CHAPTER-III

MAINTENANCE ACTIVITY SINGLE-MACHINES SCHEDULING ANDDUE-DATE ASSIGNMENT SIMULTANEOUSLY

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3.1 INTRODUCTION

The majority of machine scheduling literature assumes that the machines are available for processing jobs at all times during the planning horizon. However, this assumption may not be valid in a real production situation due to preventive maintenance (a deterministic event) or breakdown of machines (a stochastic phenomenon). Uncertain breakdowns will make the shop behavior hard to predict, thus reducing the efficiency of the production system. Maintenance activity can reduce the breakdown rate with minor sacrifices in production time. The role and importance of industrial maintenance has increasingly been recognized by decision makers (Pinjala et al., 2006). Therefore, scheduling the maintenance in manufacturing systems has gradually become a common practice in many companies. However, according to practical experience, it sometimes can be found that some of the machines are awaiting maintenance while there are jobs waiting to be processed by these machines. This is due to the lack of coordination between maintenance planning and production scheduling. Therefore, there is a need to develop efficient scheduling methods to improve the situation by deriving a satisfactory schedule that considers both jobs and maintenance activity simultaneously. With proper planning of the maintenance activities, the shop can improve production efficiency and safety, resulting in increased productivity and heightened safety awareness.

Recently, machine scheduling problems with the effect of learning or aging have received increasing attention. In scheduling with the learning effect, the actual processing time of a job decreases as a function if it is scheduled later in a sequence, while in scheduling with the aging effect the actual processing time of a job increases as a function if it is scheduled later in a sequence. Up to now, numerous papers have been investigated on scheduling problems with the effect of learning or aging to minimize performance measures. For details on this stream of research, the reader may refer to the comprehensive surveys by **Janiak** and **Mikhail** [1], **Biskup** [2], and **Janiak** and **Rudek** [3].

Applications of the common due-date problem in real-life situations can readily be found. For example: a common due-date might reflect an assembly environment in which the components of a product should all be ready at the same time in order to avoid staging delays, or a shop where several jobs constitute a single customer's order. **Panwalkar et al.** [4] first introduced a due-date assignment problem in scheduling. They considered that all the jobs have a common due-date. The objective was to find an optimal common due-date and an optimal schedule which minimizes the total earliness, tardiness and due-date costs. Their study provided a $O(n\log n)$ solution for the problem. Additionally, due to the many applications, production scheduling problems with a maintenance activity planning to improve the production efficiency or in preventing the machine from malfunction have been one of the most popular topics among researchers. During the maintenance activity, the machine becomes unavailable for processing jobs. Scheduling under such an environment is known as scheduling with availability constraints. As of now, plentiful research has been conducted on availability constraints under different environments, such as **Chen** and **Yang** [5], **Yao** and **Huang** [6], **Gawiejnowicz** [7], **Chen** and **Tsou** [8], **Yang** and **Yang** [9]**, Schmidt** [10] and **Ma et al.** [11] provided extensively surveys related to machine scheduling problems with availability constraints.

Recently **L. Chen, J.** [12] represented transmission constraints, but did not recognize interconnection failures and Maintenance,

Silva, E. L., et al. [13] recognized the composite generation and transmission reliability but did not consider transmission maintenance. **Mosheiov** and **Sidney** [14] addressed the problems of minimizing makespan with precedence relations, minimizing makespan with learning effect, and minimizing the number of tardy jobs. In this note we study a classical due-date assignment problem with the option of scheduling a maintenance activity. **Panwalker, Smith** and **Seidmann** [4] addressed the following single machine scheduling and common due-date assignment problem: All jobs have a common (but unknown) due-date. The objective is to find an optimal value of the due-date and optimal sequence which minimizes the total penalty based on the due-date value and the earliness or tardiness of each job. Panwalker et al. consider a set of n jobs available at time zero. The common due-date d is a decision variable. The processing time of job j is denoted by p_j, $j = 1, 2, ..., n$. For a given schedule, the completion time of job j is denoted by C_j. The earliness and tardiness of job j are defined as $E_{[j]} = \max\{0, d - C_j\}$ and $T_j = \max\{0, C_j - d\}$, $j = 1, 2, ..., n$, respectively. Three cost components are assumed: for earliness, for tardiness and for (delaying the) due-date. The unit penalties for earliness, tardiness and due-date are denoted by α , β and γ respectively. The objective is to minimize the total cost, i.e.

