3. SYSTEM DEVELOPMENT

3.1 Statement

Electrical power engineers have always been concerned about power quality. They see power quality as anything that affects the voltage, current, and frequency of the power being supplied to the end user, i.e., the ultimate user or consumer of electricity. Power quality problems occur when the alternating-voltage power source’s 50-Hz sine wave is distorted. In the past, most power-consuming equipment tolerated some distortion. Today highly sensitive computers and computer-controlled equipment require a power source of higher quality and more reliability than standard, less sensitive electricity-consuming equipment of the past, like motors and incandescent lights. As loads have become more sensitive to variations in the quality of power, the definition of power quality has become important but somewhat confusing.

Disturbances produced in electrical networks by electric arc furnaces can significantly affect the voltage quality supplied by electrical power companies. In fact, an electric arc furnace is a non-linear, time-varying load, which gives rise to harmonics, inter harmonics and voltage fluctuations (flicker). The cause of harmonics is mainly related to the non-linear voltage-current characteristic of the arc while the voltage fluctuations are due to the arc length changes that occur during the melting of the scrap. The current and voltage harmonic distortion causes several problems in electrical power systems, such as incorrect operation of devices, premature ageing of equipment, additional losses in transmission and distribution networks, over voltages and over currents. The flicker phenomenon does not very much affect the electric equipments, but a physiological uneasiness in vision occurs due to electric lightning flux fluctuations, which are particularly important with incandescent lamps. Therefore, it is of crucial importance to predict these effects when an arc furnace is to be connected to a network or when an existing furnace is to be upgraded. Whenever the emission limits are exceeded mitigation techniques are to be used in order to correct such disturbances.
Hence situations and disturbances deliberately selected from practical experience for system development for study.

3.2 Methodology Of System Development

The system development is addressed by the following methods of analysis as per IEEE standards on power quality.
1. Equivalent circuit
2. Circle diagram development
3. Electrode Control

The power quality study of arc plant is carried out in the following steps for selected steel plant.
1. Select appropriate definition of Power Quality and criteria
2. Make assumptions and describe a mathematical model and phenomenon under study
3. Analyze and / or simulate to determine power quality, using a scenario of events
4. Review results in light of assumptions and engineering experience
5. Draw conclusions
6. Evolve new ideas leading to contributions

First two steps are addressed in this section with a view to develop analytical base for system development and create ground for actual analysis.

3.3. Power Quality Issue and Problem Formulation

The rapid change in the electric load profile from being mainly a linear type to greatly nonlinear, has created continued power quality problems which are difficult to detect and is in general complex. The most important contributor to power quality problems is the customers’ (or end-user electric loads) use of sensitive type nonlinear load in all sectors (Industrial, Commercial and Residential). Power Quality issues can be roughly broken into a number of sub-categories:
Power Quality issues can be roughly broken into a number of sub-categories:

- Harmonics (integral, sub, super and interharmonics)
- Voltage swells, sags, fluctuations, flicker and Transients
- Voltage magnitude and frequency, voltage imbalance
- Hot grounding loops and ground potential rise (GPR)
- Monitoring and measurement of quasi-dynamic, quasi-static and transient type phenomena

Nonlinear type loads contribute to the degradation in the electric supply Power Quality through the generation of harmonics. The increased use of nonlinear loads makes the harmonic issue (waveform distortion) a top priority for all equipment manufacturers, users and electric utilities. Severe Power System harmonics are usually the steady state problem not the transient or intermittent type, and these
harmonics can be mitigated by using the new family of modulated/switched power filters.

Lower order harmonics cause the greatest concern in the electrical distribution/utilization system. Harmonics interfere with sensitive-type electronic communications and networks. Low order triplen harmonics cause hot-neutrals, grounding potential rise (GPR), light flickering, malfunction of computerized data processing equipment and computer networks and computer equipment.

There are several defined measures commonly used for indicating the harmonic severity and content of a waveform. One of the most common measures is total harmonic distortion in current $(THD_i)$.

\[
(THD_i) = \left[ \frac{\sum_{n=2}^{\infty} I_n^2}{I_1} \right]
\]

Where $I_1$: Fundamental (60Hz) Current; $n$: Harmonic order and $I_n$: Harmonic current.

Total Harmonic Distortion (THD) and Power Factor (PF)

The power factor PF for any non-sinusoidal quantities is defined by:

\[
PF = \frac{V_s I_{s1} \cos \phi_1}{V_s I_s} = \frac{I_{s1}}{I_s} \cos \phi_1
\]

$I_{s1}$ is the rms value of the fundamental 60Hz component of the current. The displacement power factor (DPF, which is the same as the power factor in linear circuits with pure sinusoidal voltage and current) is defined as the cosine of the angle $\phi_1$ (angle between the fundamental-frequency (60Hz) current and voltage waveforms) which could be written as: $DPF = \cos \phi_1$, therefore, the power factor PF with a nonsinusoidal current is:

\[
PF = \frac{I_{s1}}{I_s} DPF
\]
in terms of total harmonic current distortion ($THD_i$), the PF and $I_s$ (the rms value of the total current) could be written as:

$$PF = \frac{1}{\sqrt{1 + THD_i^2}} DPF$$

(3.4)

Where $I_s = I_{s1} \sqrt{1 + THD_i^2}$

(3.5)

From an examination of (3.4) and (3.5), we can conclude that the power factor value decreases with any high current harmonic content or distortion ($THD_i$). These definitions assume that the source voltage is near sinusoidal of fundamental frequency (maximum allowable ($THD_v$)=5%).

Any steady state periodic time domain waveform can be expressed by infinite summation of sinusoidal waveforms at integer multiples of a fundamental frequency. The definition of power system harmonics is based on the application of Fourier transform and superposition of voltage and current waveforms. There are two basic ways to obtain the Fourier transform of a voltage or current waveform. One way is to use a spectrum analyzer and measure the harmonics directly in an on line mode, however this method does not provide harmonic phase angle information. Another method is to sample and store the time domain specifications. As discrete data points and compute the harmonic components digitally using a microprocessor in an off line type mode. A popular technique for computing the harmonic magnitudes and phase angles as a sample time domain waveform is a discrete Fourier transform (DFT).

$$F_n = \frac{1}{N} \sum_{k=0}^{N-1} f_k e^{-j2\pi n k/N}$$

(3.6)

Where $F_n$. Fourier coefficient of $n^{th}$ harmonic

$F_k$. Data point of the $k^{th}$sample.

