all 13 points of CCC are occupied. The constellation points of CCC are divided into four different groups, these groups contains the constellation points located at (i) origin, \(G_1\) (ii) on circle of radius $2d$, \(G_2\) (iii) on circle of radius $4d$ with phase angles $\left\{0, \frac{\pi}{2}, \pi \text{ or } \frac{3\pi}{2}\right\}$, \(G_3\)
and (iv) on circle of radius $4d$ with phase angles $\left(\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \text{ or } \frac{7\pi}{4}\right)$, $(G_4)$. The constellation points obtained after multiplication with phase factor are unique and can be easily de-mapped to original quaternary data as per table 5.2, without requiring any side information. Like MPSM, in CCM also no data rate loss takes place due to the constellation extension and the only price paid for this advantage is requirement of increased $E_b/N_0$.

![Concentric circle constellation mapping and effect of phase rotation factors (1, j, -1, -j) on data symbols](image)

**Figure 5.4:** Concentric circle constellation mapping and effect of phase rotation factors (1, j, -1, -j) on data symbols

### 5.5 SER ANALYSIS CCM-PTS AND MPSM-PTS SCHEMES

In this section we evaluate the SER performance of CCC by minimum distance decoding and circular boundary decoding [89] over AWGN channel. The mathematical analysis of SER performance of MPSM-PTS by using minimum distance decoding is also presented here.

#### 5.5.1 CALCULATION OF PROBABILITY OF ERROR FOR CCC USING MINIMUM DISTANCE DECODING

Before going into the detail, first we discuss the various notations used in this analysis. Let $P_{el}^1$ denote the conditional probability of making an error when the transmitted point belongs to group $G_i$. Here, the superscript 1 is used for the case of minimum distance decoding and
subscript $l$ denotes the group number. In concentric circle constellation, all constellation points are divided into 4 groups $G_1$ to $G_4$, so we need to calculate 4 different error probabilities $P_{e1}^1$ to $P_{e4}^1$. The decoding regions for recovering quaternary data points (0, 1, 2 and 3) are shown in Fig. 5.5.

![Diagram](image)

**Figure 5.5:** Decoding of quaternary data points (a) ‘0’ (b) ‘1’ (c) ‘2’ and (d) ‘3’ from concentric circle constellation points using minimum distance decoding rule
The constellation points belonging to group $G_1$ have four nearest neighbours belonging to group $G_2$, and have a Euclidean distance of $2d$, as shown in Fig. 5.5. The probability of error $P_{e1}^1$ can be calculated using union bound approximation [90] as

$$P_{e1}^1 \leq 4 \times \frac{1}{2} \text{erfc} \left( \sqrt{\frac{d^2}{\eta}} \right)$$  \hspace{1cm} (5.1)$$

Here $\eta$ is single sided PSD of complex Gaussian white noise and $\text{erfc}(.)$ is complementary error function [90].

Similarly, for the constellation points belonging to group $G_2$, there are four nearest neighbours, one of them belongs to group $G_1$ and has a distance of $2d$, one in group $G_3$ at also at a distance of $2d$ and the remaining two belong to the group $G_4$ and are at a distance of $2d\sqrt{5-\sqrt{2}}$. The probability of error $P_{e2}^1$ can be calculated as

$$P_{e2}^1 \leq 2 \times \frac{1}{2} \text{erfc} \left( \sqrt{\frac{d^2}{\eta}} \right) + 2 \times \frac{1}{2} \text{erfc} \left( \sqrt{\frac{(5-\sqrt{2})d^2}{\eta}} \right)$$  \hspace{1cm} (5.2)$$

Similarly, for the constellation points belonging to group $G_3$, there are three nearest neighbours; one of them belongs to group $G_2$ and is at a distance $2d$ and remaining two belong to group $G_4$ and are at a distance of $8d \sin(\pi/8)$. The probability of error $P_{e3}^1$ can be calculated as follows

$$P_{e3}^1 \leq \frac{1}{2} \text{erfc} \left( \sqrt{\frac{d^2}{\eta}} \right) + 2 \times \frac{1}{2} \text{erfc} \left( \sqrt{\frac{8d \sin(\pi/8)^2}{4\eta}} \right)$$  \hspace{1cm} (5.3)$$

