Chapter 5
Coloring of a Fuzzy Graph

5.1 Introduction
Coloring of a crisp graph is an age old problem. This problem is concerned with determining the minimum number of colors required for coloring the vertices of a graph so that no two adjacent vertices will have the same color. If k is this minimum number, then k is called the chromatic number and the graph is said to be k-colorable. A very famous problem called four color problem states that any planar graph is 4-colorable. This problem had remained as an open problem for long. The problem has been finally solved after years of unsuccessful attempts by veterans.

It is rather easy to prove that every graph is 5-colorable. Strictly speaking, the above problem should have been called vertex coloring since we are considering the coloring of the vertices. We can also look at the problem as the problem of coloring the edges of a graph so that no two adjacent edges will have the same color. The minimum number of colors required is called the edge chromatic number of the graph. We can combine vertex coloring and edge coloring. In other words, we would like to know the minimum number of colors required for coloring the vertices and edges so that no two adjacent vertices will have the same color, no two adjacent edges will have the same color and also a vertex and an edge which is incident on that vertex are differently colored. This problem is called total coloring. These problems have been studied by various authors [22-23].

Vertex coloring: Let N denote the set of all natural numbers and assume that a unique natural number is associated with each color. Given a graph $G = (V, E)$, a vertex coloring function is a function $C_V : V \to N$ such that $C_V(u) \neq C_V(v)$ for any two adjacent vertices $u$ and $v$. A vertex k-coloring $C_V^k$ is a vertex coloring function in which no more than k different colors are used. In other words $C_V^k : V \to \{1, 2, ..., k\}$. 
A graph is said to be vertex $k$-colored if it admits a vertex $k$-coloring. The minimum value of $k$ for which $G$ is vertex $k$-colored is called vertex chromatic number of $G$ and is denoted by $\chi_v(G)$.

**Edge coloring:** Given a graph $G = (V, E)$, an edge coloring function is a function $C_E : E \to \mathbb{N}$ such that $C_E(i, j) \neq C_E(i, k)$ and $C_E(i, j) \neq C_E(l, j)$ for all edges $(i, j)$, $(i, k)$ and $(l, j) \in E$. An edge $k$-coloring $C_E^k$ is an edge coloring function in which no more than $k$ different colors are used. In other words, $C_E^k : E \to \{1, 2, \ldots, k\}$.

A graph is said to be edge $k$-colored if it admits an edge $k$-coloring. The minimum value of $k$ for which $G$ is edge $k$-colored is called edge chromatic number of $G$ and is denoted by $\chi_e(G)$.

It is known that every graph $G$ can be edge colored with at most $\delta(G) + 1$ colors and at least $\delta(G)$ colors are always necessary where $\delta(G)$ denotes the maximum degree of $G$.

**Total coloring:** Given a graph $G = (V, E)$, a total coloring function is a function $C_T : V \cup E \to \mathbb{N}$ which satisfies the following conditions.

- $C_T(u) \neq C_T(v)$ for any two adjacent vertices $u, v \in V$.
- $C_T(i, j) \neq C_T(i, k)$ and $C_T(i, j) \neq C_T(l, j)$ for all edges $(i, j)$, $(i, k)$ and $(l, j) \in E$.
- $C_T(u) \neq C_T(u, v)$ and $C_T(v) \neq C_T(u, v)$ for any two adjacent vertices $u, v \in V$.

A total $k$ coloring $C_T^k$ is a total coloring function in which no more than $k$ different colors are used. In other words, $C_T^k : V \cup E \to \{1, 2, 3, \ldots, k\}$.

A graph is said to be total $k$-colored if it admits a total $k$-coloring. The minimum value of $k$ for which $G$ is total $k$-colored is called total chromatic number of $G$ and is denoted by $\chi_T(G)$.
We now consider fuzzy graphs. Again, vertex coloring of fuzzy graphs has been extensively studied [24 -28]. In this chapter, we have made an attempt to extend the concepts of vertex coloring of a fuzzy graph to edge coloring of a fuzzy graph. We then consider total coloring of a fuzzy graph. All the concepts introduced in this chapter have been illustrated by examples.

Analogous to vertex coloring given in [26], we introduce the concepts of \((d, f)\) extended \(k\) - coloring (both for edge coloring and total coloring). With examples, we show that some of the results of edge coloring / total coloring of a crisp graph do not carry over to our set up. We also develop algorithms for determining the \((d, f)\) edge chromatic number and \((d, f)\) total chromatic number of a fuzzy graph.

The results of this chapter have been published in *Advances in Fuzzy Mathematics, Volume 4, Number 1 (2009), P. 49 – 58* and in *International Journal of Computational and Applied Mathematics, Volume 5, Number 1 2010, P. 11 – 22*.

### 5.2 The \((d, f)\) – extended edge coloring function of a fuzzy graph

Let \(S\) denote the set of all available colors. As already mentioned, every color is associated with a unique natural number. Thus \(S\) can be thought of as a set of natural numbers. If \(n\) denotes the number of available colors, then without loss of generality, we can take \(S = \{1, 2, \ldots, n\}\). \(S\) is referred to as the color set.

A **dissimilarity measure** defined on \(S\) is a function \(d: S \times S \rightarrow [0, \infty)\) which satisfies the following properties for all \(r, s \in S\).

1. \(d (r, s) = 0 \iff r = s\)
2. \(d (r, s) = d (s, r)\)

The term dissimilarity measure stems from the fact that it measures the dissimilarity (incompatibility) between edges. The more incompatible two edges are, the more distant their associated colors are.
Consider a fuzzy graph $G = (V, \rho)$. Here we have taken $\mu(v) = 1$ for all $v \in V$. We note that for $u, v \in V$, $\rho(u, v)$ (also denoted by $\rho_{u,v}$) need not take only real values between 0 and 1. $\rho(u, v)$ can assume fuzzy values such as low, high etc. Let $I$ denote the image set of $\rho$. In other words, $I$ denotes the set of all membership grades assigned to the edges. We assume there is a relation $<$ defined on the elements of $I$.