N - Total no. of cycles per 50 Hz cycle

n - Harmonic index

k - Data point index

As a example, if are using a 5 KHz sample rate on a 60 Hz waveform, this means that $83^{1/3}$ samples are needed to complete one cycle. Since the total number of cycles used
in the DFT must be an integer, $83^{1/3}$ must be rounded to either 83 or 84. Rounding in this fashion is referred to as windowing the time domain function and it produces leakage errors in the output of the harmonic components when compared to the continuous-time Fourier transform. But if high sample rates are used, the error is not significant compared to the levels of harmonics being measured. The errors associated with windowing can be reduced further by increasing the sample rate or increasing the window size. For harmonic analysis of voltage and current waveforms on a power system, a 5 KHz sample rate can produce reasonably accurate results with a minimum amount of computational efforts. Individual harmonic distortion (HD) and total harmonic distortion (THD) are defined as follows [29, 30].

$$\text{HD}_n = \frac{|F_n|}{|F_1|}$$

(3.7)

$$\text{THD} = \frac{1}{F_1} \sqrt{\sum_{n=2}^{N/2} F_n^2}$$

(3.8)

where:
- $\text{THD}$ = Total (harmonic) Voltage Distortion
- $\text{HD}_n$ = Individual (harmonic) Voltage Distortion
- $\text{THD}_i$ = Total (harmonic) Current Distortion
- $\text{HD}_i$ = Individual (harmonic) Current Distortion

Computing the harmonics using the fast Fourier transform is widely used when computational time is important, but the sample rate must be adjusted to integer powers of two data samples per cycle. Zero padding at the end of sample test can be used to adjust the number of samples to equal an integer power of two when the sample rate cannot be adjusted easily in the test equipment. This technique affects the magnitude and phase angles of the harmonic components.

### 3.4.1 Transmission Line Modeling

High voltage transmission lines may contain several conductors per phase (bundle conductors) and ground wires. Distribution lines may include a neutral wire as a return path. Transmission and primary distribution circuits may be responsible for introducing considerable geometric imbalances, even at the fundamental frequency, depending on their electrical distance.
Figure 3.4.1: Transmission Line Representation in the Form of an Equivalent Pi Circuit

The phase conductors of a three-phase transmission line, with ground as the return path and negligible capacitive effects, are illustrated schematically in Figure 3.4.1. If the circuit terminal conditions enable current to flow in conductors a, b, c, and in the ground return path, the voltage-drop equation of the transmission line shown in Figure 3.2, at a given frequency, may be expressed in matrix form as follows:

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c \\
\end{bmatrix} = \begin{bmatrix}
R_{aa-g} + j\omega L_{aa-g} & R_{ab-g} + j\omega L_{ab-g} & R_{ac-g} + j\omega L_{ac-g} \\
R_{ba-g} + j\omega L_{ba-g} & R_{bb-g} + j\omega L_{bb-g} & R_{bc-g} + j\omega L_{bc-g} \\
R_{ca-g} + j\omega L_{ca-g} & R_{cb-g} + j\omega L_{cb-g} & R_{cc-g} + j\omega L_{cc-g} \\
\end{bmatrix} \begin{bmatrix}
I_a \\
I_b \\
I_c \\
\end{bmatrix} + \begin{bmatrix}
V'_a \\
V'_b \\
V'_c \\
\end{bmatrix}
\]

\[(3.9)\]

The subscript g indicating that the ground return effect has been included.

Figure 3.4.2: Phase Conductors of a Three Phase Transmission Line
The series impedance matrix $Z_{\text{series}}$ of a multi conductor transmission line, which takes account of geometric imbalances and frequency dependency but not long line effects, may be assumed to consist of the following components:

$$
Z_{\text{series}} = Z_{\text{internal}} + Z_{\text{geometric}} + Z_{\text{ground}}. \quad (3.10)
$$

In Equation (3.10), $Z_{\text{internal}}$ is the impedance inside the conductors, $Z_{\text{ground}}$ is the impedance contribution of the ground return path, and $Z_{\text{geometric}}$ is the impedance contribution from the magnetic fluxes in the air surrounding the conductors. For most practical purposes, the parameters $Z_{\text{geometric}}$ may be taken to be linear functions of the potential coefficients $P$.

Unlike $Z_{\text{internal}}$, the parameters $Z_{\text{ground}}$, $Z_{\text{geometric}}$, and $P$ are a function of the physical geometry of the conductor’s arrangement in the tower. The capacitive effects are incorporated in the shunt admittance matrix $Y_{\text{shunt}}$, which is a linear function of $P$. It should be noted that potential coefficients are dimensionless and reciprocal. The geometric impedance matrix for the circuit of is

$$
Z_{\text{geometric}} = j \frac{\omega \mu \omega}{4\pi} P \omega \Omega \text{km}^{-1}, \quad (3.11)
$$

Where $Z_{\text{geometric}}$ varies linearly with the base frequency $\omega = 2\pi f$ and the permeability of free space is $\mu = 4\pi \times 10^{-4}$ Hkm$^{-1}$.

Shunt admittance parameters vary linearly with frequency and are completely defined by the inverse potential coefficients.

$$
Y_{\text{shunt}} = j \omega 2\pi \omega P \Omega^{-1} S \text{ km}^{-1}, \quad (3.12)
$$

where $\varepsilon_0$, equal to $8.85 \times 10^{-9}$ F/m, is the permittivity of free space.

It has long been recognized that the internal resistance and inductance of conductors vary with frequency in a nonlinear manner. The reason for this effect is attributed mainly to the non uniform distribution of current flow over the full area available, with current tending to flow on the surface. This trend increases with frequency and is termed the ‘skin effect’. The overall effect is an increase in resistance and a decrease in internal inductance.
Hence, the conductor impedance matrix for this circuit is

\[
Z_{\text{internal}} = R_{\text{ac}} + j \frac{\omega \mu 0}{4\pi} P_{\text{internal}} \Omega km^{-1}
\]  

(3.13)

Where \( R_{\text{ac}} \) is a diagonal matrix with entries corresponding to the AC power frequency resistances (50 or 60 Hz) of the various conductors in the transmission circuit.

The impedance of the ground return path varies nonlinearly with frequency and exhibits an effect similar to that of the skin effect in conductors, where the effective area available for the current to flow reduces with frequency. It should be noted that the use of Equations (3.11) and (3.12) yields combined information of \( Z_{\text{geometric}} \& Z_{\text{ground}} \). Moreover, if the transmission-line parameters are intended for power frequency applications (50 or 60 Hz) then the skin effect inside the conductors can be ignored and Equation (3.12) can be combined with Equation (3.13) to take account of the impedance contribution from \( Z_{\text{internal}} \):

\[
Z_u = R_{\text{ac}} l + j \frac{\omega \mu 0}{2\pi} \ln \left[ \frac{2(h1 + p)}{r_{gw} l} \right] \Omega km^{-1}
\]

(3.14)

Using the same notation as in Section, we may express the voltage-drop equation of a three-phase transmission line with two ground wires, \( w \) and \( v \), as follows

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c \\
V_w \\
V_v
\end{bmatrix} =
\begin{bmatrix}
Z_{aa-g} & Z_{ab-g} & A_{ac-g} & Z_{aw-g} & Z_{av-g} \\
Z_{ba-g} & Z_{bb-g} & A_{bc-g} & Z_{bw-g} & Z_{bv-g} \\
Z_{ca-g} & Z_{cb-g} & A_{cc-g} & Z_{cw-g} & Z_{cv-g} \\
Z_{wa-g} & Z_{wb-g} & A_{wc-g} & Z_{ww-g} & Z_{wv-g} \\
Z_{va-g} & Z_{vb-g} & A_{vc-g} & Z_{vw-g} & Z_{vv-g}
\end{bmatrix}
\begin{bmatrix}
V_a \\
V_b \\
V_c \\
V_w \\
V_v
\end{bmatrix}
\]  

