Appendix A
The various displacement fields of corresponding theories used in chapter 3.1 and 3.2 are as follows:

A.1 Classical Plate Theory

\[ u = u_0 - z \frac{\partial w}{\partial x} \]
\[ v = v_0 - z \frac{\partial w}{\partial y} \]
\[ w = w(x, y) \]  \hspace{1cm} (A.1)

where \( u_0 \), \( v_0 \) and \( w \) are the unknown functions of position \((x, y)\) to be determined.

A.2 First order Shear Deformation Theory

\[ u = u_0 + z \phi(x, y) \]
\[ v = v_0 + z \psi(x, y) \]
\[ w = w(x, y) \]  \hspace{1cm} (A.2)

where \( u_0 \), \( v_0 \) and \( w \) are the unknown functions of position \((x, y)\) to be determined and \( \phi \) and \( \psi \) are the rotations of a transverse normal about the \( y \) and \( x \) axes respectively.

A.3 Trigonometric Shear Deformation Theory

\[ u = -z \frac{\partial w}{\partial x} + \frac{h}{\pi} \sin \frac{\pi z}{h} \phi(x, y) \]
\[ v = -z \frac{\partial w}{\partial y} + \frac{h}{\pi} \sin \frac{\pi z}{h} \psi(x, y) \]
\[ w = w(x, y) \]  \hspace{1cm} (A.3)

where \( u_0 \), \( v_0 \) and \( w \) are the unknown functions of position \((x, y)\) to be determined and \( \phi \) and \( \psi \) are the rotations of a transverse normal about the \( y \) and \( x \) axes respectively.
A.4 Higher-order Shear Deformation Theory

\[
\begin{align*}
    u &= u_0 - z \frac{\partial w}{\partial x} + z \left[ 1 - \frac{4}{3} \left( \frac{z}{h} \right)^2 \right] \phi(x, y) \\
    v &= v_0 - z \frac{\partial w}{\partial y} + z \left[ 1 - \frac{4}{3} \left( \frac{z}{h} \right)^2 \right] \psi(x, y) \\
    w &= w(x, y)
\end{align*}
\] (A.4)

where \( u_0, v_0 \) and \( w \) are the unknown functions of position \((x, y)\) to be determined and \( \phi \) and \( \psi \) are the rotations of a transverse normal about the \( y \) and \( x \) axes respectively.

A.5 Hyperbolic shear deformation Theory

\[
\begin{align*}
    u &= u_0 - z \frac{\partial w}{\partial x} + z \left[ z \cosh \frac{1}{2} - h \sinh \left( \frac{z}{h} \right) \right] \phi(x, y) \\
    v &= v_0 - z \frac{\partial w}{\partial y} + z \left[ z \cosh \frac{1}{2} - h \sinh \left( \frac{z}{h} \right) \right] \psi(x, y) \\
    w &= w(x, y)
\end{align*}
\]

where \( u_0, v_0 \) and \( w \) are the unknown functions of position \((x, y)\) to be determined and \( \phi \) and \( \psi \) are the rotations of a transverse normal about the \( y \) and \( x \) axes respectively.

In case of one dimensional analysis, in each set of equations, the second equation is absent, and \( u_0, \) and \( w \) are the unknown functions of position \((x)\) only and \( \phi \) is the rotation of a transverse normal about the \( y \) axis.
Appendix B

The equations obtained for free vibration of plate in Example 4) of section 3.2 can be written in the matrix form as:

\[
\begin{bmatrix}
K_{11} & K_{12} & K_{13} \\
K_{21} & K_{22} & K_{23} \\
K_{31} & K_{32} & K_{33}
\end{bmatrix}
- \omega^2 \begin{bmatrix}
M_{11} & M_{12} & M_{13} \\
M_{21} & M_{22} & M_{23} \\
M_{31} & M_{32} & M_{33}
\end{bmatrix} \begin{bmatrix}
\phi_{mn} \\
\psi_{mn} \\
w_{mn}
\end{bmatrix} = 0
\]