Let $f : I \to [0, \infty)$ be a non decreasing function which means $f(\mu) \leq f(\mu')$ for all $\mu, \mu' \in I$ such that $\mu < \mu'$. $f$ is called a scale function.

For our purpose, we assume that a fuzzy graph $G$ has five components namely $(V, \rho, S, d, f)$ where $V$ denotes the vertex set, $\rho$ is a fuzzy subset of $V \times V$ which can assume fuzzy values, $S$ denotes the available color set, $d$ denotes the dissimilarity measure and $f$ denotes the scale function.

The dissimilarity measure and scale function introduced above lead to the following definition.

**Definition 5.2.1:** Given a fuzzy graph $G = (V, \rho, S, d, f)$, a $(d, f)$ - extended edge coloring function of $G$ denoted by $C_{df}$ or simply as $C$ is a mapping $C : E \to S$ satisfying the following.

\[
d(C(i,j), C(i,l)) \geq \land \{f(\rho_{i,j}), f(\rho_{i,l})\} \text{ for all edges } (i,j) \text{ and } (i,l).
d(C(i,j), C(l,j)) \geq \land \{f(\rho_{i,j}), f(\rho_{l,j})\} \text{ for all edges } (i,j) \text{ and } (l,j).
\]

A $(d, f)$ - extended edge $k$ - coloring $C_{df}^k$ or simply $C^k$ is a $(d, f)$ - extended edge coloring function which takes maximum $k$ different colors. In other words, $C^k : E \to S$ where $S = \{1, 2, \ldots, k\}$ which satisfies the following.

\[
d(C^k(i,j), C^k(i,l)) \geq \land \{f(\rho_{i,j}), f(\rho_{i,l})\} \text{ for all edges } (i,j) \text{ and } (i,l),
d(C^k(i,j), C^k(l,j)) \geq \land \{f(\rho_{i,j}), f(\rho_{l,j})\} \text{ for all edges } (i,j) \text{ and } (l,j).
\] (5.1)
Note that the \((d, f)\) - extended edge \(k\) - coloring of a fuzzy graph \(G\) is nothing but a generalization of the \(k\) - coloring of a crisp graph \(G = (V, E)\). Take \(I = \{0, 1\}\), \(f(0) = 0, f(1) = 1\) and \(d = d^o\) where \(d^o\) is defined as

\[
d^o(r, s) = \begin{cases} 
1 & \text{if } r \neq s \\
0 & \text{if } r = s
\end{cases}
\]

Edge coloring of a fuzzy graph differs from edge coloring of a crisp graph in the following sense.

Given a crisp graph \(G = (V, E)\), an edge coloring function for \(G\) always exists whereas given a fuzzy graph \(G = (V, \rho, S, d, f)\), a \((d, f)\) - extended coloring function for \(G\) need not exist as the following example shows.

**Example 5.2.1:**
Let \(G = (V, \rho, S, d, f)\) be a fuzzy graph where \(V = \{A, B, C, D\}\). Let \(I = \text{Image} (\rho) = \{n, l, m, h\}\) where \(n, l, m\) and \(h\) stand for null, low, medium and high respectively. We assume that \(n < l < m < h\). \(\rho\) is defined by the following matrix.

\[
\rho = \begin{pmatrix}
_ & h & m & m \\
h & _ & l & h \\
m & l & _ & l \\
m & h & l & _
\end{pmatrix}
\]

Let \(S = \{1, 2, 3, 4\}\) be the available color set and \(d\) be the dissimilarity measure defined as \(d(r, s) = |r - s|\).

Let the scale function \(f\) be defined as shown below.

<table>
<thead>
<tr>
<th>(I)</th>
<th>(n)</th>
<th>(l)</th>
<th>(m)</th>
<th>(h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f(I))</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 5.1.Scale function for example 5.2.1
This fuzzy graph is shown below.

![Fig. 5.1 Fuzzy graph of example 5.2.1](image)

Consider the following cases.

**Case 1:** $C^4(A, B) = 1$ and $C^4(A, C) = 3$. This assignment of colors is possible because $d(C^4(A, B), C^4(A, C)) = 2$ whereas $\hat{f}(\rho_{A,B}) \land \hat{f}(\rho_{A,C}) = 2$. It is not possible to color $(A, D)$ with either 2 or 4 because $d(C^4(A, C), C^4(A, D)) = 1$ when $C^4(A, D) = 2$ or when $C^4(A, D) = 4$. However $\hat{f}(\rho_{A,C}) \land \hat{f}(\rho_{A,D})$ is 2. We thus note that inequality (5.1) is not satisfied.

**Case 2:** $C^4(A, B) = 1$ and $C^4(A, C) = 4$. This assignment of colors is possible because $d(C^4(A, B), C^4(A, C)) = 3$ whereas $\hat{f}(\rho_{A,B}) \land \hat{f}(\rho_{A,C})$ is 2. It is not possible to color $(A, D)$ with either 2 or 3. Note that $d(C^4(A, C), C^4(A, D)) = 1$ when $C^4(A, D) = 3$ whereas $\hat{f}(\rho_{A,C}) \land \hat{f}(\rho_{A,D})$ is 2, $d(C^4(A, B), C^4(A, D)) = 1$ when $C^4(A, D) = 2$ whereas $\hat{f}(\rho_{A,B}) \land \hat{f}(\rho_{A,D})$ is 2 violating inequality (5.1).