(3.15)

It is assumed that the individual impedance elements are calculated by using Equations
\[ \Delta V_{abc} = \begin{bmatrix} V_a - V_a^\prime \\ V_b - V_b^\prime \\ V_c - V_c^\prime \end{bmatrix}, \quad I_{abc} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}, \quad (3.16) \]

\[ \Delta V_{wv} = \begin{bmatrix} V_w - V_w^\prime \\ V_v - V_v^\prime \end{bmatrix}, \quad I_{wv} = \begin{bmatrix} I_w \\ I_v \end{bmatrix}, \]

\[ A = \begin{bmatrix} Z_{aa-g} & Z_{ab-g} & Z_{ac-g} \\ Z_{ba-g} & Z_{bb-g} & Z_{bc-g} \\ Z_{ca-g} & Z_{cb-g} & Z_{cc-g} \end{bmatrix}, \quad B = \begin{bmatrix} Z_{aw-g} & Z_{aw-g} \\ Z_{hw-g} & Z_{bv-g} \\ Z_{cw-g} & Z_{cv-g} \end{bmatrix} \]

\[ C = \begin{bmatrix} Z_{wa-g} & Z_{wb-g} & Z_{wc-g} \\ Z_{wb-g} & Z_{wb-g} & Z_{wc-g} \\ Z_{wh-g} & Z_{vh-g} & Z_{vc-g} \end{bmatrix}, \quad D = \begin{bmatrix} Z_{ww-g} & Z_{ww-g} \\ Z_{ww-g} & Z_{ww-g} \end{bmatrix} \quad (3.17) \]

In compact notation, we have,

\[ \Delta V_{abc} = AI_{abc} + BI_{wv} \quad (3.18) \]

\[ \Delta V_{abc} = [A - BD^{-1}C]I_{abc} = Z_{abc-wv-g} I_{abc} \quad (3.19) \]

As it is normal practice to connect ground wires to earth at both ends of every transmission span, \( \Delta V_{wv} = 0 \), and it is possible to simplify Equation (3.15) to

\[ 0 = CI_{abc} + DI_{wv} \quad (3.20) \]

And substitution of equation 3.20 into 3.18 we have,

\[ \Delta V_{abc} = [A - BD^{-1}C]I_{abc} = Z_{abc-wv-g} I_{abc} \quad (3.21) \]

\[ Z_{abc-wv-g} = A - BD^{-1}C \quad (3.22) \]

Equation (3.21) can be written in expanded form as

\[ \begin{bmatrix} \Delta V_a \\ \Delta V_b \\ \Delta V_c \end{bmatrix} = \begin{bmatrix} Z_{aa-wv-g} & Z_{ab-wv-g} & Z_{ac-wv-g} \\ Z_{ba-wv-g} & Z_{bb-wv-g} & Z_{bc-wv-g} \\ Z_{ca-wv-g} & Z_{cb-wv-g} & Z_{cc-wv-g} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (3.23) \]
3.4.2 Power Transformer Modeling

Power transformers are essential plant components of the power system. In general, they provide the interface between sections of the network with different rating voltages. Transformers consist of two or three copper windings per phase and one or more iron cores. They are normally contained in metallic enclosures (i.e. tanks), and are immersed in high-grade oil for insulation purposes. Based on nodal analysis, quite general models for multi windings, multiphase transformers can be derived. The essence of the method is to transform the short-circuit parameters of the transformer windings, suitably arranged in a matrix of primitive parameters $Y_{\psi\psi}$, into nodal parameters $Y_{\alpha\alpha}$. This is done with the help of appropriate connectivity matrices, namely, $C_{\alpha\psi}$ and $C_{\psi\alpha}$. The connectivity matrices relate the voltages and currents in the unconnected transformer windings to the phase voltages and currents when the three-phase transformer is actually connected. The primitive and nodal parameters are related by the following matrix expression [30].

$$Y_{\alpha\alpha} = C_{\alpha\psi}Y_{\psi\psi}C_{\psi\alpha}$$

(3.24)

The primitive parameters of three identical single-phase transformers, for which the terminals between transformers are not connected in any way but contain off-nominal tapping facilities on the primary winding, have the following arrangement:

$$\begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6 \end{bmatrix} = \begin{bmatrix} Y_{w} & -T_{w}Y_{w} \\ -T_{v} & T_{v}^{2}Y_{w} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix},$$

(3.25)

$$\begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6 \end{bmatrix} = \begin{bmatrix} Y_{w} & -T_{w}Y_{w} & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_{w} & -T_{w}Y_{w} & 0 & 0 \\ 0 & 0 & -T_{w}Y_{w} & T_{w}^{2}Y_{w} & 0 & 0 \\ 0 & 0 & 0 & Y_{w} & -T_{w}Y_{w} & 0 \\ 0 & 0 & 0 & -T_{w}Y_{w} & T_{w}^{2}Y_{w} & 0 \\ 0 & 0 & 0 & 0 & Y_{w} & -T_{w}Y_{w} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}.$$
In general, these matrix equations may be expressed in compact form:

$$I_v = Y_{vv} V_v$$  \hspace{1cm} (3.26)

The three-phase connection is shown in Figure 3.4 when the windings are connected in star– star configuration, with both star points grounded through admittances, YN and Yn, respectively [31].

![Figure 3.4.3: Star–Star Between Neutral Grounded Through Reactor](image)

The transformation matrix, which relates the voltages existing in the unconnected transformer to the voltages in the connected three-phase transformer shown in Figure 3.4.3, is given explicitly in Equation (3.27)

$$
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5 \\
V_6 \\
V_7 \\
V_8
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\
0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
V_a \\
V_b \\
V_c \\
V_d \\
V_e \\
V_f \\
V_N \\
V_n
\end{bmatrix}
$$  \hspace{1cm} (3.27)

In compact form, we have,

$$V_{\psi} = C_\psi \alpha \ V \alpha$$  \hspace{1cm} (3.28)
The nodal matrix representation of this transformer connection is obtained by substituting Equations (3.24) and (3.25) into Equation (3.29):

\[
\begin{bmatrix}
I_A \\
I_B \\
I_C \\
I_a \\
I_b \\
I_c \\
I_N \\
I_n \\
\end{bmatrix}
= 
\begin{bmatrix}
Y_{sc} & 0 & 0 & -T_u^{sc}Y_{sc} & 0 & 0 & -Y_{sc} & T_Y^{sc} \\
0 & Y_{sc} & 0 & 0 & -T_Y Y_{sc} & 0 & -Y_{sc} & T_Y^{sc} \\
0 & 0 & Y_{sc} & 0 & 0 & -T_Y Y_{sc} & -Y_{sc} & T_Y^{sc} \\
-T_u^{sc}Y_{sc} & 0 & 0 & -T_u^{sc}Y_{sc} & 0 & 0 & T_Y^{sc} & -T_Y^{sc} \\
0 & -T_u^{sc}Y_{sc} & 0 & 0 & -T_u^{sc}Y_{sc} & 0 & T_Y^{sc} & -T_Y^{sc} \\
0 & 0 & -T_u^{sc}Y_{sc} & 0 & 0 & T_Y^{sc} & -T_Y^{sc} & T_Y^{sc} \\
-T_u^{sc}Y_{sc} & -Y_{sc} & -Y_{sc} & T_u Y_{sc} & T_Y Y_{sc} & T_Y Y_{sc} & 3Y_{sc}^{2} Y_{N} & -3T_Y Y_{sc} \\
T_u^{sc} Y_{sc} & T_u^{sc} Y_{sc} & T_u Y_{sc} & -T_u^{sc} Y_{sc} & -T_u^{sc} Y_{sc} & 3T_Y^{2} Y_{sc} & 3T_Y^{2} Y_{sc} & 3T_Y^{2} Y_{sc} \\
\end{bmatrix}
\begin{bmatrix}
V_A \\
V_B \\
V_C \\
V_a \\
V_b \\
V_c \\
V_N \\
V_n \\
\end{bmatrix} 
\tag{3.29}
\]