Similarly, for constellation points belonging to group $G_4$, there are four nearest neighbours, two of them belong to group $G_2$ and are at a distance of $2d\sqrt{5-\sqrt{2}}$, while remaining two belong to group $G_3$ and are at a distance of $8d \sin(\pi/8)$. The probability of error $P_{e4}^1$ can be calculated using
The overall average probability of symbol error ($P_e^1$) is calculated as follows

$$P_e^1 \leq \frac{1}{4}(P_{e1}^1 + P_{e2}^1 + P_{e3}^1 + P_{e4}^1)$$ (5.5)

$$P_e^1 \leq \frac{1}{4} \left( \frac{7}{2} \text{erfc} \left( \sqrt{\frac{d^2}{\eta}} \right) + 2 \times \text{erfc} \left( \sqrt{\frac{(5-\sqrt{2})d^2}{\eta}} \right) + 2 \times \text{erfc} \left( \sqrt{\frac{8d \sin \left( \frac{\pi}{8} \right)}{4\eta}} \right) \right)$$ (5.6)

5.5.2 CALCULATION OF ERROR PROBABILITY USING CIRCULAR BOUNDARY DECODING RULE

As shown in Fig. 5.4, even after multiplication with phase factors in $B$, the constellation points are distinct and lie on the same concentric circle (except 0, which remains unchanged). This fact enables us to apply a simplified method to decode the data symbols, without any need of SI and is known as “Circular Boundary Decoding”. In this scheme, decoding of constellation points are performed circle wise as discussed in [89]. The following decoding rules are utilized for decoding the received constellation point and corresponding decoding regions are shown if Fig. 5.6.

For a received complex-valued data point of the form $re^{j\phi}$, we can perform the decoding as

$$r < d \Rightarrow \text{Quaternary data 0},$$

$$d \leq r < 3d \Rightarrow \text{Quaternary data 1}$$

$$r \geq 3d, \frac{n\pi}{2} - \frac{\pi}{8} \leq \phi < \frac{n\pi}{2} + \frac{\pi}{8}, 0 \leq n \leq 3 \Rightarrow \text{Quaternary data 2}$$

Otherwise, it is decoded as quaternary data 3.

Here $r$ and $\phi$ are the magnitude and the phase angle of complex valued received data point.
Figure 5.6: Decoding of quaternary data points (a) ‘0’ (b) ‘1’ (c) ‘2’ and (d) ‘3’ from concentric circle constellation points using circular boundary decoding rule

In this decoding scheme, groups $G_1, G_2, G_3$ and $G_4$, considered in previous section, are also used here for calculating probability of error. Let $P_{e1}^2, P_{e2}^2, P_{e3}^2$ and $P_{e4}^2$ represents probability of errors for constellation points belonging to groups $G_1, G_2, G_3$ and $G_4$ respectively.
To calculate probability of error \( P_{e_l}^2 \) for received signal with amplitude \( r(t) \) (transmitted symbol plus complex Gaussian white noise \( n(t) \)) and phase \( \phi(t) \), let \( R \) and \( \phi \) be the random variables corresponding to \( r \) and \( \phi \) respectively and their probability density functions as \( f_R(r) \) and \( f_\phi(\phi) \) respectively. Here, the superscript 2 denotes the circular boundary decoding rule and subscript \( l \) denotes the group number.

The constellation point belonging to group \( G_i \), is located at origin. In this particular case, the magnitude and phase of \( r(t) \) will denote nothing but the magnitude and phase of noise. Therefore, the probability density functions of magnitude and phase of \( r(t) \) will be same as that of noise. So, in this particular case the probability density of phase, considered as a random variable \( \phi \), has a uniform distribution in the interval \([0, 2\pi]\) and the probability density function of the magnitude of \( r(t) \) taken as a random variable \( R \) has a Rayleigh distribution, given by

\[
f_{1_R}(r) = \frac{r}{\sigma_n^2} \exp\left(\frac{-r^2}{2\sigma_n^2}\right), \quad r \geq 0
\]  