(B.1)

where

\[
K_{11} = \left[6 \frac{m^2}{a^2} + 3 \frac{(1-\mu)}{h^2}\right]
\]

\[
K_{12} = K_{21} = \left[3 \frac{(1-\mu)mn}{ab}\right]
\]

\[
K_{13} = K_{31} = 24 \left[\frac{m^3}{a^3} + \frac{mn^2}{ab^2}\right]
\]

\[
K_{22} = \left[6 \frac{(1-\mu)m^2}{a^2} + 6 \frac{m^2}{a^2} + 3 \frac{(1-\mu)}{h^2}\right]
\]

\[
K_{23} = K_{32} = \left[24 \left(\frac{n^3}{b^3} + \frac{m^2n}{a^3b}\right)\right]
\]

\[
K_{33} = \left[\frac{m^3\pi^4}{a^4} + 2 \frac{m^3n^2}{a^2b^3} + \frac{n^3\pi^4}{b^4}\right]
\]

\[
M_{11} = \left(\frac{\rho h^3}{2\pi^2}\right), \quad M_{12} = M_{21} = 0
\]

\[
M_{13} = M_{31} = -\left(\frac{2\rho h^3m}{\pi^2a}\right)
\]

\[
M_{22} = \left(\frac{\rho h^3}{2\pi^2}\right)
\]

\[
M_{23} = M_{32} = \left(\frac{2\rho h^3n}{\pi^2b}\right)
\]

\[
M_{33} = -\left(\rho h + \frac{\rho h^3}{12} \left(\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2}\right)\right)
\]

where \(\rho\) is the density of the plate material, \(\mu\) is Poisson’s ratio, \(a, b, h\) are the length, breadth and thickness of the plate and \(m\) and \(n\) are positive integers.
Appendix C

The set of algebraic equations obtained for Example 6) in Section 3.2 can be expressed in the following form:

\[
\begin{align*}
A_{11}w_{mn} + A_{12} \phi_{mn} + A_{13} \psi_{mn} + A_{14} u_{mn} + A_{15} v_{mn} &= q_{mn} \\
A_{21}w_{mn} + A_{22} \phi_{mn} + A_{23} \psi_{mn} + A_{24} u_{mn} + A_{25} v_{mn} &= 0 \\
A_{31}w_{mn} + A_{32} \phi_{mn} + A_{33} \psi_{mn} + A_{34} u_{mn} + A_{35} v_{mn} &= 0 \\
A_{41}w_{mn} + A_{42} \phi_{mn} + A_{43} \psi_{mn} + A_{44} u_{mn} + A_{45} v_{mn} &= 0 \\
A_{51}w_{mn} + A_{52} \phi_{mn} + A_{53} \psi_{mn} + A_{54} u_{mn} + A_{55} v_{mn} &= 0 \\
\end{align*}
\]

\[\text{(C.1)}\]

where

\[
A_{ij} = D_{ij} \left[ \frac{m^4 \pi^4}{a^4} Q_{11} + \frac{m^2 n^2 \pi^4}{a^2 b^2} (Q_{12} + 2Q_{66}) + \frac{n^4 \pi^4}{b^4} Q_{22} \right]
\]

\[
A_{21} = A_{12} = AS_{ij} \left[ \frac{m^3 \pi^3}{a^3} Q_{11} + \frac{mn^2 \pi^3}{ab^2} Q_{12} + 2 \frac{mn^2 \pi^3}{ab^2} Q_{66} \right]
\]

\[
A_{13} = A_{31} = AS_{ij} \left[ \frac{m^2 n \pi^3}{a^2 b} \frac{Q_{12}}{} + \frac{n^3 \pi^3}{b^3} Q_{22} + 2 \frac{m^2 n \pi^3}{a^2 b} Q_{66} \right]
\]

\[
A_{41} = A_{14} = BS_{ij} \left[ \frac{m^3 \pi^3}{a^3} Q_{11} + \frac{mn^2 \pi^3}{ab^2} Q_{12} + 2 \frac{mn^2 \pi^3}{ab^2} Q_{11} \right]
\]

\[
A_{51} = A_{15} = BS_{ij} \left[ \frac{m^2 n \pi^3}{a^2 b} \frac{Q_{12}}{} + \frac{n^3 \pi^3}{b^3} Q_{22} + 2 \frac{m^2 n \pi^3}{a^2 b} Q_{66} \right]
\]

\[
A_{22} = ASS_{ij} \left[ \frac{m^2 \pi^2}{a^2} Q_{11} + \frac{n^2 \pi^2}{b^2} Q_{66} \right] + ACC_{ij} Q_{55}
\]

\[
A_{23} = A_{32} = ASS_{ij} \left[ \frac{mn \pi^3}{ab} (Q_{12} + Q_{66}) \right]
\]

\[
A_{24} = A_{42} = CS_{ij} \left( \frac{m \pi^2}{a^2} Q_{11} + \frac{n \pi^2}{b^2} Q_{66} \right)
\]

\[
A_{25} = A_{32} = CS_{ij} \left[ \frac{mn \pi^2}{ab} (Q_{12} + Q_{66}) \right]
\]