**Case 3:** $C^4(A, B) = 2$ and $C^4(A, C) = 4$. This assignment of colors is possible because $d(C^4(A, B), C^4(A, C)) = 2$ whereas $\hat{f}(\rho_{A,B}) \land \hat{f}(\rho_{A,C})$ is 2. Now it is not possible to color $(A, D)$ with either 1 or 3. Note that $d(C^4(A, C), C^4(A, D)) = 1$ when $C^4(A, D) = 3$ whereas $\hat{f}(\rho_{A,C}) \land \hat{f}(\rho_{A,D})$ is 2, $d(C^4(A, B), C^4(A, D)) = 1$ when $C^4(A, D) = 1$ whereas $\hat{f}(\rho_{A,B}) \land \hat{f}(\rho_{A,D})$ is 2 again violating inequality (5.1).

**Case 4:**

- $C^4(A, B) = 1$ and $C^4(A, C) = 2$ or $C^4(A, B) = 2$ and $C^4(A, C) = 3$ or $C^4(A, B) = 3$ and $C^4(A, C) = 4$. 

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Studies in Fuzzy Graphs

Fig. 5.1 Fuzzy graph of example 5.2.1
Above assignments of colors is not possible because in each case
d(C^k(A, B), C^k(A, C)) = 1 whereas f(p_{A, B}) \land f(p_{A, C}) = 2 which violates
inequality (5.1).
Thus it is not possible to color the above graph with the available colors.

In a (d, f) - extended k coloring, it might happen that there are some colors which are
not assigned to any of the edges as the following example shows.

Example 5.2.2:
Let G = (V, p, S, d, f) be a fuzzy graph where V = {A, B, C, D, E}, I = Image (p) =
{n, l, m, h} where n, l, m and h have the same meanings as in Example 5.2.1.
\(\rho\) is defined by the following matrix.

\[
\rho = \begin{pmatrix}
- & l & l & n & m \\
1 & - & m & n & h \\
l & - & m & n & h \\
l & m & - & h & n \\
n & n & h & - & m \\
m & h & n & m & - \\
\end{pmatrix}
\]

Let S = \{1, 2, 3, 4, 5\} be the available color set. The dissimilarity measure d is
defined below.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>2</td>
<td>3</td>
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<td>0</td>
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<td>2</td>
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<tr>
<td>4</td>
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<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Let the scale function \(f\) be defined as shown below.

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>n</th>
<th>l</th>
<th>m</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f(I))</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>
This fuzzy graph is shown below.

![Fuzzy Graph of Example 5.2.2]

It can be easily seen that a \((d, f)\) – extended 5 – coloring is
\[
C^5(A, B) = 1, C^5(A, C) = 2, C^5(A, E) = 3, C^5(B, C) = 3, C^5(B, E) = 5, C^5(C, D) = 5, \\
C^5(D, E) = 1.
\]

Note that color 4 is not assigned to any of the edges of G.

5.3 Algorithm to determine the \((d, f)\) - edge chromatic number

We read the graph in the form of an adjacency matrix \texttt{adjMatrix [i][j]}.

The membership grades on the edges of the graph are stored in a two dimensional array \texttt{weight [i][j]}.

The number of colors \texttt{k} available to color the edges of the graph and their color difference table \texttt{colorTable [i][j]} also read.

class node with ‘index’ and ‘deg’ as member variables is created. class edge with member variables ‘a’, ’b’ (end vertices) and ‘color’ are created. The vertices are sorted in descending order of their degree and stored in vertex array. An array of edges \texttt{e [totaldeg]} is created and array \texttt{e [enumber]} results with enumber of edges after removing the redundant edges.

The program calls the procedure \texttt{graphColor (adjMatrix, n, k)} to properly color the graph. This procedure in turn calls the procedure \texttt{edgeColor (e, eNumber, 0)}, to start coloring the edges of the graph. The importance of this procedure is that, it is recursively called to color all the edges of the graph. For each edge, it get list of colors from procedure \texttt{getavailcol ( )}, after removing all the colors already used to
color the edges incident on the vertex considered and by removing those colors which
do not satisfy the colors difference table. Color the edge with the first available color
(preferred color). If not possible to color any particular edge, it backtracks to change
the color of the already colored previous edge with next preferred color. The
procedure edgecolor() returns true to graphcolor() only when all edges are colored.
Colors used to color the edges are now stored in the finalcolorset[ ] array.

sub graphColor (adjMatrix, n, k) // To Properly color the graph
    node vertex // array of vertices sorted in 'Descending' order of their degree
    edge e // array of eNumber of edges with 'a', 'b' and color as elements
if (edgeColor(e, eNumber, 0)) // start coloring edges of the graph
    for i = 0 to eNumber
        for j = 0 to finalK
            if (finalColorSet[j] = getAvailColor(e, i, n, 0, finalColorSet|j|))
                e[i].color = finalColorSet[j] //assign new color toe [i] and update colMatrix
            end if
        end for
    end for
    get finalColorSet array // array of colors used to color e with finalK numbers of colors
    return true // returns to main with finalColorSet indicating successful coloring
end if
return false // returns to main indicating coloring not possible
end sub

int edgeColor (edge e, eNumber, index) // To color all edges of the graph
    c = getAvailColor(e, index, n, ic, k +1)
    if (c = = -1) break
    e [index].color = c // color e[index] with color c and update colMatrix
    edgeColor(e, eNumber, index + 1) // call recursively edgeColor
if (all edges in e colored)
    return true // returns to graphColor
else
    return false // back track to edgeColor to color previous edge
end if
end sub
int getAvailColor (edge e, index, n, ic, preferredColor)