(5.7)

where \( \sigma_n^2 = \eta/2 \) is the variance of complex Gaussian white noise \( n(t) \). If any of the constellation point belonging to \( G_i \) is transmitted then it will be detected incorrectly if \( |r(t)| \geq d \), and the probability of error \( P_{e_l}^2 \) can be calculated as follows

\[
P_{e_l}^2 = 1 - \left( \frac{d}{\sigma_n^2} \int_0^\infty \exp\left(-\frac{r^2}{2\sigma_n^2}\right) dr \right)
\]  

(5.8)

\[
= \exp\left( -\frac{d^2}{2\sigma_n^2} \right)
\]  

(5.9)

For any transmitted point belonging to group \( G_2 \), the constellation points are located on the circle of radius \( 2d \), the joint probability density function \( f_{2R,\phi}(r,\phi) \) of magnitude \( r \) and phase \( \phi \) for the received signal [91] (transmitted symbol plus noise) is given by

\[
f_{2R,\phi}(r,\phi) = \frac{r}{2\pi\sigma_n^2} \exp\left(\frac{-r^2 + 4d^2 - 4d \cos(\phi)}{2\sigma_n^2}\right)
\]  

(5.10)
According to the circular boundary decoding rule for the quaternary data 1, we have

\[
P_{e2} = 1 - \left[ \frac{2\pi}{d} \int_0^3 r \exp \left( -\frac{r^2 + 4d^2 - 4d\cos(\phi)}{2\sigma_n^2} \right) dr \right]
\]

\[
= 1 - \left( \frac{1}{\sigma_n^2} \exp \left( -\frac{2d^2}{\sigma_n^2} \right) \int_0^3 \frac{r}{d} \exp \left( -\frac{r^2}{2\sigma_n^2} \right) I_0 \left( \frac{2rd}{\sigma_n^2} \right) dr \right)
\]

(5.11)

where \( I_0(z) \) is modified Bessel function of the first kind of zero order, \( I_0(z) \) can be expanded in terms of following series [92]

\[
I_0(z) = \sum_{m=0}^\infty \frac{z^{2m}}{2^m \times (m!)^2}
\]

(5.12)

Using (5.12) and (5.13) we have

\[
P_{e2} = 1 - \left( \frac{1}{\sigma_n^2} \exp \left( -\frac{2d^2}{\sigma_n^2} \right) \int_0^3 \frac{r}{d} \exp \left( -\frac{r^2}{2\sigma_n^2} \right) \sum_{m=0}^\infty \frac{(d)^{2m}}{2^m \times (m!)^2} r^{2m+1} dr \right)
\]

(5.13)

By interchanging the order of the summation and integration, (5.14) can be written as

\[
P_{e2} = 1 - \left( \frac{1}{\sigma_n^2} \exp \left( -\frac{2d^2}{\sigma_n^2} \right) \int_0^3 \frac{r}{d} \exp \left( -\frac{r^2}{2\sigma_n^2} \right) \sum_{m=0}^\infty \frac{(d)^{2m}}{2^m \times (m!)^2} \int_0^\infty r^{2m+1} \exp \left( -\frac{r^2}{2\sigma_n^2} \right) dr \right)
\]

(5.14)

Let \( u = \frac{r^2}{2\sigma_n^2} \) and by substituting the value of \( u \) in (5.15), we have

\[
P_{e2} = 1 - \left( \frac{1}{\sigma_n^2} \exp \left( -\frac{2d^2}{\sigma_n^2} \right) \sum_{m=0}^\infty \frac{(d)^{2m}}{2^m \times (m!)^2} \int_0^\infty (u)^m \exp(-u) du \right)
\]

(5.15)