\[
A_{33} = ASS_{ij} \left[ \frac{n \pi^2}{b^2} Q_{22} + \frac{m \pi^2}{a^2} Q_{66} \right] + ACC_{ij} Q_{44}
\]

\[
A_{34} = A_{43} = CS_{ij} \left[ \frac{mn \pi^2}{ab} (Q_{12} + Q_{66}) \right]
\]
\[ A_{35} = A_{33} = CS_y \left( \frac{n^2 \pi^2}{b^2} Q_{22} + \frac{m^2 \pi^2}{a^2} Q_{66} \right) \]
\[ A_{44} = DS_y \left( \frac{m^2 \pi^2}{a^2} Q_{11} + \frac{n^2 \pi^2}{b^2} Q_{66} \right) \]
\[ A_{45} = A_{54} = DS_y \frac{mn \pi^2}{ab} (Q_{12} + Q_{66}) \]
\[ A_{55} = DS_y \left( \frac{n^2 \pi^2}{b^2} Q_{22} + \frac{m^2 \pi^2}{a^2} Q_{66} \right) \]

where \( (DS_y, BS_y, D_y, CS_y, AS_y, ASS_y) = \int_{-h/2}^{h/2} Q_y \left( 1, z, z^2, f(z), z f(z), [f(z)]^2 \right) dz \)
for \( i, j = 1, 2, 6 \) and
\( (ACC_y) = \int_{-h/2}^{h/2} Q_0 \left( [f'(z)]^2 \right) dz \) for \( i, j = 4, 5 \)

Application of Gauss Elimination method in the above equations gives the solution of algebraic equations as follows:

\[\begin{align*}
v_{mn} &= Z2/Z1 \\
u_{mn} &= (X3 - X2.v_{mn}) / X1 \\
\psi_{mn} &= (E4 - E2.u_{mn} - E3.v_{mn}) / E1 \\
\phi_{mn} &= (A5 - A4.v_{mn} - A3.u_{mn} - A2.\psi_{mn}) / A1 \\
w_{mn} &= (q_0 - A12.\phi_{mn} - A13.\psi_{mn} - A14.u_{mn} - A15.v_{mn}) / A11
\end{align*}\]

where

\[\begin{align*}
A1 &= (A_{22} - A_{12}.A_{21} / A_{11}) \\
A2 &= (A_{23} - A_{13}.A_{21} / A_{11}) \\
A3 &= (A_{31} - A_{14}.A_{31} / A_{11}) \\
A4 &= (A_{35} - A_{15}.A_{31} / A_{11}) \\
A5 &= -q_0.A_{21} / A_{11} \quad B5 = -q_0.A_{31} / A_{11} \\
C1 &= (A_{42} - A_{12}.A_{41} / A_{11}) \\
C2 &= (A_{43} - A_{13}.A_{41} / A_{11}) \\
C3 &= (A_{44} - A_{14}.A_{41} / A_{11}) \\
C4 &= (A_{45} - A_{15}.A_{41} / A_{11}) \\
C5 &= -q_0.A_{41} / A_{11} \quad D5 = -q_0.A51 / A11 \\
E1 &= (B_{22} - A2.B1 / A1) \\
E2 &= (B_{32} - A3.B1 / A1) \\
E3 &= (B_{42} - A4.B1 / A1) \\
E4 &= (B_{52} - A5.B1 / A1) \quad F1 = (C2 - A2.C1 / A1) \\
F2 &= (C3 - A3.C1 / A1) \\
F3 &= (C4 - A4.C1 / A1) \\
F4 &= (C5 - A5.C1 / A1)
\end{align*}\]
\[ G_1 = (D_2 - A_2. D_1 / A_1) \]
\[ G_2 = (D_3 - A_3. D_1 / A_1) \]
\[ G_3 = (D_4 - A_4. D_1 / A_1) \]
\[ G_4 = (D_5 - A_5. D_1 / A_1) \]
\[ Y_1 = (G_2 - E_2. G_1 / E_1) \]
\[ Y_2 = (G_3 - E_3. G_1 / E_1) \]
\[ Y_3 = (G_4 - E_4. G_1 / E_1) \]
\[ Z_1 = (Y_2 - X_2. Y_1 / X_1) \]
\[ Z_2 = (Y_3 - X_3. Y_1 / X_1) \]
\[ X_1 = (F_2 - E_2. F_1 / E_1) \]
\[ X_2 = (F_3 - E_3. F_1 / E_1) \]
\[ X_3 = (F_4 - E_4. F_1 / E_1) \]