Start coloring edge (v, nib)  //Assume that all colors are available
remove all colors already used to color edges starting from 'v' and 'nib'.
for i = 0 to n
    for c = 0 to k
        if (colorTable [c][colMatrix [v][i]] < min (weight [v][nib], weight [v][i]))
            availCol [c] = 0  // remove all the colors which will not satisfy color difference table
        end for
    end for
    x = 0
    for c = 0 to (c < k && x < ic)  // points first preferred color
        if (availColour [c] == 1)
            x = x + 1
        end if
    end for
    if (x = ic)
        return c - 1  // return index of the color to edgeColor
    else
        return -1  // returns to edgeColor indicating coloring not possible
    end if
end sub

sub main

Read the graph of n vertices in terms of Adjacency matrix adjMatrix [i][j]
Read the membership values for the edges of the graph weight [i][j]
Read the number of available colors k and color difference table colorTable [i][j]
Initiate colMatrix.
if (edge exists between i and j)
    colMatrix [i][j] = k + 1
else
    colMatrix [i][j] = -1
if (graphColor (adjMatrix, n, k)) // calling graphColor
    print finalColorSet and K  // K total number of colors used to color the graph
else
    print "can't be colored ..."
end sub
**Definition 5.3.1:** For a given fuzzy graph $G = (V, \rho, S, d, f)$, the **minimum** value of $k$ for which a $(d, f)$ - extended edge $k$ - coloring exists is called the $(d, f)$ - edge chromatic number of $G$ and is denoted by $\chi_{d/E}(G)$.

**Example 5.3.1:**
Let $G = (V, \rho, S, d, f)$ be a fuzzy graph where $V = \{A, B, C, D, E\}$, $I = \text{Image}(\rho) = \{n, l, m, h\}$ where $n, l, m, h$ are defined exactly the same way as in examples 5.2.1 and 5.2.2.

$\rho$ is defined by the following matrix.

\[
\begin{pmatrix}
_ & m & m & l & m \\
m & _ & l & m & m \\
m & l & _ & h & l \\
l & m & h & _ & l \\
m & m & l & l & _
\end{pmatrix}
\]

The scale function $f$ is as shown below.

<table>
<thead>
<tr>
<th>$I$</th>
<th>n</th>
<th>l</th>
<th>m</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(I)$</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

The fuzzy graph is represented by the figure given below.

Fig 5.3 Fuzzy graph of example 5.3.1
Let $S = \{1, 2, 3, 4\}$ be the available color set and $d$ be the dissimilarity measure as defined below.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
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<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>6</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5.5 Color difference table

It can be seen that it is not possible to color the edges with these four colors.

Suppose $S = \{1, 2, 3, 4, 5\}$ and $d$ is defined as shown below.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>4</td>
<td>2</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5.6 Color difference table

It can be seen that one $(d, f)$ - extended 5-coloring is

$C^5(A, B) = 1$, $C^5(A, C) = 4$, $C^5(A, D) = 2$, $C^5(A, E) = 5$, $C^5(B, C) = 3$, $C^5(B, D) = 5$, $C^5(B, E) = 4$, $C^5(C, D) = 1$, $C^5(C, E) = 2$, $C^5(D, E) = 3$.

We note that all the five colors are used and the $(d, f)$ edge chromatic number is 5.

**Example 5.3.2:**

Consider the graph given below. Its adjacency matrix is given in Table 5.7.

![Fuzzy graph of example 5.3.2](image)
Table 5.7. Adjacency matrix for example 5.3.2

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
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<td>A</td>
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<tr>
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<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>1</td>
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<td>0</td>
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<tr>
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<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
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</tbody>
</table>

The membership grades on the edges are given below.

Table 5.8 Membership table for example 5.3.2

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.0</td>
<td>0.2</td>
<td>0.0</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>B</td>
<td>0.2</td>
<td>0.0</td>
<td>0.4</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>C</td>
<td>0.0</td>
<td>0.4</td>
<td>0.0</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>D</td>
<td>0.4</td>
<td>0.0</td>
<td>0.6</td>
<td>0.0</td>
<td>0.2</td>
</tr>
<tr>
<td>E</td>
<td>0.6</td>
<td>0.0</td>
<td>0.4</td>
<td>0.2</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The number of available colors is three and their color difference table is given below.

Table 5.9 Color difference table

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.8</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>1.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

It can be seen that it is **not possible** to color the graph with these three colors.

Now assume that the number of available colors is 4 and their color difference table is as shown below.

Table 5.10 Color difference table

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.8</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
<td>0.0</td>
<td>1.0</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>1.0</td>
<td>0.0</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>0.0</td>
</tr>
</tbody>
</table>

**Outputs:**

- The edge (A, B) is colored with 4
- The edge (A, D) is colored with 2
- The edge (A, E) is colored with 3
- The edge (C, B) is colored with 2
- The edge (C, D) is colored with 3
- The edge (C, E) is colored with 1
- The edge (D, E) is colored with 4
- The (d, f) edge chromatic number is 4 and the colors used are 1, 2, 3 and 4.
Example 5.3.3:
Consider the graph in Example 5.3.2. Assume there are five colors whose color difference table is as shown below.

Table 5.11 Color difference table

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.4</td>
<td>0.5</td>
<td>0.7</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
<td>0.0</td>
<td>0.9</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>0.9</td>
<td>0.0</td>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>4</td>
<td>0.7</td>
<td>0.6</td>
<td>0.2</td>
<td>0.0</td>
<td>0.3</td>
</tr>
<tr>
<td>5</td>
<td>0.3</td>
<td>0.5</td>
<td>0.6</td>
<td>0.3</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The output produced by the algorithm is as follows.
- The edge (A, B) is colored with 4
- The edge (A, D) is colored with 2
- The edge (A, E) is colored with 3
- The edge (C, B) is colored with 2
- The edge (C, D) is colored with 3
- The edge (C, E) is colored with 1
- The edge (D, E) is colored with 4
- The (d, f) edge chromatic number is 4 and the colors used are 1, 2, 3 and 4.