Using the standard result [92] for integration of (5.16), we can write (5.16) as
\[
P_{c2} = 1 - \left( \exp\left( -\frac{2d^2}{\sigma_n^2} \sum_{m=0}^{\infty} (d^2)^m \left( \frac{2}{\sigma_n^2} \right)^m \right) \exp(-u) \left( \sum_{m=0}^{\infty} u(m-1)(m-2)\ldots(m-k+1) \right) \left( \frac{d^2}{2\sigma_n^2} \right)^{m-k} \right)
\]

\[
P_{c2} = 1 - \left( \exp\left( -\frac{2d^2}{\sigma_n^2} \sum_{m=0}^{\infty} (d^2)^m \left( \frac{2}{\sigma_n^2} \right)^m \right) \exp\left( -\frac{d^2}{2\sigma_n^2} \right) \left( \frac{d^2}{2\sigma_n^2} \right)^m \sum_{k=1}^{m} u(m-1)(m-2)\ldots(m-k+1) \left( \frac{d^2}{2\sigma_n^2} \right)^{m-k} \right)
\]

\[
- \exp\left( -\frac{9d^2}{2\sigma_n^2} \right) \left( \frac{9d^2}{2\sigma_n^2} \right)^m \sum_{k=1}^{m} u(m-1)(m-2)\ldots(m-k+1) \left( \frac{d^2}{2\sigma_n^2} \right)^{m-k} \right)
\]

(5.17)

The constellation points of group \( G_3 \) are located on the circle of radius \( 4d \) and at an angle \( \left\{ 0, \frac{\pi}{2}, \pi \text{ or } \frac{3\pi}{2} \right\} \). The joint probability density function \( f_{3R,\phi}(r,\phi) \) for received signal (transmitted symbol plus noise) is given by

\[
f_{3R,\phi}(r,\phi) = \frac{r}{2\pi}\exp\left( -\frac{r^2 + 16d^2 - 8d\cos(\phi)}{2\sigma_n^2} \right)
\]

(5.18)

\[
P_{c3} = 1 - \left( \int_{\frac{\pi}{8}}^{\frac{\pi}{2}} \int_{\frac{r}{2d}}^{\infty} \exp\left( -\frac{r^2 + 16d^2 - 8d\cos(\phi)}{2\sigma_n^2} \right) dr d\phi \right)
\]

(5.19)

\[
P_{c3} = 1 - \left( \int_{\frac{\pi}{8}}^{\frac{\pi}{2}} \exp\left( -\frac{16d^2\sin^2(\phi)}{2\sigma_n^2} \right) \left( \int_{3d}^{\infty} \exp\left( -\frac{(r - 4d\cos(\phi))^2}{2\sigma_n^2} \right) dr \right) d\phi \right)
\]

(5.20)

By substituting \( u = r - 4d\cos(\phi) \) in (5.20), we have

\[
P_{c3} = 1 - \left( \int_{\frac{\pi}{8}}^{\frac{\pi}{2}} \exp\left( -\frac{16d^2\sin^2(\phi)}{2\sigma_n^2} \right) \left( \int_{3d-4d\cos(\phi)}^{\infty} (u + 4d\cos(\phi)) \exp\left( -\frac{u^2}{2\sigma_n^2} \right) du \right) d\phi \right)
\]

(5.21)

Using the standard result for the inner integral of (5.21), (5.21) can be written as

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In general the close form solution of the integration given in (5.22) is not possible to derive and it should be evaluated numerically. At high SNR and for $|\phi|<\pi$, the first term of the integration given in (5.22) is very small and is therefore neglected and the value of $Q(.)$ function is approximately equals to 1, thus (5.22) reduces to

$$P_{e3}^2 \approx 1 - \frac{\pi}{8} \int_{\frac{\pi}{8}}^{\infty} \frac{1}{2\pi} \exp\left(\frac{-2\pi^2 + 2\pi^2 \cos(\phi)}{2\sigma_n^2}\right) + \frac{4d \cos(\phi)}{\sqrt{2\pi\sigma_n^2}} \exp\left(-\frac{16d^2 \sin^2(\phi)}{2\sigma_n^2}\right) Q\left(\frac{3d - 4d \cos(\phi)}{\sigma_n}\right) d\phi$$

(5.23)

Let $\frac{4d \sin(\phi)}{\sigma_n} = \lambda$, (5.23) can now be written as

$$P_{e3}^2 \approx 1 - \frac{2}{\sqrt{\pi}} \int_0^{4d \sin(\pi/8)} e^{-\frac{\lambda^2}{2}} d\lambda$$