If both star points N and n are solidly grounded then the nodal voltages \(V_N\) and \(V_n\) become zero. Hence, the rows and columns corresponding to bus N and bus n become redundant and are deleted from matrix Equation (3.24):

\[
\begin{bmatrix}
I_A \\
I_B \\
I_C \\
I_a \\
I_b \\
I_c \\
\end{bmatrix}
= 
\begin{bmatrix}
Y_{sc} & 0 & 0 & -T_u^{sc}Y_{sc} & 0 & 0 \\
0 & Y_{sc} & 0 & 0 & -T_Y Y_{sc} & 0 \\
0 & 0 & Y_{sc} & 0 & 0 & -T_Y Y_{sc} \\
-T_u^{sc}Y_{sc} & 0 & 0 & -T_u^{sc}Y_{sc} & 0 & 0 \\
0 & -T_u^{sc}Y_{sc} & 0 & 0 & -T_u^{sc}Y_{sc} & 0 \\
0 & 0 & -T_u^{sc}Y_{sc} & 0 & 0 & T_u^{sc}Y_{sc} \\
\end{bmatrix}
\begin{bmatrix}
V_A \\
V_B \\
V_C \\
V_a \\
V_b \\
V_c \\
\end{bmatrix} 
\tag{3.30}
\]

### 3.5 Equivalent Circuit for AC ArcFurnace

#### 3.5.1 Single phase equivalent circuit

The electrical behavior of AC furnaces depends directly on transformer voltages, the geometric distribution of high current conductors and the associate reactance. The output voltage of the transformer \(V_H\) is divided in the electrical arc voltage \(V_B\), the voltage \(V_V\) on the ohmic resistance \(R_v\) and the voltage \(V_L\) induced to the self inductance due to variable magnetic fields. According to Kirchhoff’s voltage law, the relation between current and voltages can be described by the following differential equation:
\[ V_n = V_v + V_L + V_B = I R_v + L \frac{dI}{dt} + V_B \]  

(3.31)

Specially for high power circuits, the voltage and current signals no sinusoidal components in addition to the fundamental one that makes the time domain analysis more difficult. Thus the analysis is developed within those restrictions.

For clarification purposes, all the analysis will be done based on the equivalent circuit diagram by replacing the non linear characteristics of the arc by a linear resistance \( R_B \). By this way electrical variables can be calculated by means of complex calculation and represented by complex phasors.

<table>
<thead>
<tr>
<th>Time domain</th>
<th>Complex domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_v = i R_v V_v = I R_v )</td>
<td>( V_L = L \frac{dI}{dt} V_x = I j \omega L = I jX )</td>
</tr>
<tr>
<td>( V_B = I R_B )</td>
<td></td>
</tr>
</tbody>
</table>

According to voltage complex equation, the current \( I \) can be calculated:

\[ V_{tr} = V_v + V_B + V_X \]  

(3.32)

\[ V_n = I (R_v + R_B + jX) = I Z \]  

(3.33)

The magnitude and phase of the impedance are:

\[ |Z| = \sqrt{(R_v + R_B)^2 + X^2} \]  

(3.34)

\[ \phi = \tan^{-1} \frac{X}{R_v R_B} \]  

(3.35)

\[ I = \frac{V_o}{|X|} \Rightarrow \frac{V_o}{\sqrt{(R_v + R_B)^2 + X^2}} \]  

(3.36)

Figure 3.5.1: Phasor Diagram of Voltages and Current
3.5.2 Three Phase Equivalent Circuit Diagram of an Arc Furnace

The equivalent circuit of a three phase arc furnace consists of three phase with inductance, loss resistance and arc voltage source connected in series meeting at free star point(o) in the furnace bath. The effective inductances $L_1 = M_{12,13}$, $L_2 = M_{23,21}$, $L_3 = M_{31,32}$ are the mutual inductances between any two high current loops. The voltage induced in loop 1-2 comprises two components which are produced by the fictive circuit currents $i_{13}$ and $i_{23}$. Thus the mutual inductances act like concentrated circuit elements $L_1, L_2$, in phases 1 and 2. The equivalent circuit of the three phase furnace can be liberalized. By introducing the operational reactances $X_1, X_2$ and $X_3$. With the phase impedances:

$$Z_1 = R_{R1} + R_{A1} + jX_1$$
$$Z_2 = R_{R2} + R_{A2} + jX_2$$
$$Z_3 = R_{R3} + R_{A3} + jX_3$$

![Figure 3.5.2: Equivalent Circuit of Three Phase Furnace](image)

Figure 3.5.2: Equivalent Circuit of Three Phase Furnace

$$V_{12} = I_1Z_1 - I_2Z_2, V_{23} = I_2Z_2 - I_3Z_3, V_{31} = I_3Z_3 - I_1Z_1$$

The furnace phase voltages are $V_{10} = I_1Z_1$

$$V_{20} = I_2Z_2$$

$$V_{30} = I_3Z_3$$
3.5.3 Balanced Furnace Operation

The furnace is in balanced state when most of the electric variables are within certain symmetry. The target of the matter is to achieve this state during the whole process. When the furnace works in operation mode, the distribution of the power inside is better and melting is smaller. The balanced state entails three phases similar operation. Thus it is assumed that the reactance, loss resistances, arc lengths, and impedances are of the same magnitude.

\[ X_1 = X_2 = X_3 = X \]
\[ R_{V1} = R_{V2} = R_{V3} = R_V \]
\[ R_{B1} = R_{B2} = R_{B3} = R_B \]
\[ Z_1 = Z_2 = Z_3 = Z \]

In addition, the phase voltages are identical and smaller than phase to phase voltages \( V \) by the factor \( \sqrt{3} \).

\[ V_{10} = V_{20} = V_{30} = V_{tr} = V / \sqrt{3} \]

3.5.4 Effect of Reactance on Furnace

Problems Caused by Reactance

Reactance problems are associated with large furnaces, for in small furnaces the effects are usually insignificant. The reason for this can easily be seen from an examination of the effects of furnace size on the average resistance, \( R \), and reactance, \( X \). \( R \) varies inversely with electrode diameter, \( D \), for a given specific electrode resistance. For a fixed ratio of electrode diameter to electrode spacing, \( X \) is proportional to the electrode length (if the effects of arcing are ignored), which is related to electrode diameter, Obviously, as \( D \) increases, a point is reached where the reactance becomes noticeable, and the problem then rapidly becomes worse with increasing \( D \). The term low power is synonymous with high reactance relative to the resistance. Problems caused by a low power factor can be divided into two groups: (i) problems with the power supply, i.e., transformer size, phase-correction capacitors, and the supply authority (ii) problems with the operation and control of the furnace. The problems with the power supply are relatively well known and well understood,
and were often believed to be the only trouble arising from high reactance. The problems with operation and control are more difficult, and there are three main problems:

(a) The dead-phase and live-phase phenomenon
(b) The lack of sensitivity to electrode movement
(c) The interaction effects between electrodes

a) Dead and Live Phases

Dead and live phases are caused by imbalances in the reactance and are better known in open-arc steelmaking furnaces, where they cause uneven erosion of the refractory. They also occur in submerged-arc furnaces but tend to be misinterpreted under normal operation. The asymmetry could be avoided if the transformers were placed in a triangular arrangement round the furnace so that the bus bars were of equal length, but this could make access to the electrode structure difficult. An additional asymmetry occurs if the furnace is tapped from one side, since the metal tends to flow downwards towards the tap hole from the rear electrodes so that the current path for the front electrode tends to be longer than that for the other two electrodes. A third source of asymmetry in reactance results from an imbalance in the relative amount of arcing in each phase. When a furnace is run with equal currents in the phases, the resistances must be unbalanced to compensate for the unequal reactance. If the phase rotation is 1-2-3 and electrode one has a higher reactance than the other two phases, then the power in phase two (dead phase) will be lower than the power in phase one, and phase three (live phase) will have a higher power.

This phenomenon can be seen from the phasor diagrams which are based on a simple star-equivalent circuit. If the furnace is as completely balanced, with the reactance equal and the resistances equal, the result being equal currents in each phase. If there is an increase of reactance in phase one at the same time as the reactance in the other two phases are reduced, the reactance become asymmetrical. The resistances in phases two and three are adjusted so that the currents remain equal.
When the neutral has shifted from the centre, N, to N' and the resistances are unbalanced so that
\[ I_1R_1' = I_1R_1 \text{(no change)} \]
\[ I_2R_2' = I_2R_2 \text{(dead phase)} \]
\[ I_3R_3' = I_3R_3 \text{(live phase)} \]
and, since the currents are the same, \( R_3' > R_1' > R_2' \). Consequently, with the power in each phase will be unbalanced.

Another effect of unbalanced resistances is that one electrode will consistently ride higher in the furnace than the other two. In extreme cases, such an electrode may be so far out that the metal bath freezes up and causes difficulties in tapping. If this occurs, that electrode should be run at a lower resistance. The main danger with dead and live phases is that they easily lead to gross imbalances on large furnaces, and this is most undesirable as is shown below.

**b) Lack of Sensitivity to Electrode Movement**

The lack of sensitivity resulting from high reactance can be observed. The current is relatively less sensitive to changes in resistance in a furnace where \( X \) is large than in a furnace where \( X \) is small. This means that, with a furnace under current control, the electrodes have to move further to correct for changes in the currents. By far the most serious problem with this sensitivity occurs during a bad imbalance of the furnace. During such an imbalance, it can happen that the current in an electrode cannot be maintained; even though the electrode is so far down that the variable resistance in that phase has been reduced to zero. This is because the reactance in that phase is limiting the current.

With a furnace under current control during unbalanced conditions, the low ratio of resistance to reactance results in a situation in which a small imbalance in the currents can cause relatively large changes in the resistances (and hence powers) in each phase, thus aggravating the unbalanced conditions.
c) Interaction Effect

The interaction effect is the somewhat unexpected interaction between the movement of one electrode and the currents and powers in the other electrodes. In the most common furnace arrangement, three-phase power is fed through three electrodes to a bath of molten metal, which forms the neutral point for the circuit. With only three connections to the neutral point, changes in one phase have a significant effect on the other two phases, particularly when the ratio of reactance to resistance is high. This interaction can be seen on a furnace when one of the electrodes is moved deliberately. If the electrode is raised (thus increasing the resistance), the current in that phase drops as expected. However, at the same time, the current in the previous phase in the phasor rotation also drops, but the third phase remains almost unaffected. The interaction with the powers in each phase (as shown) is even more difficult to predict intuitively. To show how the amount of interaction increases with reactance, the following example can be considered. Assume that the furnace circuit can be represented by the star-equivalent circuit, that the circuit is driven by live voltages $V_{12}, V_{23}, V_{31}$, which remain constant at 300 V, and that the normal operating current for the furnace is 100 kA in each phase. The circuit is then deliberately unbalanced by the forcible reduction of current in phase two to 80 kA while the currents in the other two phases remain at 100 kA. Now consider the effect of an increase in the relative reactance in the circuit by setting $X_1 = X_2 = X_3 = X$, changing X from 0 to 1.4 ms and calculating the power in each phase as a percentage of the total circuit power.

![Figure 3.5.4.1: Furnace 1 - Variation of Operating Reactance with Current](image-url)
At low reactance the current imbalance has little effect on the power balance. However, as the reactance is increased, the power imbalance becomes increasingly more severe until, at a reactance of about 1, 35 ms (there is no power at all in phase one and a large amount of power in phase three. It should also be noted that the phase with the low current is not the phase with the lowest power, and in fact this phase is relatively insensitive to changes in reactance. The basic trouble with interactions is that, if one electrode is abnormal, it affects the other two phases, and these phases in turn may become abnormal. Unfortunately, in the day-to-day operation of a furnace, the effects of interactions are difficult to predict without fairly sophisticated calculating facilities. Because of this difficulty, it is better to try to keep a large furnace as balanced as possible, thereby avoiding the difficulty. However, if an imbalance does occur, it must be carefully handled to avoid any subsequent troubles, and the following is a convenient rule of thumb that works satisfactorily. If an electrode is short so that the resistance in that phase is high, the current for that electrode will be lower than normal. If this lower current persists, then the current set point in the previous phase in the phasor rotation should be set to about midway between the normal current and the current in the low-current phase. This will keep the resistances under the two normal electrodes reasonably balanced, and therefore should not introduce any further troubles through these electrodes in turn becoming abnormal.

Figure 3.5.4.2: Phasor Diagrams Showing Dead and Live Phases
3.6 Development of Arc Furnace Model

In this research work, instead of using single valued piece-wise linear v-i characteristics of the arc furnace load, a dynamic and multi-valued v-i characteristics are obtained by solving the corresponding differential equation, whose parameter is the arc length and the forcing function is the arc current. In order to represent the flicker effect, a low frequency signal is modulated with the arc voltage. The arc furnace model is connected to the system as a controlled voltage source.

