CHAPTER 10

BEHAVIOUR OF HYDROMAGNETIC PARALLEL SQUEEZE FILMS BETWEEN TWO CONDUCTING POROUS SURFACES

10.1 Introduction 211
10.2 Basic equations 213
   10.2.1 Circular plates 222
   10.2.2 Annular discs 224
   10.2.3 Elliptical plates 225
   10.2.4 Triangular plates 228
   10.2.5 Infinitely long rectangular plate 229
   10.2.6 Complete cone 231
10.3 Results and discussion 233
In this chapter, the problem of squeeze films between electrically conducting porous surfaces with electrically conducting lubricant in the presence of transverse magnetic field is investigated for various geometrical shapes of the bearing surfaces.

10.1 INTRODUCTION

A considerable attention has been paid to the potentiality of liquid metals as lubricants utilized under the high temperature at which conventional lubricants would undergo some undesirable physical changes. Although the liquid metals such as Mercury, Sodium and Sodium-Potassium alloy, etc. have a defect as lubricants, at high temperature these may be preferred as lubricant because of their highly thermal conductivity and low viscosity [Elco and Hughes (1962)]. These can still be considered to be suitable as lubricants for a bearing operating at high temperature because of their stability at high temperature. Moreover, since the liquid metals are good electrical conductors it is possible to increase the load capacity by utilizing the electromagnetic force, thus overcoming the above defect sufficiently and there by alleviating the drawback of low viscosity. When a conducting fluid flows across a magnetic field, the electromagnetic pressurisation may be substantial.
This effect is utilised to improve the lubricating properties of electrically conducting lubricants.

Wu (1970; 1972) and Prakash and Vij (1973) analysed and discussed the behaviour of the squeeze film when one surface was porous and backed by an impermeable solid wall. Patel and Gupta (1979) analysed the effect of a transverse magnetic field on the behaviour of a squeeze film between porous plates of different geometries. Shukla (1965) and Kuzma (1965) investigated independently the hydromagnetic theory of squeeze films for conducting lubricants between two non-conducting non-porous surfaces in the presence of a transverse magnetic field. Shukla and Prasad (1965) discussed hydromagnetic squeeze films between two conducting non-porous surfaces and studied the effect of the conductivities of surfaces on the load capacity and time of approach. They considered two geometrical shapes, infinitely long rectangular and parabolically curved circular. It was observed that when the bearing surfaces are conducting, load carrying capacity decreased in comparison to the corresponding hydromagnetic case when the bearing surfaces are non-conducting. However, the increase in load capacity, pressure, and time of approach are possible by increasing the conductivities of the surfaces.
10.2 BASIC EQUATIONS

We consider the problem of squeeze film between two parallel plates. The lower plate with a porous facing is assumed to be fixed while the upper plate moves along its normal towards the lower plate (Fig. 10.1). The plates are considered electrically conducting and the clearance space between them is filled by a electrically conducting lubricant. A uniform transverse magnetic field $B_0$ is applied between the plates.

The flow in the porous medium obeys the modified form of Darcy's law [Ene (1969)] while in the film region the equations of hydromagnetic lubrication theory hold. Following the usual assumptions of hydromagnetic lubrication, the basic equations governing the hydromagnetic flow of the lubricants are [Shukal and Prasad; (1965)]:

\[
\frac{\partial u}{\partial y^2} - \frac{M^2}{h^2} u = \left[ \frac{M}{h} \sqrt{\frac{\sigma}{\mu}} E_z + \frac{1}{\mu} \frac{\partial p}{\partial x} \right] \quad (10.1)
\]

\[
0 = \frac{\partial p}{\partial y} \quad (10.2)
\]
Figure 10.1 Configuration of bearing
\[
\frac{\partial^2 v}{\partial y^2} - \frac{M}{h^2} v = \left[ \frac{M}{h} \sqrt{\frac{\sigma}{\mu}} \cdot E_x + \frac{1}{\mu} \frac{\partial p}{\partial z} \right]
\] (10.3)

where \(M = \text{Boh} (\frac{\sigma}{\mu})^{1/2}\) (10.4)

is the Hartmann-number, \(\sigma\) is the electrical conductivity of the lubricant and \(h\) is the lubricant film thickness.