(5.24)

$$P_{e3}^2 \approx \text{erfc} \left(\frac{2d \sin\left(\frac{\pi}{8}\right)}{\sigma_n}\right)$$

(5.25)

The constellation points belonging to group $G_4$ are located on circle of radius $4d$ and at an angle $\left(\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \text{ or } \frac{7\pi}{4}\right)$, the error probability $P_{e4}^2$ is same as $P_{e3}^2$ because decision boundaries of constellation points in group $G_4$ and $G_3$ are similar.

$$\therefore P_{e4}^2 = P_{e3}^2$$

(5.26)

The overall average probability of symbol error for concentric circle constellation using circular boundary decoding can be calculated using (5.9), (5.17), (5.25) and (5.26). The average probability of error ($P_e^2$) can be calculated as follows
\[
P_e^2 = \frac{1}{4} (P_{e1}^2 + P_{e2}^2 + P_{e3}^2 + P_{e4}^2)
\]

(5.27)

\[
P_e^2 = \frac{1}{4} \left[ \exp \left( -\frac{d^2}{2\sigma_n^2} \right) \right] + 1
\]

- \exp \left( -\frac{4d^2}{\eta} \sum_{m=0}^{\infty} \left( \frac{d^2}{\eta} \right)^m \right) \left[ \exp \left( -\frac{d^2}{\eta} \right) + \sum_{k=1}^{m} \frac{m(m-1)\cdots(m-k+1)}{k!} \left( \frac{d^2}{\eta} \right)^{m-k} \right] + 2 \sum_{k=1}^{m} \frac{m(m-1)\cdots(m-k+1)}{k!} \left( \frac{d^2}{\eta} \right)^{m-k}
\]

(5.28)

By substituting the variance of AWGN \( \sigma_n^2 = \eta / 2 \), the (5.28), reduces to the form

\[
P_e^2 = \frac{1}{4} \left[ \exp \left( -\frac{d^2}{\eta} \right) \right] + 1
\]

- \exp \left( -\frac{9d^2}{\eta} \sum_{k=1}^{m} \frac{m(m-1)\cdots(m-k+1)}{k!} \left( \frac{9d^2}{\eta} \right)^{m-k} \right) + 2 \sum_{k=1}^{m} \frac{m(m-1)\cdots(m-k+1)}{k!} \left( \frac{9d^2}{\eta} \right)^{m-k}
\]

(5.29)

For \( d = 1 \), the average symbol energy of concentric circle constellation points shown in Fig. 5.4 is \( E_s = 9 \).

### 5.5.3 Calculation of Error Probability for MPSM Scheme

As discussed in section 5.3, MPSM scheme maps the quaternary data points on 16-QAM constellation by using four phase rotation factors (1, j, -1, -j). As seen from Fig. 5.3, the constellation points of 16-QAM are divided into four groups \( G_1^1, G_2^1, G_3^1 \) and \( G_4^1 \). Each of these four groups contains 4 constellation points. The constellation points belonging to the groups \( G_1^1, G_2^1, G_3^1 \) and \( G_4^1 \) are located at \{3 + 3j, -3 + 3j, -3 - 3j, 3 - 3j\}, \{-3 + j, -1 - 3j, 3 - j, 1 + 3j\}, \{-1 + j, 1 - j, 1 + j, -1 + j\} and \{1 - 3j, 3 + j, -1 + 3j, -3 - j\} respectively. The minimum Euclidean distance between any two constellation points is 2, same as in concentric
circle constellation. The constellation points in groups $G_1^\#$, $G_2^\#$, $G_3^\#$ and $G_4^\#$ have 2, 3, 2 and 3 nearest neighbours with an Euclidean distance of 2. The average symbol energy of constellation points shown in Fig. 5.3 is $E_s = 10$. Let $P_{e1}^\#$, $P_{e2}^\#$, $P_{e3}^\#$ and $P_{e4}^\#$ represent error probabilities of the 16-QAM constellation points belonging to groups $G_1^\#$, $G_2^\#$, $G_3^#$ and $G_4^\#$ respectively.