Example 5.3.4:
Consider the graph given in Example 5.3.1. Assume there are six colors. Membership grades on the edges and color difference table are given below.

Table 5.12 Membership table for example 5.3.4

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.0</td>
<td>0.5</td>
<td>0.0</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>B</td>
<td>0.5</td>
<td>0.0</td>
<td>0.8</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>C</td>
<td>0.0</td>
<td>0.8</td>
<td>0.0</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>D</td>
<td>0.1</td>
<td>0.0</td>
<td>0.4</td>
<td>0.0</td>
<td>0.9</td>
</tr>
<tr>
<td>E</td>
<td>0.3</td>
<td>0.0</td>
<td>0.2</td>
<td>0.9</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Output produced by the algorithm:

The edge (A, B) is colored with 6
The edge (A, D) is colored with 4
The edge (A, E) is colored with 3
The edge (C, B) is colored with 3
The edge (C, D) is colored with 1
The edge (C, E) is colored with 4
The edge (D, E) is colored with 6

The (d, f) edge chromatic number is 4 and the colors used are 1, 3, 4 and 6.

5.4 The (d, f) \(-\) extended total coloring function of a fuzzy graph

Definition: Consider a fuzzy graph \(G = (V, p, S, d, f)\) where \(V\) denotes the vertex set and \(p\) is a (partial) function defined on \(V \times V\). Note that \(p(u, v)\) is defined only for those \(u, v \in V\) for which \((u, v) \in E\). As before, \(p\) can take its values from \([0, 1]\) or from a fuzzy set. \(S\) denotes the available color set, \(d\) denotes the dissimilarity measure and \(f\) denotes the scale function. A \((d, f)\) \(-\) extended vertex coloring of \(G\) denoted by \(C_{\text{vdf}}\) or simply \(C_v\) is a mapping \(C_v: V \rightarrow S\) which satisfies the condition

\[d(C_v(i), C_v(j)) \geq f(p_{i,j})\] for all \(i, j \in V\) for which \((i, j) \in E\).

A \((d, f)\) extended vertex k-coloring \(C_{\text{vdf}}^k\) or simply \(C_v^k\) is a \((d, f)\) \(-\) extended vertex coloring function which takes maximum \(k\) different colors. In other words, \(C_v^k: V \rightarrow S\) where \(S = \{1, 2, \ldots, k\}\) satisfying the following.

\[d(C_v^k(i), C_v^k(j)) \geq f(p_{i,j})\] for all \(i, j \in V\) for which \((i, j) \in E\).
A - (d, f) extended total coloring of G denoted by $C_{\text{tot}}$ or simply $C_T$ is a mapping from $V \cup E$ to $S$ which satisfies the following.

- $d(C_T(i), C_T(j)) \geq f(p_{i,j})$ for all $i, j \in V$. (5.2)
- $d(C_T(i), C_T(i, j)) \geq A(f(p_{i,j}), f(p_{i,l}))$ and (5.3)
- $d(C_T(i, j), C_T(i, l)) \geq A(f(p_{i,j}), f(p_{i,l}))$ for all edges $(i, j)$ and $(i, l)$. (5.4)

- $d(C_T(i, j), C_T(i, l)) > A(f(p_{i,j}), f(p_{i,l}))$ for all edges $(i, j) \in E$.

A (d, f) extended total k-coloring $C_{\text{tot}}^k$ or simply $C_T^k$ is a (d, f) - extended total coloring function which takes maximum k different colors. In other words, $C_T^k : V \cup E \rightarrow S$ where $S = \{1, 2, \ldots, k\}$ satisfying the inequalities (5.2), (5.3) and (5.4) given above.

Following examples illustrate that total coloring of a fuzzy graph differs from total coloring of a crisp graph.

**Example 5.4.1:**

Let $G = (V, \rho, S, d, f)$ be a fuzzy graph where $V = \{A, B, C, D\}$ and $I = \{n, l, m, h\}$ where $n$, $l$, $m$ and $h$ stand for null, low, medium and high respectively. We assume that $n < l < m < h$. $\rho$ is defined by the following matrix.

\[
\rho = \begin{pmatrix}
- & l & n & h \\
- & - & l & 1 \\
n & l & - & m \\
h & l & m & -
\end{pmatrix}
\]

Let $S = \{1, 2, 3, 4\}$ be the available color set. $d$ is defined as $d(r, s) = |r - s|$. The scale function $f$ is defined as shown below.

<table>
<thead>
<tr>
<th>I</th>
<th>n</th>
<th>l</th>
<th>m</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(I)$</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 5.14 Scale function for example 5.4.1
This fuzzy graph is given below.

![Fuzzy Graph](image)

Fig 5.5 Fuzzy graph of example 5.4.1

It is possible to vertex color the above fuzzy graph with the following assignment of colors.