The continuity equation is,

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = 0
\] (10.5)

Solving equations (10.1) and (10.3) with boundary conditions,

\(u = 0, v = 0\) at \(y = h\) (10.6)

gives:

\[
u = \frac{h^2}{M^2} \left[ \frac{M}{h} \sqrt{\frac{\sigma}{\mu}} \cdot E_x + \frac{1}{\mu} \frac{\partial p}{\partial z} \right] \left[ \frac{\cosh \frac{M}{Z} \left( \frac{2v}{h} - 1 \right)}{\cosh \frac{M}{Z}} - 1 \right]
\] (10.7)

and

\[
v = \frac{h^2}{M^2} \left[ -\frac{M}{h} \sqrt{\frac{\sigma}{\mu}} \cdot E_x + \frac{1}{\mu} \frac{\partial p}{\partial z} \right] \left[ \frac{\cosh \frac{M}{Z} \left( \frac{2v}{h} - 1 \right)}{\cosh \frac{M}{Z}} - 1 \right]
\] (10.8)
The electric field components $E_x$, $E_z$ and induced magnetic field components $H_x$, $H_z$ are obtained from,

$$\frac{\partial H_x}{\partial y} = -\sigma [E_z + B_0 u]$$

(10.9)

$$\frac{\partial H_z}{\partial y} = \sigma [E_x - B_0 v]$$

(10.10)

The boundary conditions for induced magnetic field are those obtained by Snyder (1962), i.e.

$$\frac{\partial H_x}{\partial y} - \frac{1}{\phi_0 h}, H_x = 0 \text{ and } \frac{\partial H_z}{\partial y} - \frac{1}{\phi_0 h} H_z = 0 \text{ at } y = 0$$

(10.11)

$$\frac{\partial H_x}{\partial y} + \frac{1}{\phi_1 h}, H_x = 0 \text{ and } \frac{\partial H_z}{\partial y} + \frac{1}{\phi_1 h} H_z = 0 \text{ at } y = h$$

(10.12)

$\sigma_0$ and $\sigma_1$ are the electrical conductivities of lower and upper bearing surfaces respectively, and $h'_0$ and $h'_1$ are the surface widths of the lower and upper plates. We have

$$\phi_0(h) = \frac{\sigma_0 h'_0}{\sigma h}$$

(10.13)

$$\phi_1(h) = \frac{\sigma_1 h'_1}{\sigma h}$$

(10.14)
Solving equations (10.9) and (10.10) with (10.7) and (10.8) using the boundary conditions (10.11) and (10.12) one gets,

\[ H_x = \frac{\hbar}{M^2} \sqrt{\sigma u} \left[ \frac{M}{h} \sqrt{\sigma u} E_z + \frac{1}{u} \frac{\partial p}{\partial x} \right]. \]

\[ \left[ \frac{My}{h} - \tanh \frac{M}{2} - \frac{\sinh \frac{M}{2} \left( \frac{2y}{h} - 1 \right)}{\cosh \frac{M}{2}} \right] - \sigma E_z (y + h\phi_o) \quad (10.15) \]

\[ H_z = \frac{\hbar}{M^2} \sqrt{\sigma u} \left[ \frac{M}{h} \sqrt{\sigma u} E_x + \frac{1}{u} \frac{\partial p}{\partial z} \right]. \]

\[ \left[ \frac{My}{h} - \tanh \frac{M}{2} - \frac{\sinh \frac{M}{2} \left( \frac{2y}{h} - 1 \right)}{\cosh \frac{M}{2}} \right] - \sigma E_x (y + h\phi_o) \quad (10.16) \]
\[ E_x = \frac{h}{M} \frac{1}{\sqrt{\sigma u}} \frac{\partial p}{\partial z} \left[ \frac{1 - \tanh \left( \frac{M}{2} \right)}{\frac{M}{2}} \right] \left( \phi_0 + \phi_1 + \frac{\tanh(M/2)}{(M/2)} \right) \] (10.17)

\[ E_z = \frac{h}{M} \frac{1}{\sqrt{\sigma u}} \frac{\partial p}{\partial x} \left[ \frac{1 - \tanh \left( \frac{M}{2} \right)}{\frac{M}{2}} \right] \left( \phi_0 + \phi_1 + \frac{\tanh(M/2)}{(M/2)} \right) \] (10.18)