The constellation points belonging to group $G_1^\#$ have 2 nearest neighbours at a Euclidean distance of 2. The error probability $P_{e1}^\#$ using union bound approximation is given by

$$P_{e1}^\# = \left[ 4 \times 2 \times \frac{1}{2} \text{erfc} \left( \frac{0.1E_s}{\eta} \right) \right]$$  \hspace{1cm} (5.30)

Similarly, the constellation points belonging to groups $G_2^\#$, $G_3^#$ and $G_4^\#$ have 3, 2 and 3 nearest neighbours respectively and are at a Euclidean distance of 2 from them and their error probabilities $P_{e2}^\#$, $P_{e3}^\#$, and $P_{e4}^\#$ can be calculated using union bound approximation as

$$P_{e2}^\# = \left[ 4 \times 3 \times \frac{1}{2} \text{erfc} \left( \frac{0.1E_s}{\eta} \right) \right]$$  \hspace{1cm} (5.31)

$$P_{e3}^\# = \left[ 4 \times 2 \times \frac{1}{2} \text{erfc} \left( \frac{0.1E_s}{\eta} \right) \right]$$  \hspace{1cm} (5.32)

$$P_{e4}^\# = \left[ 4 \times 3 \times \frac{1}{2} \text{erfc} \left( \frac{0.1E_s}{\eta} \right) \right]$$  \hspace{1cm} (5.33)

The average error probability or SER ($P_e^\#$) of MPSM can be calculated using (5.30), (5.31), (5.32) and (5.32) as

$$P_e^\# = \frac{1}{16} \left[ 4 \times \text{erfc} \left( \frac{0.1E_s}{\eta} \right) + 6 \times \text{erfc} \left( \frac{0.1E_s}{\eta} \right) + 4 \times \text{erfc} \left( \frac{0.1E_s}{\eta} \right) + 6 \times \text{erfc} \left( \frac{0.1E_s}{\eta} \right) \right]$$

$$= \frac{5}{4} \text{erfc} \left( \frac{0.1E_s}{\eta} \right)$$  \hspace{1cm} (5.34)
5.6 COMPUTATIONAL COMPLEXITY ANALYSIS

In order to calculate the total computational complexity of CCM-PTS and MPSM-PTS schemes, we take both transmitter and receiver into consideration. As discussed in section 3.1, we used adjacent partitioning scheme to divide the entire data block into $S$ sub blocks. If we use oversampling by a factor $L$ then we have to calculate $LN$ point IFFT of each sub block. This requires $LN \log_2(LN)$ complex additions and $\frac{LN}{2} \log_2(LN)$ complex multiplications. For $S$ sub-blocks, we require $SLN \log_2(LN)$ and $\frac{SLN}{2} \log_2(LN)$ complex additions and multiplications respectively. In order to combine $S$ partial transmit sequences obtained from $S$ IFFT operations, $(S-1)LN$ complex additions are required. Here, we have to perform $W^{S-1}$ number of searches to get the OFDM signal with minimum PAPR; therefore, we require $W^{S-1}(S-1)LN$ number of complex additions. At the receiver, we have to perform one $LN$ point FFT for the subcarrier demodulation and for decoding each of the symbols we require $2DN$ additions and $DN$ multiplications, where $D$ is the number of decision regions.

The overall computational complexity can be calculated as

\[ n_{add} = \text{Total no. of complex additions at Tx. + Total no. of complex additions at Rx.} \]
\[ = SLN \log_2(LN) + W^{S-1}(S-1)LN + 2DN \]

\[ n_{mul} = \text{Total no. of complex multiplications at Tx. + Total no. of complex multiplications at Rx.} \]
\[ = \frac{SLN}{2} \log_2(LN) + DN \]

5.7 RESULTS AND DISCUSSION

In this chapter, the PAPR and error performance of all PTS based schemes under consideration are evaluated by computer simulations using MATLAB and verified with their corresponding mathematical results presented in section 5.5. In order to check the validity of simulation results, 10,000 OFDM symbols are considered. To compare the SER performance of the proposed CCM-PTS scheme with MPSM-PTS, over AWGN channel, we have
considered complex AWGN with zero mean. Fig. 5.7 shows the PAPR performance of original OFDM signal without PAPR reduction, scheme proposed in [84]-[86], [88], CCM-PTS with two phase factors, CCM-PTS with four phase factors and MPSM-PTS with four phase factors.