**Alternative a:** \( C^4_T (A) = 1, C^4_T (B) = 2 \) or \( 3, C^4_T (C) = 1, C^4_T (D) = 4. \)

**Alternative b:** \( C^4_T (A) = 4, C^4_T (B) = 2 \) or \( 3, C^4_T (C) = 4, C^4_T (D) = 1. \)

The assignment of colors in alternative a satisfies the inequality (5.2) as shown below.

\[
\begin{align*}
& d (C^4_T (A), C^4_T (B)) = 1 \text{ or } 2 \text{ whereas } f (\rho_{A,B}) = 1 \\
& d (C^4_T (A), C^4_T (D)) = 3 \text{ whereas } f (\rho_{A,D}) = 3 \\
& d (C^4_T (B), C^4_T (D)) = 2 \text{ or } 1 \text{ whereas } f (\rho_{B,D}) = 1 \\
& d (C^4_T (C), C^4_T (D)) = 3 \text{ whereas } f (\rho_{C,D}) = 2 \\
& d (C^4_T (C), C^4_T (B)) = 1 \text{ or } 2 \text{ whereas } f (\rho_{B,C}) = 1.
\end{align*}
\]

But with this assignment of colors to vertices, it is not possible to color the edges. Note that the edge AD can be colored with either 2 or 3 only and the edge CD can be colored with either 2 or 3 only. The edge BD can be colored with either 1 or 3 when B is colored with 2 and D is colored with 4. Alternatively, the edge BD can be colored with either 1 or 2 when B is colored with 3 and D is colored with 4.

**Case 1:** \( C^4_T (AD) = 2, C^4_T (BD) = 1. \) This assignment of colors is possible because

\[
\begin{align*}
& d (C^4_T (AD), C^4_T (BD)) = 1 \text{ whereas } f (\rho_{A,D}) \land f (\rho_{B,D}) = 1. \text{ But now it is not possible } \text{to color CD with 2 or 3. The former is ruled out because AD is already colored with 2 and the latter is not possible because in that case } d (C^4_T (AD), C^4_T (CD)) = 1 \text{ whereas } f (\rho_{A,D}) \land f (\rho_{C,D}) \text{ is 2 which violates inequality (5.3).}
\end{align*}
\]
Case 2: $C^4_T(AD) = 2, C^4_T(BD) = 3$. This assignment of colors is possible because $d(C^4_T(AD), C^4_T(BD)) = 1$ whereas $f(\rho_{A,D}) \wedge f(\rho_{B,D}) = 1$. However, it is now not possible to color CD with either 2 or 3 because these colors are already assigned to edges AD and BD.

Case 3: $C^4_T(AD) = 3$. Then $C^4_T(BD)$ cannot be 3 and the only possibility is that $C^4_T(BD) = 1$. This assignment of colors is possible because $d(C^4_T(AD), C^4_T(BD)) = 2$ whereas $f(\rho_{A,D}) \wedge f(\rho_{B,D}) = 1$. Now it is not possible to color CD with 3 (which is already assigned to AD). It is also not possible to color CD with 2 because in this case $d(C^4_T(AD), C^4_T(CD)) = 1$ whereas $f(\rho_{A,D}) \wedge f(\rho_{C,D}) = 2$ which violates inequality (5.3).

Case 4: $C^4_T(AD) = 3, C^4_T(BD) = 2$. This assignment of colors is possible because $d(C^4_T(AD), C^4_T(BD)) = 1$ whereas $f(\rho_{A,D}) \wedge f(\rho_{B,D}) = 1$. However, it is now not possible to color CD with either 2 or 3 because these colors are already assigned to edges AD and BD.

The assignment of colors in alternative b satisfies the inequality (5.2) as shown below.

$d(C^4_T(A), C^4_T(B)) = 2$ or $1$ where as $f(\rho_{A,B}) = 1$

$d(C^4_T(A), C^4_T(D)) = 3$ where as $f(\rho_{A,D}) = 3$

$d(C^4_T(B), C^4_T(D)) = 1$ or $2$ where as $f(\rho_{B,D}) = 1$

$d(C^4_T(C), C^4_T(D)) = 3$ where as $f(\rho_{C,D}) = 2$

$d(C^4_T(C), C^4_T(B)) = 2$ or $1$ where as $f(\rho_{B,C}) = 1$.

But with this assignment of colors to vertices, it is not possible to color the edges. Note that the edge AD can be colored with either 2 or 3 only and the edge CD can be colored with either 2 or 3 only. Edge BD can be colored with either 3 or 4 when B is colored with 2 and D is colored with 1. Again, the edge BD can be colored with either 2 or 4 when B is colored with 3 and D is colored with 1.
Case 1: \( C_T^4(AD) = 2, C_T^4(BD) = 3 \). This assignment of colors is possible because 
\[ d(C_T^4(AD), C_T^4(BD)) = 1 \] whereas \( f(\rho_{A,D}) \land f(\rho_{B,D}) = 1 \). However, it is now not possible to color CD with either 2 or 3 because these colors are already assigned to edges AD and BD.

Case 2: \( C_T^4(AD) = 2, C_T^4(BD) = 4 \). This assignment of colors is possible because 
\[ d(C_T^4(AD), C_T^4(BD)) = 2 \] whereas \( f(\rho_{A,D}) \land f(\rho_{B,D}) = 1 \). But now it is not possible to color CD with 2 or 3. The former is ruled out because AD is already colored with 2 and the latter is not possible because in that case 
\[ d(C_T^4(AD), C_T^4(CD)) = 1 \] whereas \( f(\rho_{A,D}) \land f(\rho_{C,D}) = 2 \) which violates inequality (5.3).

Case 3: \( C_T^4(AD) = 3, C_T^4(BD) = 4 \). This assignment of colors is possible because 
\[ d(C_T^4(AD), C_T^4(BD)) = 1 \] whereas \( f(\rho_{A,D}) \land f(\rho_{B,D}) = 1 \). Now it is not possible to color CD with 3 (which is already assigned to AD) and also with 2 because in this case 
\[ d(C_T^4(AD), C_T^4(CD)) = 1 \] whereas \( f(\rho_{A,D}) \land f(\rho_{C,D}) = 2 \) which violates inequality (5.3).