Substitution from equation (10.17) and (10.18) in equation (10.7) and (10.8), yields,

\[
u = \frac{h}{\mu M^2} \frac{\partial p}{\partial x} \left[ \frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \tanh(M/2)} \right] \left[ \frac{\cosh \left( \frac{2y}{h} \right) - 1}{\cosh \left( \frac{M}{2} \right)} \right] - 1
\] (10.19)

\[
v = \frac{h}{\mu M^2} \frac{\partial p}{\partial z} \left[ \frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \tanh(M/2)} \right] \left[ \frac{\cosh \left( \frac{2y}{h} \right) - 1}{\cosh \left( \frac{M}{2} \right)} \right] - 1
\] (10.20)

The velocity of the lubricant in the porous region satisfies the modified Darcy's law, equation of continuity and generalised Ohm's Law [Ene (1969)].

In the present case, we have for the porous region:
\[ \bar{u} = \left[ -\frac{K}{A_1} \frac{\partial P}{\partial x} - K \sqrt{\frac{\sigma}{A_1 \frac{M}{h} E_z}} \right] \frac{1}{C^2} \] (10.21)

\[ \bar{w} = -\frac{K}{A_1} \frac{\partial P}{\partial y} \] (10.22)

\[ \bar{v} = \left[ -\frac{K}{A_1} \frac{\partial P}{\partial z} - K \sqrt{\frac{\sigma}{A_1 \frac{M}{h} E_x}} \right] \frac{1}{C^2} \] (10.23)

where \( C^2 = 1 + \frac{K}{m} \frac{M}{h^2} \), (10.24)

\[ K \) is the permeability and \( m \) is the porosity of the porous matrix

\[ \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial z} + \frac{\partial \bar{w}}{\partial y} = 0 \] (10.25)

Using equations (10.21 - 10.23) in equation (10.25) and simplifying one gets,

\[ \frac{\partial^2 P}{\partial y^2} = -\frac{1}{C^2} \left[ \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} + \sqrt{\frac{\sigma}{A_1 \frac{M}{h}} \left( \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \right)} \right] \] (10.26)

Using Taylor's series expansion for \( \frac{\partial P}{\partial y} \) at \( y = -H \) and neglecting quadratic power's of \( H \),
The boundary conditions are

\[ \frac{\partial P}{\partial y} = P \text{ at } y = 0 \]

(10.28)

and

\[ \frac{\partial P}{\partial y} = 0 \text{ at } y = -H \]

gives,

\[ \left( \frac{\partial P}{\partial y} \right)_{y=0} = H \left( \frac{\partial^2 P}{\partial y^2} \right)_{y=0}, \text{ using equation (10.26)} \]

(10.29)

Substituting \( E_x \) and \( E_z \) from equations (10.17) and (10.18)

\[ \left( \frac{\partial P}{\partial y} \right)_{y=0} = -\frac{H}{C^2} \left[ \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} + \sqrt{\sigma \mu} \frac{M}{H} \left( \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \right) \right] (10.30) \]

Now integrating the equation (10.5) across the film thickness \( h \) and using the condition that,
Substitution for \( u \) and \( v \) from equations (10.19) and (10.20) in equation (10.31) and using (10.30) the modified Reynold's equation takes the form:

\[
\frac{\partial^2 \Phi}{\partial \phi^2} = \frac{\partial h}{\partial t} \left[ \frac{2h^3}{\Phi \lambda M^2} \left( \tanh \frac{M}{2} - \frac{M}{2} \right) - \frac{\Psi h^3}{\lambda C^2} \right] \left[ \frac{\Phi_0 + \Phi_1 + 1}{\Phi_0 + \Phi_1 + \frac{\tanh (M/2)}{M/2}} \right]
\]

(10.32)

where \( \Psi = \frac{K_H}{h^3} \)

(10.33)

Hence the problem reduces to the solution of equation (10.32) with the appropriate boundary conditions.