$$z = f(d,\pi) = \sum_{j=1}^{n} (\alpha E_{[j]} + \beta T_{[j]} + fd).$$

The objective is to minimize the total earliness, tardiness and due-date costs. We show that there exists a polynomial time solution for the proposed problem. We also discuss two special cases of the problem and show that they can be optimally solved by a lower order algorithm.

3.2 MAINTENANCE ACTIVITY

Consider a single machine setting and n independent jobs. All jobs are available for processing at time zero. The (basic) processing time of job j is denoted by $a_j, j = 1,...,n.$ The scheduler has an option to perform a maintenance activity denoted by MA. As mentioned, the impact of performing this activity is reflected in a reduction of the job processing times: if job j is scheduled after the maintenance activity (called by Lee and Leon (2001), the rate modifying activity), then its processing time becomes $b_j \left(b_j \le a_j\right),\ j = 1,...,n.$ The basic and shorter processing times of the j^{th} job in the sequence are denoted by $a_{(j)}$ and $b_{(j)}$, respectively.

The length of time required to perform the maintenance activity T_{MA} increases linearly with its starting time: $T_{MA} = T_0 + \delta t$, (where T_0 and δ are positive constants and t is the MA's starting time). It should be noted that in the context of scheduling a maintenance activity (or in general, when machine non-availability periods are assumed), the scheduling literature (see e.g. Lee, 2004) considers two important settings: (i). The non-resumable case: if a job is interrupted by the maintenance, it needs to be restarted from the beginning when the machine becomes available again, and (ii). The resumable case: when such a job does not need to restart and will complete its processing after the maintenance.

For a given schedule, C_j denotes the completion time of job j, $j = 1,\ldots,n$. If d_j is the due-date of job j, then its lateness is $L_j = C_j - d_j$, $j = 1,\ldots,n$. The earliness and tardiness of job j are defined by $E_j = \max\{d_j - C_j, 0\}$ and $T_j = \max\{C_j - d_j, 0\}$, respectively. The earliness and tardiness costs per unit of time are denoted by α and β respectively. In the case of a common due date $(d_j = d)$, we denote the cost per unit of time for (delaying) the due-date by γ.

Finally, we define the standard indicator for tardiness: $U_j = 1$ if $C_j > d_j$ (a tardy job), and $U_j = 0$ if $C_j \le d_j$ (on-time job), $j = 1,...,n.$

Most classical objectives are considered: makespan, flow-time, maximum lateness, total earliness, tardiness and due-date cost, and minimum number of tardy jobs. Formally, if $T_{MA} = T_0 + \delta t,$ denotes the (linearly deteriorating) maintenance time, and $p_j = (a_j, b_j)$ reflects the normal and the shortened processing times, the following problems are studied:

(i) $1 / T_{MA} = T_0 + \delta t, p_j = (a_j, b_j) / C_{\max}$

(ii) $1 / T_{MA} = T_0 + \delta t, p_j = (a_j, b_j) / \sum C_j$

(iii) $1 / T_{MA} = T_0 + \delta t, p_j = (a_j, b_j) / L_{\max}$

(iv) $1 / T_{MA} = T_0 + \delta t, p_j = (a_j, b_j), d_j = d / \sum (\alpha E_j + \beta T_j + \gamma d)$

(v) $1 / T_{MA} = T_0 + \delta t, p_j = (a_j, b_j), d_j = d / \sum U_j$

In all these cases we look for the optimal schedule of both the jobs and the maintenance activity.