It can be observed from Fig. 5.7 that for a CCDF of PAPR=0.001 i.e. for 0.1% of the OFDM symbols, the original OFDM signal (without PAPR reduction) has a PAPR of 11dB or more, whereas the scheme proposed in [84]-[85] and CCM-PTS with two phase factors have PAPR of 9.1dB, 8.65dB and 8.5dB respectively and therefore achieves a PAPR reduction capability of 1.9dB, 2.35dB and 2.5dB respectively. The PAPR performances of the schemes proposed in [86],[88], CCM-PTS and MPSM-PTS with four phase factors are very close and have at least 1dB more PAPR reduction capability in comparison to the scheme proposed in [84], [85]. The scheme proposed in [84], [85] have limited number of candidate signals for PAPR reduction, which is the main reason behind these schemes to achieve limited PAPR reduction capability and hence, the schemes [84], [85] are not good choices in comparison to the remaining PAPR reduction schemes under consideration.

Figure 5.7: PAPR performance comparison of PTS based PAPR reduction schemes
As discussed earlier the PAPR performances of the schemes proposed in [86], [88] are very close to MPSM-PTS and CCM-PTS but their [86], [88] SI detection capability is very poor at lower values of SNR. Hence, the SI embedding schemes proposed in [86], [88] are also not good choices in comparison to MPSM-PTS [87] and CCM-PTS [89]. Therefore, we will have more focus on MPSM-PTS and CCM-PTS in the subsequent discussion.

In CCM-PTS with circular boundary decoding, we have 10 decision regions, one each for decoding the constellation point located at zero and on a circle of radius 2. Further to decode the eight constellation points located on the circle of radius 4 we require 8 decision regions. So, in this scheme we require a total of 10 different decoding regions (\(D\)). In CCM-PTS with minimum distance decoding rule, we have only 13 points and we require one separate decision region for each of them, so we have a total of 13 decoding regions (\(D\)). In MPSM-PTS with four phase factors quaternary data is mapped to a 16-QAM constellation which has 16 distinct points. To decode all of them, we require only 13 decision regions (\(D\)). A comparison of computational complexity of CCM-PTS and MPSM-PTS scheme for \(S = W = L = 4\) and \(N=256\) is given in Table 5.5. As seen from Table 5.5, the CCM-PTS with circular boundary decoding requires least computational complexity in comparison to all the schemes under consideration.

Table 5.5: Computational complexity of CCM-PTS and MPSM-PTS

<table>
<thead>
<tr>
<th></th>
<th>CCM-PTS</th>
<th>MPSM-PTS ((D=13))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum Distance Decoding ((D=13))</td>
<td>Circular Boundary Decoding ((D=10))</td>
</tr>
<tr>
<td>(n_{add})</td>
<td>244224</td>
<td>242688</td>
</tr>
<tr>
<td>(n_{mul})</td>
<td>23808</td>
<td>23040</td>
</tr>
</tbody>
</table>

The plot of the analytical results for SER performance of CCM-PTS (using minimum distance decoding and circular boundary decoding) and MPSM-PTS schemes over AWGN channel, discussed in section 4 are shown in Fig. 5.8. These results, in Fig. 5.8 are denoted by ‘CCM-PTS (1) Theoretical’, ‘CCM-PTS (2) Theoretical’ and ‘MPSM-PTS Theoretical’ respectively. To confirm the validity of results for schemes under consideration, simulations are performed using MATLAB. In Fig. 5.8, the simulation results for SER performance of CCM-PTS (using minimum distance decoding and circular boundary decoding) and MPSM-
PTS scheme over AWGN channel are denoted by ‘CCM-PTS(1) Simulated’, ‘CCM-PTS(2) Simulated’ and ‘MPSM-PTS Simulated’ respectively. All simulation results are coinciding with their analytical results and therefore the validity of simulation and theoretical results is confirmed. The results denoted by ‘CCM-PTS(1) Theoretical’ is the upper bound of SER and the results denoted by ‘CCM-PTS(1) Simulated’ will never exceed it.