10.2-1 Circular Plates

For circular plates the differential equation is

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) = \frac{\frac{2h^3}{\lambda M^3} \left( \tanh \left( \frac{M}{2} \right) - \frac{M}{2} \right) - \frac{\Psi h^3}{\mu C^2} }{\phi_0 + \phi_1 + \frac{\tanh (\phi_1/2)}{M/2}} \]

(10.34)

and the boundary conditions are

\[
p(a) = 0 \text{ and } \frac{\partial p}{\partial r} = 0 \text{ at } r = 0
\]

(10.35)

Solving the equation (10.34) using the boundary conditions (10.35) gives the pressure distribution in dimensionless form:

\[
\bar{p} = -\frac{bh^2}{\mu h^2 \pi a^2}
\]

\[
(1 - \frac{r^2}{a^2})
\]

\[
= \frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \frac{\tanh (\phi_1/2)}{M/2}}
\]

(10.36)
and the load carrying capacity is

\[ W = 2\pi \int_0^a \rho \, dr \]

It is in dimensionless form:

\[ \bar{W} = -\frac{W \, h^3}{\lambda h^2 \pi^2 a^4} \]

\[ = \frac{1}{8\pi \left[ \frac{2 \lambda}{M^3} \left\{ \tanh \left( \frac{M}{2} \right) - \frac{M}{2} \right\} \right] - \frac{\Psi}{C^2} \left[ \frac{\phi_0 + \phi_1 + 1}{\phi_0 \phi_1 + \tanh \left( \frac{M}{2} \right)} \right]} \]

(10.37)

The time height relation is

\[ \Delta \tilde{t} = \int_0^{h_0} \frac{h_0^2 dt}{\lambda \pi^2 a^4} \]

\[ \Delta \tilde{t} = \frac{1}{8} I \]

(10.38)

where \( a \) is the radius of the plate and \( I \) is given by,

\[ I = -h_0^2 \int \frac{dh}{h_0} \left[ \frac{2h^3}{M^3} \left\{ \tanh \left( \frac{M}{2} \right) - \frac{M}{2} \right\} \right] - \frac{K}{C^2}\left[ \frac{\phi_0 + \phi_1 + 1}{\phi_0 \phi_1 + \tanh \left( \frac{M}{2} \right)} \right] \]

(10.39)
10.2.2 Annular discs

Solving equation (10.34) with the boundary conditions for $b \leq r \leq a$:

$$\bar{p}(a) = \bar{p}(b) = 0 \quad (10.40)$$

The expression for the pressure distribution in dimensionless form is:

$$\bar{p} = -\frac{3h}{a b} \frac{1}{\sinh \left(\frac{a^2 - b^2}{2}\right)} \pi$$

$$= \frac{1}{\left[\frac{2}{M^2} \left\{ \tanh \left(\frac{M}{2}\right) - \left(\frac{M}{2}\right)^2 \right\} \right\} \left[ \frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \frac{\tanh (M/2)}{M/2}} \right]}$$

$$= \frac{1}{4 \pi} \left[ \frac{\ln \left(\frac{r}{b}\right)}{\ln (a/b)} - \left(\frac{r}{b}\right)^2 - 1 \right] \quad (10.41)$$

The load carrying capacity is:

$$W = 2\pi \int_b^a \bar{p} dr$$

in dimensionless form,

$$\bar{W} = -\frac{3h}{a b} \frac{W}{\sinh \left(\frac{a^2 - b^2}{2}\right) \pi}$$
For constant load \( \nu \) the film thickness and the time relation can be obtained by integrating equation (10.42) which yields.

\[
\Delta t = \int_{0}^{\frac{t}{t_{0}}} \frac{\nu h_0}{\nu} dt
\]

\[
\Delta t = \frac{1}{8\pi} \left[ \frac{(a/b)^2 + 1}{(a/b)^2 - 1} - \frac{1}{\ln(a/b)} \right] I
\]

where \( I \) is given by equation (10.39).