3.3 PRELIMINARY RESULTS

In the following we show that several properties of an optimal solution for the original due-date assignment problem, which were proved by **Panwalkar et al.** (1982), continue to hold when the job-dependent aging effect and the deteriorating maintenance activity are considered simultaneously. First, it is clear that an optimal schedule starts at time zero. In addition, an optimal schedule exists with no idle time between consecutive jobs. In Lemma 1 we show that there exists an optimal schedule in which the due-date coincides with some job completion time for the $1/ma, p_{jr} = p_j r^{aj} / \sum_{j=1}^{n}\left(\alpha E_j + \beta T_j + \gamma d\right)$ problem.

3.4 FORMULATION OF THE PROBLEM

There are n independent jobs to be processed on a single machine. All the jobs are non-resumable and available for processing at time zero. Job j has a normal processing time P_j and a job-dependent aging factor a_j, where $a_j > 0$. Let job j is scheduled in the r^{th} position is a sequence, its actual processing time is $P_{jr} = P_j r^{a_j}$. All the jobs share a common due-date d.

The position and the starting time of the maintenance activity are not known in advance. It can be scheduled immediately after the processing of any one job has been completed. We further assume that: (1) after the maintenance activity, the machine will revert to its initial condition and the aging effect will start a new (2) the machine maintenance duration is a linear function of its starting time and is represented as $f(t)=b+ct$, where $b>0$ and $c\geq 0$ are constants, and t is the scheduled in the r^{th} position after the maintains activity in a sequence, the its actual processing time is given by $P_{jr}=P_j(r-i)^{a_j}$, where i denotes the position of a job preceding the maintain activity (i.e. position $(i+a)$ is the first position after the maintains activity). The problem under consideration is to find jointly the optimal common due date d. The optimal maintenance position and the optimal job sequence π such that the following cost function is minimized.

$$z=f(d,\pi)=\sum_{j=1}^{n}(\alpha E_{[j]}+\beta T_{[j]}+fd) \qquad (3.4.1)$$

where $\alpha>0$, $\beta>0$ and $\gamma>0$ $(\beta>\gamma)$ are the earliness tardiness and due-date unit time penalties respectively, i.e. a job is finished on the due date, it will incur the manufacturing penalty cost only. If a job is finished earlier than its due date, it will incur the manufacturing penalty cost and the earliness penalty cost. On the other hand, if a job a finished later then its due-date, if will incur the manufacturing penalty cost and the tardiness penalty cost, the problem can be denoted as $1/ma, P_{jr} = P_j r^{a_j} / \sum_{j=1}^{n} (\alpha E_j + \beta T_j + \gamma d)$, where ma in the second field denotes the maintenance activity.

***Property 1**: For any specified sequence π there exists an optimal due-date d in which coincides with some job completion times.*

Proof. Suppose that there exists a job sequence π starting at time zero and containing jobs at the k^{th} positions such that $C_{[k-1]} \le d \le C_{[k]}$, where $0 \le k \le n$. We focus on the case of $i < k$, the complementary case can be similarly proved. Without loss of generality, we assume that a job sequence $\pi = (J_1, J_2, ..., J_n)$. Let sequence $\delta = d - C_{k-1}$. Note that $0 \le k \le p_k (k-i)^{ak}$. Then, the machine maintenance duration is

$$f(t) = b + ct = b + c\left(p_1 + p_2.2^{a2} + p_3.3^{a3} + ... + p_i.i^{ai}\right)$$

And the earliness cost (denoted by Z_i) associated with job $j,\ j = k-1, k-2, ..., 1$ is given by

$$Z_{k-1} = \alpha\delta$$

$$Z_{k-2} = \alpha\left[\delta + p_{k-1}(k-1-i)^{ak-1}\right]$$

$$Z_{k-3}=\alpha\left[\delta+p_{k-1}(k-1-i)^{ak-1}+p_{k-2}(k-2-i)^{ak-2}\right]$$

$$Z_{i+1}=\alpha\left[\delta+p_{k-1}(k-1-i)^{ak-1}+p_{k-2}(k-2-i)^{ak-2}+\ldots+p_{i+2}.2^{a_{i+2}}\right]$$