Case 4: \( C_T^4(AD) = 3, C_T^4(BD) = 2 \). This assignment of colors is possible because 
\[ d(C_T^4(AD), C_T^4(BD)) = 1 \] whereas \( f(\rho_{A,D}) \land f(\rho_{B,D}) = 1 \). However it is now not possible to color CD with either 2 or 3 because these colors are already assigned to edges AD and BD.

Thus it is not possible to color the above graph with the available colors.

In a \((d, f)\) extended total k coloring, it might happen that there are some colors which are not assigned to any of the vertices or edges as shown in the following example.

Example 5.4.2:
Let \( G = (V, \rho, S, d, f) \) be a fuzzy graph where \( V = \{A, B, C, D, E\} \). Let \( I = \{n, l, m, h\} \) where \( n, l, m \) and \( h \) have the same meanings as in the above example.
$\rho$ is defined by the following matrix.

$$
\rho = \begin{pmatrix}
  - & h & h & m \\
  h & - & h & m \\
  h & h & - & m \\
  m & m & m & - \\
\end{pmatrix}
$$

Let $S = \{1, 2, 3, 4, 5, 6\}$ be the available color set. The dissimilarity measure $d$ and the scale function $f$ are as follows.

**Table 5.15 Color difference table**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>6</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>0</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 5.16 Scale function for example 5.4.2**

<table>
<thead>
<tr>
<th></th>
<th>l</th>
<th>n</th>
<th>l</th>
<th>m</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(l)</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

This fuzzy graph is as shown below.

![Fig 5.6 Fuzzy graph of example 5.4.2](image)

A $(d, f)$ - extended total coloring is

$C_T^4(A) = 1$, $C_T^4(B) = 4$, $C_T^4(C) = 6$, $C_T^4(D) = 2$,

$C_T^4(A, B) = 2$, $C_T^4(A, C) = 4$, $C_T^4(A, D) = 3$, $C_T^4(B, C) = 3$, $C_T^4(B, D) = 6$,

$C_T^4(C, D) = 1$.

Note that color 5 is not assigned to any of the vertices or edges of $G$. 

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5.5 Algorithm to determine the (d, f) - total chromatic number

The graph is read in the form of adjacent matrix adjMatrix [i][j]. The membership values for the edges of the graph are read in terms of two dimensional array weight [i][j]. The number of colors k provided to color the graph and their color difference table colorTable [i][j] are read.

class node with 'index' and 'deg' as member variables is created. class edge with 'a', 'b' and 'color' as member variables is created. The vertices are sorted in descending order and stored in vertex array vertex[v]. An array of edges e [totaldeg] is created and array e [enumber] is obtained after removing the redundant edges.

The program first vertex color the graph by calling procedure greedycol (v0, C [i], vMax, cMax) where v0, the first node to be colored, C [i] array to store the color assigned to nodes, vMax, the first node with highest assigned color and cMax, the highest assigned color.

To edge color the graph, program calls the procedure grapgColor (adjMatrix, n, k) to properly color the graph. This procedure in turn calls the procedure edgeColor (e, eNumber, 0 ), to start coloring the edges of the graph. The importance of this procedure is that, it is recursively called to color all the edges of the graph. For each edge, it get list of colors from procedure getavailcol ( ), after removing all the color already used to color the edges incident on the vertex, color used to color the vertex considered and by removing those color which does not satisfy the color difference table. Color the edge with first available color (preferred color). If not possible to color the edge considered, it backtrack to change the color of the already colored previous edge with next preferred color and continue to color the edges. The procedure edgecolor ( ) returns true to grapgcolor ( ) only when all edges are colored. Colors used to color the edges are now stored in the finalcolorset [i] array. The procedure grapgcolor ( ) uses this finalcolorset [i] to check whether it is possible to color the edges of the graph (proper coloring) using colors of this finalcolorset.
Studies in Fuzzy Graphs

sub greedycol (v0, C [i], vMax, cMax) // Vertex coloring
    istop = 0, i = v0, c = C [i] // i th node, c color
    do while (istop = 0 )
        j = 0 // previous node
        do while (j < i)
            if (d [c][C[j] < weight [i][j]]) // c and C[j] are not compatible
                c = c + 1 // consider next color
                if (c > cMax) // indicating not possible to color i th node
                    j = i
                else
                    else // c and C[j] are compatible
                        j = j + 1 // Another previous node
                    endif
            endif
        enddo
        if (c < cMax) // c is compatible
            C [i] = c, i = i + 1, c = 0 // color i th node with c, start coloring next node
            if (i > n) then istop = 1 // all nodes are colored
                else // c is not compatible
                    i = i - 1, c = C [i] + 1 // backtrack to change already colored ith node color
                endif
        endif
    enddo

sub grapgColor (adjMatrix, n, k) // To Properly edge color the graph
    node vertex // array of vertices sorted in ‘Descending’ order of their degree
    edge e // array of eNumber of edges with ‘a’, ‘b’ and ‘color’ as elements
    if (edgeColor (e, eNumber, 0)) // start coloring edges of the graph
        for i = 0 to eNumber
            for j = 0 to finalK
                if (finalColorSet [j] = getAvailColor (e, i, n, 0, finalColorSet[j]))
                    e[i].color = finalColorSet [j] // assign new color to [i] and update colMatrix
                    endif
                get finalColorSet array
                return true // returns to main with finalColorSet indicating successful coloring
            end if
            return false // returns to main indicating coloring not possible
        end sub
int edgeColor (edge e, eNumber, index) // To color all edges of the graph
c = getAvailcolor (e, index, n, ic, k+1)
if (c == -1) break
    e[index].color = c // color e[index] with color c and update colMatrix
eedgeColor (e, eNumber, index +1) // call recursively edgeColor
if (all edges in e colored)
    return true // returns to grapgColor
else
    return false // back track to edgeColor to color previous edge
end if
end sub

sub main
Read the graph of n vertices in terms of Adjacency matrix adjMatrix [i][j]
Read the membership values for the edges of the graph weight [i][j]
Read the number of available colors k and color difference table colorTable [i][j]
Initialize cMax = k + 1 and C [0] = -1.
    Initiate Colmatrix.
    if (edge exists between i and j)
        colMatrix [i][j] = k + 1
    else
        colMatrix [i][j] = -1
    greedycol (0, C, vMax, cMax) // Vertex coloring
    if (grapgColor (adjMatrix, n, k)) // calling grapgColor
        print finalColorSet and K // K final total number of colors used to color the graph
    else
        print "can't be colored ..."
end sub