Arc Furnaces Model

The dynamic v-i characteristics of arc furnace load are obtained by using a general dynamic arc model in the form of a differential equation derived in. The approach taken in is fundamentally different from the previous methods that represent the electric arc in terms of their static v-i characteristics. The dynamic arc model will capture changes in the v-i characteristics as the operating conditions change in the power system. Therefore, starting from the power balance equation for the electric arc, the following differential equation that represents the general dynamics of the arc model is derived.

\[ K_1 r'' + K_2 r \frac{dr}{dt} = \frac{K_3}{r^{m+2}} i^2 \]  

(3.37)

Here "r\(^n\)" stands for the arc radius, is chosen as a state variable instead of taking arc resistance or conductance. While solving equation 1, the parameters are chosen as \(k_1=3000\), \(k_2=l\), and \(k_3=12.5\). And the arc voltage is then given by:

\[ V = \frac{i}{g} \]  

(3.38)

where \(g\) is defined as arc conductance and given by the following equation:

\[ g = \frac{r^{m+2}}{K_3} \]  

(3.39)

Figure 3.9 shows the dynamic v-i characteristics of 250 V, 70-kAac electric arc obtained by solving equations 1 and 2 in time domain. The simulated v-i
characteristics of electric arc matches well with the measured characteristics. The simulated arc voltage waveform is shown in Figure 3.6.1.

![Figure 3.6.1: Dynamic Characteristics of Arc Furnace](image1)

Figure 3.6.1: Dynamic Characteristics of Arc Furnace

![Figure 3.6.2: Voltage Waveform of Arc furnace](image2)

Figure 3.6.2: Voltage Waveform of Arc furnace

The model is built in PSCAD and implemented. The sample power system is test system for the arc furnace model. This system consists of linear loads and electric arc furnace loads connected to the secondary of the step down transformer. The test system comprises of source, transformer, transmission line, furnace transformer, arc furnace as load.
Figure 3.6.3: System Model Configuration
The test system model is implemented in PSCAD and the results are obtained and verified. Test System Specifications are as follows.

Table 3.1: Test System Specifications

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Specifications</th>
<th>Parameters</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source-Base MVA</td>
<td>80 MVA</td>
<td>Inductance of transmission line</td>
<td>2.38 e-3</td>
</tr>
<tr>
<td>Base Voltage</td>
<td>220 kV</td>
<td>Three Phase Furnace Transformer</td>
<td>40 MVA</td>
</tr>
<tr>
<td>Three phase transformer MVA</td>
<td>80</td>
<td>Basic operation frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Positive sequence leakage reactance</td>
<td>0.15 p.u.</td>
<td>Positive sequence leakage reactance</td>
<td>0.077 p.u.</td>
</tr>
<tr>
<td>Winding voltages 1&amp;2</td>
<td>220/34.5 kV</td>
<td>Winding voltages 1&amp;2</td>
<td>33/0.55 kV</td>
</tr>
<tr>
<td>Magnetizing current</td>
<td>0.4%</td>
<td>Magnetizing current</td>
<td>0.4%</td>
</tr>
<tr>
<td>Inrush decay time constant</td>
<td>1.0 sec</td>
<td>Inrush decay time constant</td>
<td>1.0 sec</td>
</tr>
<tr>
<td>Arc Furnace-K&lt;sub&gt;1&lt;/sub&gt;</td>
<td>2750</td>
<td>Magnitude of phase B</td>
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</tr>
<tr>
<td>K&lt;sub&gt;2&lt;/sub&gt;</td>
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<td>Modulation Phase B</td>
<td>8.8 Hz</td>
</tr>
<tr>
<td>K&lt;sub&gt;3&lt;/sub&gt;</td>
<td>1.0</td>
<td>Magnitude of Phase C</td>
<td>0.01</td>
</tr>
<tr>
<td>Magnitude of phase A</td>
<td>0.01</td>
<td>Modulation of Phase C</td>
<td>8.8 Hz</td>
</tr>
<tr>
<td>Modulation Phase A</td>
<td>8.8 Hz</td>
<td>Arc length&lt;sub&gt;initial&lt;/sub&gt;</td>
<td>6 ohms</td>
</tr>
</tbody>
</table>
3.7 Electrode Control System Model

With energy applied to the arc furnace the position of raw material and scrap steel inside the bath is highly heterogeneous and will cause irregular variations in the lengths of electric arcs. This will cause electrical energy input to vary significantly if the electrodes are kept static. Automatic and manual control of the electric power is necessary to achieve effective production of a single bath of molten steel. Worked out electrical specification, according to the required steel grade is normally used to determine the tap settings for each period within a single tap. Automatic regulation is then used to control the electrode tip positions and to minimize the variations in lengths of electric arcs. The lengths of electric arcs are defined as the distances between the electrode tips and the metallic charge in the furnace bath. Hence constant arc lengths will help to ensure that the electric energy input remain constant at some reference value [39, 40].

3.7.1 Electrode Position Control System

The arc lengths are controlled by vertically adjusting the electrode positions whenever it differs from specified reference values. The system responsible for the automatic control of the electric arc lengths are known as the electrode system. This is the point of importance for analysis. Each of these electrodes is connected to its own actuator system, which acts according to an error input signal, to move it up or down. The most common actuators are either hydraulic or electromechanical. 50% of the industrial actuators used to move electrodes are hydraulics and 50% are electromechanical. However a greater percentage of hydraulics electrode control systems are in use for larger furnaces with large capacities. Others use of use of arc furnace process includes the tilting of furnace for tapping purpose and lifting and swinging out the furnace roof. All the hydraulic for an electric furnace process are usually installed together in a single room. A hydraulic spool valve is used to control
the flow of inflammable fluid to a mast cylinder. The main spool valve is driven by an amplifying valve which operates with lubricating oil which is under pressure. A low level electric signal is applied to this amplifying valve which has the effect of moving the piston in the mast cylinder up or down. The mast cylinder is connected to graphite electrode. Note that three separator actuators are used for three phase electric arc furnace, one for each phase/electrode [41, 42, 43].

With graphite electrodes being oxidized and production being almost continuous, the lengths of electrodes will decrease with time. The electrodes are clamped into electrode arms and then can be slipped down manually by an operator while energy is being supplied to the furnace. This ensures longer electrodes lifetimes with resulting monetary savings. This phenomenon is another source of monetary savings. This phenomenon is another source of manual control in the electric arc furnace process [44, 45].

![Figure 3.7.1: Electrode Position Controllers for Electric Arc Furnace](image)

The electrode position control system of a one phase of electric arc furnace can be modeled in a block diagram as shown in Figure 3.13 which can represent the electrode position control system under either arc impedance control principle or arc current control principle.
3.7.2 Block Diagram of Electrode Control System

In block diagram of electrode position controller for one phase of EAF the dead zone nonlinearity represents the static nonlinear characteristic existing in the actuator motor of the electrode position controller. $\Delta$ is the dead zone parameter. $G$ is the pulse transfer function of the power amplifier and actuator

$$A(q^{-1}) = 1 + a_1 q^{-1} + ... + a_n q^{-n} \quad (3.40)$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + ... + b_n q^{-n} \quad (3.41)$$

are polynomials in the backward shift operator, $d$ is the time delay of the system ($t$) is the control variable corresponding to the control voltage signal of the actuator motor ($t$), $\xi_1(t)$, $\xi_2(t)$ represent various disturbance in electric arc furnace. $\xi_1(t)$, $\xi_2(t)$ are assumed to be white noise with zero mean and $L(t)$, random step disturbance. The random step disturbance will properly represent those large disturbances that the electrode regulation process is subject to in electric arc furnace [46, 47, 48].