$$Z_{i}=\alpha\left[\delta+p_{k-1}(k-1-i)^{ak-1}+p_{k-2}(k-2-i)^{ak-2}+\ldots+p_{i+1}+f(t)\right]$$

$$Z_{i-1}=\alpha\left[\delta+p_{k-1}(k-1-i)^{ak-1}+p_{k-2}(k-2-i)^{ak-2}+\ldots+p_{i+1}\right.$$
$$\left.+f(t)+p_i i^{a_i}\right]$$

$$Z_{1}=\alpha\left[\delta+p_{k-1}(k-1-i)^{ak-1}+p_{k-2}(k-2-i)^{ak-2}+\ldots+p_{i+1}\right.$$
$$\left.+f(t)+p_i i^{a_i}+\ldots+p_2.2^{a_2}\right]$$

The tardiness cost (denoted by Z_i) associated with job $j,\ j=k,k+1,\ldots,n,$ is given by

$$Z_k=\beta\left[p_k(k-1)^{a_k}-\delta\right]$$

$$Z_{k+1}=\beta\left[p_k(k-i)^{a_k}+p_{k+1}(k+1-i)^{a_{k+1}}-\delta\right]$$

$$Z_n=\beta\left[p_k(k-i)^{a_k}+p_{k+1}(k-1-i)^{a_{k+1}}+\ldots+p_n(n-i)^{a_n}-\delta\right].$$

The due-date cost (denoted by Z_d) is given by

$$Z_d=\sum_{j=1}^{n}\gamma d=n\gamma\left[\delta+p_{k-1}(k-1-i)^{ak-1}+p_{k-2}(k-2-i)^{ak-2}\right.$$
$$\left.+\ldots+p_{i+1}+f(t)+p_i i^{a_i}+\ldots+p_2.2^{a_2}+p_1\right]$$

The total cost is given by

$$Z = \sum_{j=1}^{n} \left(\alpha E_j + \beta T_j + \gamma d \right).$$

It is easily to see that

$$Z = A\delta + B,$$

Where $A = \alpha(k-1) - \beta(n-k+1) + n\gamma$ and

$$B = \alpha\left[b_i + \sum_{j=1}^{i} (j-1+ci) p_j j^{a_j} + \sum_{j=i+1}^{k-1} (j-1) p_j (j-i)^{a_j} \right]$$

$$+ \beta \sum_{j=k}^{n} (n-j+1) p_j (j-i)^{a_j}$$

$$+ n\gamma\left[b + \sum_{j=1}^{i} (1+c) p_j j^{a_j} + \sum_{j=i+1}^{k-1} p_j (j-i)^{a_j} \right].$$

Clearly, A and B are constants and independent of δ. We conclude that Z is a liner function of δ, and is, therefore, minimized either for $\delta = 0$ or for $\delta = p_k (k-i)^{a_k}$. Thus, an optimal schedule exists in which the common due date d coincides with some job completion times.

In property 2, we determine the index k of the job whose completion time coincides with the due date, i.e., $C_{[k]} = d$. We show that the job-dependent aging effect and the deteriorating maintenance activity do not affect the value of k obtained by Panwalkar, et al. (1982).

Property 2: *For any specified sequence* π, *there exists an optimal common due-date equal to* $c_{[k]}$ *where* $k = \lceil n(\beta-\gamma)/(\alpha+\beta) \rceil$.

Proof. Consider an optimal schedule and an optimal due-date such that $C_{[k]} = d$ for some jobs k. Using the small perturbation technique introduced by Panwalkar, et al. (1982), we will investigate the change in the total cost when a due date is shifted. When the due-date is moved to the right δ units of time, change (ΔZ_1) in the total cost is given by

$$\Delta Z_1 = \alpha k\delta + n\gamma\delta - \beta(n-k)\delta \tag{3.4.2}$$

On the contrary, when the due-date is moved to the left δ units of time, change (ΔZ_2) in the total cost is given by

$$\Delta Z_2 = -\alpha(k-1)\delta + n\gamma\delta + \beta(n-k+1)\delta. \tag{3.4.3}$$