Figure 5.8: SER performance comparison of CCM-PTS and MPSM-PTS over AWGN channel

It can be seen from Fig. 5.8 that SER performances of CCM-PTS using both the decoding schemes are better than MPSM-PTS. If we compare the two decoding techniques of CCM-PTS then minimum distance decoding technique provides the lower SER. Its justification is given as follows:

The CCM approach has only 13 constellation points and their average power is 9, whereas MPSM schemes has 16 constellation points and having an average power equal to 10.

Two main reasons associated with CCM to achieve better BER performance in comparison to MPSM are lesser number of constellation points with lesser average power while maintaining same Euclidean distance.
The CCM approach has only 13 constellation points and their average power is 9, whereas MPSM schemes has 16 constellation points and having an average power equal to 10.

Two main reasons associated with CCM to achieve better BER performance in comparison to MPSM are lesser number of constellation points with lesser average power while maintaining same Euclidean distance. In Fig. 5.9, the green coloured regions indicate the extra decoding areas provided by minimum distance decoding over circular boundary decoding for constellation points having lower power (located at zero and located at circumference of the circle with radius $2d$), whereas red coloured regions indicate the extra decoding areas provided by circular boundary decoding over minimum distance decoding for constellation points located on the circumference of the circle with radius $2d$.

![Figure 5.9: Comparisons of decoding regions of CCM using minimum distance and circular boundary decoding](image-url)
But the area occupied by green colour is larger than that of red coloured regions therefore CCM-PTS using minimum distance decoding rule achieves better BER performance in comparison to circular boundary decoding.

To achieve a SER=10^{-5}, CCM-PTS with circular boundary decoding and minimum distance decoding, we require a SNR of 19.5 and 19.8 dB. To achieve the same SER performance MPSM-PTS requires a SNR of 20dB. Hence for same SER performance, MPSM-PTS requires a SNR which is about 0.5dB and 0.8dB more, as compared to that required by CCM-PTS with circular boundary decoding and with minimum distance decoding, respectively.

The CCM-PTS with circular boundary decoding requires a SNR which is 0.3dB more as compared to CCM-PTS with minimum distance decoding. CCM-PTS with minimum distance decoding requires the least SNR amongst all the schemes under consideration, but CCM-PTS with circular boundary decoding is the simplest to decode and its computational complexity is the least among all the three.

The SER performance of the methods under consideration over fading channel diverges in comparison to their respective SER performance over AWGN because the intra symbol interference cannot be avoided using cyclic prefix of sufficient length (CP only eliminates inter symbol interference(ISI)).
5.6 CONCLUSION

In PTS based methods, SER performance depends on how SI is encoded with the OFDM symbol, and if it gets corrupted, then entire OFDM symbol may be erroneous. Existing SI embedding schemes eliminate the requirement of SI transmission but these suffer from one drawback or the other, whether in terms of computational complexity, poor PAPR reduction capability or incorrect SI detection. In this chapter, we have considered SI free PTS based methods (MPSM-PTS, CCM-PTS), which do not require SI at the receiver and therefore found to be the good alternative of SI embedding schemes. The PAPR reduction capabilities of CCM-PTS and MPSM-PTS schemes are found to be almost the same. The SER performances of such PAPR reduction schemes over AWGN are derived analytically and confirmed by using MATLAB simulations. However, their SER performances over fading channel are evaluated by doing simulations. The SER performances of CCM-PTS using both the decoding schemes over AWGN and fading channels are better than MPSM-PTS. The CCM-PTS with minimum distance decoding performs the best among them. In CCM-PTS, with circular boundary decoding, we require only 10 decoding regions, whereas other
requires 13 decoding regions. Therefore, the decoding complexity of CCM-PTS with circular boundary decoding is the least. At the same time, it is also seen that, this method merely results in 0.3 dB SER loss in comparison to CCM-PTS with minimum distance decoding. Hence, CCM-PTS with circular boundary decoding has better performance in comparison to MPSM-PTS both in terms of computational complexity and SER.