Taking into account the above considerations, the discrete time model of the electrode control system is obtained as

$$A(q^{-1})y(t) = k_0 q^{-d} B_0(q^{-1}) u(t) + k_0 A(q^{-1}) L(t) + k_0 A(q^{-1}) \xi_1(t) + A(q^{-1}) k_0 \xi_2(t) + A(q^{-1}) h(t) + A(q^{-1}) \xi_3(t) \quad (3.42 \text{ a})$$

$$\bar{u}(t) = u(t) - \Delta \operatorname{sgn}[u(t)], \quad \text{I}(u(t)) I > \Delta$$

$$= 0 \quad \text{I}(u(t)) I < \Delta \quad (3.42 \text{ b})$$
Since $A(q^{-1})$ is the characteristic polynomial of $G$ and $L(t)$ is random step disturbance, they can be written as

$$A(q^{-1}) = A_1(q^{-1})(1 - q^{-1}) \quad (3.43)$$

$L(t) = \xi_L(t)(1 - q^{-1})$ is a polynomial with order $n^{-1}$ and $\xi_1(t)$ is a uncorrelated stochastic sequence with zero mean.

Evaluating 3.40 and 3.41 gives

$$A(q^{-1})t(t) = k_1q^{-1}B_1(q^{-1})u(t) + k_2A(q^{-1})A(q^{-1})\xi(t) + K_0A(q^{-1})\xi(t) + A(q^{-1}) + K_0A(q^{-1}) + A(q^{-1})\xi(t) \quad (3.44)$$

$$\bar{u}(t) = u(t) - \Delta \text{sgn}[u(t)], \quad I(\ u(t) \ ) I > \Delta$$

$$= 0 \quad I(\ u(t) \ ) I < \Delta \quad (3.45)$$

Model gives the complete description of the electrode position controller of the arc furnace. It is found that except $\xi_L(t)$, $\xi_1(t)$, $\xi_2(t)$, $k_1(t)$ and $h(t)$ the influence of the fast time varying part of the arc characteristic parameter on the electrode position control system is also considered as a kind of disturbance in this model, but its effects must be dealt with specially in the direct digital controller to be designed for the electrode position control especially when adaptive control strategies are introduced to cope with the time varying system control problem.

### 3.7.3 Digital Control strategy for Electrode Control

In regular operation single controller, impedance regulation, current regulation, constant control dynamics and digressive control dynamics are the features. The control aspects include guiding electrodes 1, 2, 3, mean current and minimum current. The auxiliary functions of electrode control are automatic adjustment for manual operation, automatic calculation of final amplification, plotting programs, software transient recorder and visualization interface. A typical value for digital electrode control response time is 17 msec. The A/D conversion and smoothing may take 4 msec, and 1 msec required for D/A conversion and so [49,50].
Figure 3.7.3: Block Diagram of Digital Control of Electric Arc Furnace
3.8 Static Var Compensator (SVC)

Static Var Compensator (SVC) can provide an excellent source of rapidly controllable reactive shunt compensation for dynamic voltage control through its utilization of high-speed thyristor switching/controlled reactive devices.

An SVC is typically made up of the following major components:

- Coupling transformer
- Thyristor valves
- Reactors
- Capacitors (often tuned for harmonic filtering)

In general, the two thyristor valve controlled/switched concepts used with SVCs are the thyristor-controlled reactor (TCR) and the thyristor-switched capacitor (TSC). The TSC provides a “stepped” response and the TCR provides a “smooth” or continuously variable susceptance.

Two “common” main SVC circuit arrangements shown in Figure 3.8.1 are:

- “FC/TCR”–fixed capacitor/filter/thyristor(phase angle)-controlled reactor (Config. A)
- “TSC/TCR”–thyristor-switched capacitor/thyristor-controlled reactor (Config. B)
An SVC is a controlled shunt susceptance (B) as defined by the SVC control settings that injects reactive power (Q) into the system based on the square of its terminal voltage. Figure 3.8.2 illustrates a TCR/FC SVC, including the operational concept. The control objective of the SVC is to maintain a desired voltage at the high-voltage bus. In the steady-state, the SVC will provide some steady-state control of the voltage to maintain it the high-voltage bus at a pre-defined level. If the high-voltage bus begins to fall below its set point range, the SVC will inject reactive power (Q<sub>net</sub>) into the system (within its controlled limits), thereby increasing the bus voltage back to its desired voltage level. If bus voltage increases, the SVC will inject less (or TCR will absorb more) reactive power (within its controlled limits), and the result will be to achieve the desired bus voltage. From Figure 3.8.2, +Q<sub>cap</sub> is a fixed capacitance value, therefore the magnitude of reactive power injected into the system, Q<sub>net</sub>, is controlled by the magnitude of -Q<sub>ind</sub> reactive power absorbed by the TCR [100].
The fundamental operation of the thyristor valve that controls the TCR is described here. The thyristor is self commutates at every current zero, therefore the current through the reactor is achieved by gating (or firing) the thyristor at a desired conduction angle (or firing angle) with respect to the voltage waveform. Figure 3.8.2 describes the relationship between the fundamental frequency TCR current and firing angle.

![Figure 3.8.2: SVC with Control Concept](image)

The relationship between the TCR current and firing angle is shown in Figure 3.8.3.

![Figure 3.8.3: TCR Current Versus Firing Angle Characteristics](image)
Figure 3.8.3 further illustrates the thyristor valve operating characteristics of a thyristor controlled reactor. The firing pulses are on the order of 10 μs. So it is concluded that as the firing angle increases above 90 degrees, the current in the TCR is reduced. Referring back to Figure 3.17, the “Pulse Generator” block after the AVR block utilizes the concepts discussed here and illustrated in Figures 3.8.3 and 3.8.4 to determine the firing angle for the thyristor valve controlling the reactor.

![TCR Operating Characteristic](image)

**TCR OPERATING CHARACTERISTIC**

Figure 3.8.4: Illustration of Relationship between TCR Current and Firing Angle

### 3.8.1 V-I characteristics of an SVC

As shown in Fig 3.3.8.5, the dynamic characteristics of an SVC are the plots of bus voltages versus current or reactive power. In Fig 3.8.5, the voltage $V_{\text{ref}}$ is the voltage at the terminals of the SVC when it is neither absorbing nor generating any reactive power. The reference voltage value can be varied between the maximum and minimum limits, $V_{\text{refmax}}$ and $V_{\text{refmin}}$ using the SVC control system. The linear range of the SVC control passing through $V_{\text{ref}}$ is the control range over which the voltage varies linearly with the current or reactive power. In this range, the power is varied from capacitive to inductive.
The slope or droop of the V-I characteristic is the ratio of change in voltage magnitude to the change in current magnitude over the linear control range. This slope is given by

\[ K_{sl} = \frac{\Delta V}{\Delta I} \Omega \]  

(3.46)

Where, \( \Delta V \) denotes the change in voltage magnitude (V) and \( \Delta I \) denotes the change in current magnitude (I). The slope \( K_{sl} \) can be changed by the control system. Ideally, for voltage regulation it is required to maintain a flat voltage profile with a slope equal to zero. In practice, it is desirable to incorporate a finite slope of about 3-5% for the following reasons:

1. It reduces the reactive power rating of the SVC substantially for achieving similar control objectives;
2. It prevents the SVC from reaching its reactive-power limits too frequently; 3. It facilitates the sharing of reactive power among multiple compensators connected or operating in parallel.