Both equations (3.4.2) and (3.4.3) are clearly non-negative due to the optimality of the original solution. From $\Delta Z_1 \geq 0$, we can have $k \geq \frac{n(\beta-\gamma)}{\alpha+\beta}$, whereas we obtain $k \leq \frac{n(\beta-\gamma)}{\alpha+\beta} + 1$ from $\Delta Z_1 \geq 0$. By property 1, we know that k coincides with some job completion times. Since k is integer, it follows that k is the smallest integer greater than or equal to $n(\beta-\gamma)/(\alpha+\beta)$, *i.e.*, $k = \lceil n(\beta-\gamma)/(\alpha+\beta) \rceil$.

Observe that form property 2, if $\beta \leq \gamma$, then the optimal due-date is set to time zero.

In general, it rarely occurs in real production settings.

Finally, a useful lemma which will be applied to solve the problem is given as follows.

Property 3*: let there be two sequences of numbers x_i and y_i the sum $\sum_{i=1}^{n} x_i y_i$ of products of the corresponding elements is the least (largest) if the sequence are monotonic in the opposite (same) sense.*

3.5 OPTIMAL SOLUTION

By property 2, we can determine the optimal position of common due date d. if the maintenance activity is performed prior to the due-date (i.e. $i < k$), then the total cost is given by

$$z = \sum_{j=1}^{n} (\alpha E_{[j]} + \beta T_{[j]} + yd)$$

$$= \alpha \left[\sum_{j=1}^{i} (d - c_{[j]}) + \sum_{j=i+1}^{k} (d - c_{[i]}) \right] + \beta \sum_{j=k+1}^{n} (c_{[j]} - d)$$

$$+ n\gamma \left[\sum_{j=1}^{i} P_{[j]} j^{a_{[j]}} + \sum_{j=i+1}^{k} P_{[j]} (j-i)^{a_{[j]}} + \left(b + c \sum_{j=1}^{i} P_{[j]} J^{a_{[j]}} \right) \right]$$

$$= \alpha \left[\sum_{j=1}^{i} (j-1) \sum_{j=1}^{i} P_{[j]} j^{a_{[j]}} + \sum_{j=i+1}^{k} (j-1) P_{[j]} (j-i)^{a_{[j]}} \right] + i \left(b + c \sum_{j=1}^{i} P_{[j]} J^{a_{[j]}} \right) \Bigg]$$

$$+\beta\sum_{j=i+1}^{k}(n-j+1)P_{[j]}(j-i)^{a_{[j]}}+n\gamma\left[\sum_{j=1}^{i}P_{[j]}j^{a_{[j]}}+\sum_{j=i+1}^{k}P_{[j]}(j-i)^{a_{[j]}}\right.$$

$$\left.+\left(b+c\sum_{j=1}^{i}P_{[j]}J^{a_{[j]}}\right)\right]+\sum_{j=k+1}^{n}\beta(n-j+1)P_{[j]}(j-i)^{a_{[j]}} \quad (3.5.1)$$

Let x_{jr} be a 0/1 variable such that $x_{jr}=1$ if job j is scheduled in the r^{th} position to be processed on the machines and $x_{jr}=0$ otherwise. Then for given $i<k$, the problem can be formulated as the following assignment problem

$$\text{Minimize } z=\sum_{j=1}^{n}(\alpha E_{[j]}+\beta T_{[j]}+\gamma d)$$

$$=b(n\gamma+\alpha i)+\sum_{j=1}^{n}\left\{\sum_{r=1}^{n}[n\gamma(1+c)+\alpha(r-1+(i))]P_j r^{a_j}x_{jr}\right\}$$

$$+\sum_{r=i+1}^{k}[n\gamma+\alpha(r-1)]P_j(r-i)^{a_j}x_{jr}+\sum_{r=k+1}^{n}[(\beta(n-r-1))]P_j(r-i)^{a_j}x_{jr} \quad (3.5.2)$$