Definition 5.5.1: For a given fuzzy graph G = (V, p, S, d, f), the minimum value of k for which a (d, f) - extended Total k - coloring exists is called the (d, f) - Total chromatic number of G and is denoted by $\chi_{d/f}^T (G)$.
Example 5.5.1:
Consider a graph with 6 vertices (figure 5.7) whose adjacency matrix is given below.

Table 5.17 Adjacency table for example 5.5.1

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.0</td>
<td>0.3</td>
<td>0.5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>B</td>
<td>0.3</td>
<td>0.0</td>
<td>0.5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>C</td>
<td>0.0</td>
<td>0.5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>D</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.3</td>
</tr>
<tr>
<td>E</td>
<td>0.0</td>
<td>0.3</td>
<td>0.0</td>
<td>0.0</td>
<td>0.3</td>
<td>0.0</td>
</tr>
<tr>
<td>F</td>
<td>0.5</td>
<td>0.0</td>
<td>0.5</td>
<td>0.0</td>
<td>0.3</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The membership grades on the edges are given in the following table.

Table 5.18 Membership table for example 5.5.1

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.0</td>
<td>0.3</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>B</td>
<td>0.3</td>
<td>0.0</td>
<td>0.5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>C</td>
<td>0.0</td>
<td>0.5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>D</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>E</td>
<td>0.0</td>
<td>0.3</td>
<td>0.0</td>
<td>0.0</td>
<td>0.3</td>
<td>0.0</td>
</tr>
<tr>
<td>F</td>
<td>0.5</td>
<td>0.0</td>
<td>0.5</td>
<td>0.0</td>
<td>0.3</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Assume that the number of available colors is four. Their color difference table is given below.

Table 5.19 Color difference table

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>
Studies in Fuzzy Graphs

Output for vertex coloring is as follows.
Vertex A is colored with 1, vertex B is colored with 2, vertex C is colored with 1, vertex D is colored with 2, vertex E is colored with 1 and vertex F is colored with 2.
But it is **not possible** to color the edges with the given four colors.

Suppose the number of available colors is five whose color difference table is as shown below.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.6</td>
<td>0.4</td>
<td>0.5</td>
<td>0.8</td>
</tr>
<tr>
<td>2</td>
<td>0.6</td>
<td>0.0</td>
<td>0.3</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>3</td>
<td>0.4</td>
<td>0.3</td>
<td>0.0</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
<td>0.4</td>
<td>0.5</td>
<td>0.0</td>
<td>0.4</td>
</tr>
<tr>
<td>5</td>
<td>0.8</td>
<td>0.6</td>
<td>0.3</td>
<td>0.4</td>
<td>0.0</td>
</tr>
</tbody>
</table>

**Outputs:**
Vertex A is colored with 1, vertex B is colored with 2, vertex C is colored with 1, vertex D is colored with 2, vertex E is colored with 1 and vertex F is colored with 2.
Edge (B, A) is colored with 5, edge (B, C) is colored with 3, edge (B, E) is colored with 4, edge (C, D) is colored with 5, edge (C, F) is colored with 4, edge (E, D) is colored with 3, edge (E, F) is colored with 5 and edge (F, A) is colored with 3.
The (d, f) Total chromatic number is 5 and the colors used are 1, 2, 3, 4 and 5.

**Example 5.5.2:**
Consider the graph given in Example 5.5.1. The number of available colors is six and the color difference table is as shown below.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.0</td>
<td>0.3</td>
<td>0.4</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>3</td>
<td>0.4</td>
<td>0.3</td>
<td>0.0</td>
<td>0.2</td>
<td>0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>0.0</td>
<td>0.4</td>
<td>0.8</td>
</tr>
<tr>
<td>5</td>
<td>0.3</td>
<td>0.6</td>
<td>0.3</td>
<td>0.4</td>
<td>0.0</td>
<td>0.9</td>
</tr>
<tr>
<td>6</td>
<td>0.5</td>
<td>0.3</td>
<td>0.6</td>
<td>0.8</td>
<td>0.9</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Outputs:

Vertex A is colored with 1, vertex B is colored with 2, vertex C is colored with 1, vertex D is colored with 2, vertex E is colored with 1 and vertex F is colored with 2. Edge (B, A) is colored with 3, edge (B, C) is colored with 5, edge (B, E) is colored with 6, edge (C, D) is colored with 3, edge (C, F) is colored with 6, edge (E, D) is colored with 5, edge (E, F) is colored with 3 and edge (F, A) is colored with 5. The (d, f) Total chromatic number is 5 and the colors used are 1, 2, 3, 5 and 6.

The membership table of example 5.5.2 is the same as that of example 5.5.1 but the number of available colors has changed from 5 to 6 and their color difference table has also changed correspondingly. Even with the change in the number of colors and the color difference table, the Total chromatic number remain the same as 5. Only thing is that this time color 4 is not being used.