Once the SVC’s operating point crosses the linear controllable range, it enters the overload zone where it behaves like a fixed inductor or capacitor.
3.8.2 Static VAR Compensator Model

The static VAR compensator under consideration is a twelve pulse system with two six pulse groups in Δ connection and coupled with the network through a single, three-winding transformer with Y and Δ secondary [105],[106].

The SVC impedances are converted to Y configuration and transferred to the primary transformer voltage. Each six-pulse group consists of the transformer model, the thyristor con-trolled reactor (TCR) and the capacitor unit in parallel with a resistance. An equivalent, six-pulse group model is shown in the single phase diagram in Figure 3.8.2.

![SVC Electrical Circuit Model](image)

Figure 3.8.2.1: SVC Electrical Circuit Model

The model can be represented in the state-space domain as follows:

\[
0 = \frac{1}{L_t} V_1 - \frac{1}{L_t} V_2 \tag{3.47}
\]

\[
0 = \frac{1}{C_{svc}} i - \frac{1}{C_{svc}} i_{scr} \tag{3.48}
\]

\[
0 = \frac{1}{L_{scr}(\phi)} V_2 \tag{3.49}
\]

Equation (3.41) is non-linear in view of the fact that the TCR reactance is dependent upon the firing angle obtained from the controller model. This equation cannot be directly linearised since the SVC model is developed in the AC frame with oscillating variables, \(i.e. v_2 = V_2 \cos(\omega t + \varphi)\) whereas the firing angle signal is derived as a signal in the controller reference frame (i.e. a non-oscillating signal).

To link the SVC model with the controller model, the approach of artificial rotating susceptance is adopted. It is firstly presumed that the AC terminal voltage has the
following value in the steady state: $v_2^0 = V_2^0 \cos(\omega t + \varphi_0^0)$, where superscript “0” denotes the steady-state variable, i.e., $V_2^0$ is a constant magnitude, $\varphi_0^0$ is a constant angle and $v_2^0$ is a rotating vector of a constant magnitude and angle. The susceptance value in the steady-state is $1/L_{tcr}^0$.

Assuming small perturbations around the steady state we have:

$$V_2 = (V_2^0 + \Delta V_2) \quad (3.50)$$

$$1/L_{tcr} = 1/L_{tcr}^0 + \Delta(1/L_{tcr}) \quad (3.51)$$

Small perturbations are justified assuming an effective voltage control at the nominal value. Multiplying the terms in (3.42) and (3.43) and substituting in (3.41) results in:

$$\Delta i_{tcr} = V_2^0 \Delta V_2/L_{tcr}^0 + \Delta V_2/L_{tcr}^0 + \Delta V_2^0 \Delta(1/L_{tcr}) \quad (3.52)$$

The susceptance in (3.41) is further represented, using only the fundamental component, as:

$$L_{tcr} = L_{tcr}^0 \frac{\pi}{\sin(2\pi - \phi)} \quad (3.53)$$

Where $L_{tcr}$ corresponds to the maximum conduction period, $\phi_0 = 90^0$. Equation (3.53) can be linearised as

$$\Delta(1/L_{tcr}) - K_{mc} \Delta \phi, K_{mc} = \partial(1/L_{tcr})/\partial \phi \quad (3.54)$$

The above linearisation is justified in practice since most modern SVC control systems will have a gain compensation scheme (look-up table) that maintains a constant system gain [107].

In view of (3.52), and neglecting the small terms, equation (3.52) is written as:

$$\Delta i_{tcr}^0 = V_2^0 K_{mc} \Delta \phi + \Delta V_2/L_{tcr}^0 \quad (3.55)$$

and it replaces the model. Equation in the AC coordinate frame, and the following term:

$$V_2^0 K_{mc} \Delta \phi = V_2^0 K_{mc} \Delta \phi \cos(\omega t + \phi_0^0) \quad (3.56)$$

is an artificial oscillating variable (susceptance) that has a varying magnitude and a constant angle equal to the voltage nominal angle. In this way, the SVC model
(6),(7),(14) has all oscillating variables that are converted to $d-q$ variables, as is done with the AC system model in (3)-(5). Subsequently, using the $d-q$ components of the inputs and outputs, this model is linked with the other model units. In order to link the $d-q$ components of the rotating susceptance (15) with the controller module, these components are further converted to magnitude angle components using the $x-y$ to polar co-ordinate transformation [108].

It should be noted that the transformer impedance ($L_t$) must be included in this model since the eigen value analysis proves that this parameter has noticeable effects on system dynamics. This conclusion is contrary to HVDC modeling principles, since it has been demonstrated [108],[109] that transformer dynamics can be excluded from system dynamic models.

### 3.8.3 Controller Model

The controller model consists of a second order feedback filter, PI controller, Phase Locked Loop (PLL) model and transport delay model, as shown in Figure 3.8.7. The PLL system is of the $d-q-z$ type and its functional diagram is given in [110] and [111], whereas the state space linearised second-order model is developed in [8].

The delay filter does not have dynamic equivalent in the actual system. It is introduced to represent the effects of the discrete nature of the signal transfer caused by thyristor firings at discrete instants in the fundamental cycle. This simplified continuous-element modeling of a discrete phenomenon has limited accuracy, but the model application value is much increased with the continuous form and, as demonstrated in the following sections, accuracy proves satisfactory for most applications. Researchers in [99] conclude that the delay filter time constant has a value of 3-6ms and reference [93] suggests 2.77ms. During the proposed model verification, simulation studies have suggested that the value of approximately $T_d = 2.85ms$ is used, which is in agreement with the above recommendations.
Model Connections

The above three models are linked to form a single system model in the state-space form. The final model has the following structure:

\[
x_i = A_s x_s + B_s u_{out}
\]

\[
y_{out} = C_s x_s + D_s u_{out}
\]

(3.57)

where “s” labels the overall system and the model matrices are:

\[
A_s = \begin{bmatrix}
A_{co} & B_{ctco} & * & C_{tcco} & B_{caco} & * & C_{acaco}
\end{bmatrix}
\]

(3.58)

\[
B_s = \begin{bmatrix}
B_{coout} \\
B_{acout}
\end{bmatrix}
\]

\[
C_s = \begin{bmatrix}
C_{coout} & C_{tcout} & C_{acout}
\end{bmatrix}
\]

(3.59)

All the subsystems’ D matrices are assumed zero in (3.59) since they are zero in the actual model and this noticeably simplifies development.

The matrix \(A_s\) has the subsystem matrices on the main diagonal, with the other sub-matrices representing interactions between subsystems. The model in this form has advantages in flexibility since, as an example, if the SVC is connected to a more complex AC system only the \(A_{ac}\) matrix and the corresponding input and output matrices need modifications. The above structure enables the model to be readily interfaced with the MATLAB model or other elements or Power Systems Blockset, for the purpose of investigating interactions and coordination.