Subject to

$$\sum_{r=1}^{n}x_{jr}=1,\ i=1,2,....,n \quad (3.5.3)$$

$$\sum_{j=1}^{n}x_{jr}=1,\ r=1,2,....,n \quad (3.5.4)$$

$$x_{jr}=1 \text{ or } 0, i=1,2,....,n, r=1,2,....,n \quad (3.5.5)$$

On the other hand, if the maintenance activity is performed after the due-date (i.e. $i\geq k$), then the total cost is given by

$$z=\sum_{j=1}^{n}(\alpha E_{[j]}+\beta T_{[j]}+\gamma d)$$

$$=\alpha\sum_{j=1}^{i}(d-c_{[j]})+\beta\left[\sum_{j=k+1}^{i}(c_{[i]}-d)+\sum_{j=i+1}^{k}(c_{[j]}-d)\right]+n\gamma\sum_{j=1}^{k}P_{[j]}j^{a_{[j]}}$$

$$=\alpha\sum_{j=1}^{k}(j-1)P_{[j]}j^{a_{[j]}}+\beta\left[\sum_{j=k+1}^{i}(n-j+1)P_{[j]}j^{a_{[j]}}+\sum_{j=i+1}^{k}(n-j+1)P_{[j]}(j-i)^{a_{[j]}}\right]$$

$$+\beta(n-i)\left(b+c\sum_{j=1}^{i}P_{[j]}j^{a_{[j]}}+n\gamma\sum_{j=1}^{k}P_{[j]}j^{a_{[j]}}\right)$$

$$=\beta b(n-i)+\sum_{j=1}^{k}\left[n\gamma+\alpha(j-1)+\beta c(n-i)\right]P_{[j]}j^{a_{[j]}}$$

$$+\sum_{j=k+1}^{i}\left[\beta(n-j+1)+\beta c(n-i)\right]P_{[j]}j^{a_{[j]}}+\sum_{j=i+1}^{n}\beta(n-j+1)P_{[j]}(j-1)^{a_{[j]}}$$

(3.5.6)

Then, for given $i\geq k$, the problem can be formulated as the following assignment problem

$$\text{Minimize } z=\sum_{j=1}^{n}(\alpha E_{[j]}+\beta T_{[j]}+\gamma d)$$

$$=\beta b(n-i)+\sum_{j=1}^{n}\left\{\sum_{r=1}^{k}\left[n\gamma+\alpha(r-1)+\beta c(n-i)\right]P_{[j]}j^{a_{[j]}}x_{jr}\right.$$

$$\left.+\sum_{r=k+1}^{i}\left[\beta(n-r+1)+\beta c(n-i)\right]P_{[j]}j^{a_{[j]}}x_{jr}+\sum_{r=i+1}^{n}\beta(n-r+1)P_{[j]}(r-1)^{a_{[j]}}x_{jr}\right\}$$

(3.5.7)

Subject to

$$\sum_{r=1}^{n} x_{jr} = 1, j = 1,2,....,n \tag{3.5.8}$$

$$\sum_{j=1}^{n} x_{jr} = 1, r = 1,2,....,n \tag{3.5.9}$$

$$x_{jr} = 1 \text{ or } 0, j = 1,2,....,n, r = 1,2,....,n \tag{3.5.10}$$

Once the position of the maintenance activity is determined. Solving the associated assignment problem requires an effort of $0(n^3)$ time. Since the maintenance activity can be scheduled immediately after any hob, n different position must be evaluated to obtain the global optimal schedule.

3.6 SPECIAL CASES

For the case where the aging factor $a_j = a, j = 1,2,..........,n$ i.e. the model with a job independent aging effect for given $i < k$, the scheduling problem above can be formulated as follows.

$$z = \sum_{j=1}^{n}(\alpha E_{[j]} + \beta T_{[j]} + \gamma d) = \alpha\left[\sum_{j=1}^{i}(d - c_{[j]}) + \sum_{j=i+1}^{k}(d - c_{[i]})\right] + \beta\sum_{j=k+1}^{k}(c_{[j]} - d)\Bigg]$$

$$+ n\gamma\left[\sum_{j=1}^{k} P_{[j]} j^{a} + \sum_{j=i+1}^{k} P_{[j]}(j - i)^{a} + \left(b + c\sum_{j=1}^{i} P_{[j]} J^{a}\right)\right]$$

$$= \alpha\left[\sum_{j=1}^{i}(j-1)P_{[j]}J^{a} + \sum_{j=i+1}^{k}(j-1)P_{[j]}(j-i)^{a} + i\left(b + c\sum_{j=1}^{i}P_{[j]}J^{a}\right)\right]$$

$$+\beta\sum_{j=k+1}^{k}(n-j+1)P_{[j]}(j-i)^{a} + n\gamma\left[\sum_{j=1}^{i}P_{[j]}j^{a} + \sum_{j=i+1}^{k}P_{[j]}(j-i)^{a} + \left(b + c\sum_{j=1}^{i}P_{[j]}J^{a}\right)\right]$$

$$= b(n\gamma + \alpha i) + \sum_{j=1}^{i}\left[n\gamma(1+c) + \alpha(j-1+ci)\right]P_{[j]}J^{a}$$

$$+\sum_{j=i+1}^{k}\left[n\gamma + \alpha(j-1)\right]P_{[j]}(j-i)^{a} + \sum_{j=k+1}^{n}\left[\beta(n-j+1)\right]P_{[j]}(j-i)^{a} \qquad (3.6.1)$$

If the maintenance activity is performed after the due-date (i.e. $i \geq k$) then the total cost is given by

$$z = \sum_{j=1}^{n}(\alpha E_{[j]} + \beta T_{[j]} + \gamma d)$$

$$= \alpha\sum_{j=1}^{k}(d - c_{[j]}) + \left[\beta\sum_{j=k+1}^{i}(c_{[i]} - d) + \sum_{j=i+1}^{k}(c_{[j]} - d)\right] + n\gamma\sum_{j=1}^{k}P_{[j]}j^{a}$$

$$= \alpha\sum_{j=1}^{k}(j-1)P_{[j]}j^{a} + \left[\beta\sum_{j=k+1}^{i}(n-j+1)P_{[j]}j^{a} + \sum_{j=i+1}^{k}(n-j+1)P_{[j]}(j-i)^{a}\right]$$

$$+\beta\sum_{j=i+1}^{k}(n-i)\left(b + c\sum_{j=1}^{i}P_{[j]}J^{a}\right) + n\gamma\sum_{j=1}^{k}P_{[j]}j^{a}$$

$$= \beta b(n-i) + \sum_{j=1}^{k}\left[n\gamma + \alpha(j-1) + \beta c(n-i)\right]P_{[j]}j^{a}$$

$$+\sum_{j=k+1}^{i}\left[\beta(n-j+1) + \beta c(n-i)\right]P_{[j]}j^{a}x_{jr} + \sum_{j=i+1}^{n}\beta(n-j+1)P_{[j]}(j-1)^{a}$$

(3.6.2)

Let $M = \begin{cases} b(ny + i\alpha) & i < k \\ \beta b(n-i) & i \ge k \end{cases}$ (3.6.3)

Then $z = \sum_{j=1}^{n} (\alpha E_{[j]} + \beta T_{[j]} + \gamma d)$

$$= \sum_{j=1}^{n} w_j P_{[j]} + M \quad (3.6.4)$$

where

$$w_j = \begin{cases} [n\gamma(1+c) + \alpha(i-1+ci)].j^a & j = 1,2,....,i \\ [n\gamma + \alpha(j-1)].(j-i)^a & j = i+1, i+2,....,k \\ [\beta(n-j+1)].(j-i)^a & j = k+1, k+2,....,n \end{cases} \quad (3.6.5)$$

For $i < k$ and

$$w_j = \begin{cases} [n\gamma + \alpha(j-1) + \beta c(n-i)].j^a & j = 1,2,....,i \\ [\beta(n-j+1) + \beta c(n-i)].j^a & j = k+1, k+2,....,k \\ [\beta(n-j+1)].(j-i)^a & j = i+1, i+2,....,n \end{cases} \quad (3.6.6)$$

for $i \ge k$ based on the above analysis and property 2 and 3, the following corollary holds.

Corollary 2. *The* $1/ma, P_{jr} = P_j r^a / \sum_{j=1}^{n} (\alpha E_{[j]} + \beta T_{[j]} + yd)$ *problem can be solved in* $0(n^2 \log n)$ *time.*

Proof. By property 2, we obtain the optimal solution of the common due date k. Once the position of the maintenance activity has been determined, the term M in equation (3.6.6) is a constant. Then, we can obtain the local optimal solution via the following steps:

- Step 1. For each position $j\,(j=1,2,\ldots,n)$, calculate the positional weights w_j mentioned above.
- Step 2. Renumber the jobs in a non-increasing order with respect to their normal processing time. By Property 3, assign the job with the largest normal processing time to the position with the smallest value of positional weight w_j, the job with the next largest normal processing time to the position with the next smallest value of positional weight $w_j 0(1)$, etc.
- Step 3. By equation (3.6.6), calculate the total cost Z. The time complexity of step 1 and step 3 is $0(1)$ and the time complexity of step 2 is $0(n\log n)$. Since the maintenance activity can be scheduled immediately after any job, n different positions must be evaluated to obtain the global optimal schedule.

3.7 COUNTER EXAMPLE

Assume $n=7$ job and the common aging factor is $a=0.3$. The processing time is $p_1=35$, $p_2=20$, $p_3=35$, $p_4=32$, $p_5=38$, $p_6=42$, and $p_7=25$. The earliness, tardiness and due-date unit cost are: $\alpha = 2, \beta = 14, \gamma = 4$. The basic time of maintenance activity is $b = 20$, the parameter of c is 0.05.

Solution. First we calculate the position of the due-date using property 2: $k = [n(\beta - \gamma)/(\alpha + \beta)]$=5

We explain the $c = 0.05$ and $a = 0.3$ present as follow.

Step 1. Calculate w_j, for $j = 1, 2, ..., 7$.

$w_1 = [n\gamma(1+c) + \alpha ci)].1^a$ =29.7

$w_2 = [n\gamma(1+c) + \alpha(1+ci)].2^a$ =32.93

$w_3 = [n\gamma(1+c) + \alpha(3+ci)].3^a$ =33.09

$w_4 = (n\gamma + i\alpha).1^a$ =34

$w_5 = [n\gamma(i+1)\alpha].2^a$ =35023

$w_6 = \beta(n-r+1).3^a$ =41.71

$w_7 = \beta(n-r+1).4^a$ =22.74

Step 2. The optimal job sequence is (3, 6, 2, 4, 7, 1, and 5).

Step 3. The total cost is Z= 11820.86

Table 1.

Optimal job sequence and total cost for all the positions of the maintenance activity under job-independent aging effect ($a = 0.3$).

Position of MA	Job Sequence	Total cost
Prior to job 1	(3,4,1,7,2,6,5)	12231.05
Prior to job 2	(3,6,4,7,2,1,5)	12201.48
Prior to job 3	**(3,6,2,4,7,1,5)**	**11820.86**
Prior to job 4	(4,6,7,2,1,3,5)	11956.25
Prior to job 5	(4,1,6,7,2,3,5)	11975.56
Prior to job 6	(3,4,6,7,2,1,5)	12248.48
Not scheduled	(3,1,6,7,2,1,5)	122485.52

3.8 CONCLUSION

In this chapter, we considered a maintenance activity single – machine scheduling problem and due-date assignment simultaneous. The objective was to find jointly the optimal common due-date, the optimal location of the maintenance activity, and the optimal job sequence for minimizing the total of earliness, tardiness and due-date costs. We showed that the problem can be optimally solved in polynomial time solution. We also discussed two special cases of the proposed problem and showed that they can be optimally solved by a lower order algorithm.

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