**10.2.3 Elliptical plates**

The differential equation is

\[
\frac{\partial}{\partial x^2} b + \frac{\partial}{\partial z^2} b = \]
and the boundary conditions are

\[ p(x_1, z_1) = 0 \]

where

\[ \frac{x^2}{a^2} + \frac{z^2}{b^2} = 1 \]  

(10.45)

The expression for the pressure distribution in dimensionless form is

\[
\bar{p} = -\frac{bh^3}{\pi ab} \frac{1}{2\pi} \left\{ \frac{a/b}{(a/b)^2 + 1} \right\} \left[ 1 - \frac{x^2}{a^2} - \frac{z^2}{b^2} \right].
\]

(10.46)
The load carrying capacity is

\[ w = \int_{x=-a}^{a} \int_{z=-\frac{b}{\sqrt{a^2-x^2}}}^{\frac{b}{\sqrt{a^2-x^2}}} \rho \, dx \, dz \]

and in dimensionless form is,

\[ M = -\frac{Wh^3}{\mu h \pi a^2 b^2} = \frac{1}{4\pi} \left[ \frac{(a/b)}{(a/b)^2 + 1} \right]. \]

\[ \frac{1}{\left[ \frac{2}{M^3} \{\tanh \left(\frac{M}{2}\right) - (\frac{M}{2})\} - \frac{\Phi}{C^2} \right] \left[ \frac{\Phi_\alpha + \Phi_1 + 1}{\Phi_\alpha + \Phi_1 + \tanh(M/2)} \right]} \]

(10.47)

The response time

\[ \Delta t = \int_0^{t_{vb}} \frac{Wh^2}{\mu h^2 a^2 b^2} \, dt \]

\[ \Delta t = \frac{1}{4\pi} \left[ \frac{(a/b)}{(a/b)^2 + 1} \right] I \]

(10.48)

where 2a and 2b are the major and minor axes of the elliptical plate and I is given by equation (10.39).
10.2.4 Triangular plates

The governing differential equation (10.44) with the boundary condition \( p(x, z) = 0 \)

\[
(x_1 - a)(x_1 - \sqrt{3} z_1 + 2a)(x_1 + \sqrt{3} z_1 + 2a) = 0
\] (10.49)

and \( a \) is the length of each side of the equilateral triangle whose equation is

\[
(x-a) (x - \sqrt{3}z + 2a) (x + \sqrt{3}z + 2a) = 0 \quad (10.50)
\]

The point of intersection of the medians of the triangle is selected as the origin.

The dimensionless pressure

\[
\bar{p} = - \frac{bh^3}{\mu h^3 \sqrt{3} a^2}
\]

\[
= \frac{1}{9 \sqrt{3}} \left[ 1 - \frac{x}{a} \right] \left[ 1 + \frac{\sqrt{3} z}{2a} + \frac{x}{2a} \right] \left[ 1 - \frac{\sqrt{3} z}{2a} + \frac{x}{2a} \right]
\]

\[
\frac{1}{\left[ \frac{2}{M^3} \{\tanh \left( \frac{M}{2} \right) - \frac{M}{2} \} - \frac{\Psi}{C^2} \right] \left[ \frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \frac{\tanh(M/2)}{M/2}} \right]}
\]

\[
(10.51)
\]

The dimensionless load-carrying capacity obtained from
\[
W = \int_{-2a}^{a} \int_{b}^{d} \frac{x + 2a}{v^3} \, dx \, dz
\]

\[
\bar{W} = -\frac{w_h^3}{27u'h'a^4} = \left[ \frac{4}{81 \sqrt{3}} \right].
\]

\[
\frac{1}{\left[ \frac{2}{M^2} \{ \tanh \left( \frac{M}{2} \right) - \left( \frac{M}{2} \right) \} - \frac{\Psi}{C^2} \right] \left[ \frac{\phi_0 + \phi_1 + 1}{\phi_0 - \phi_1 + \tanh(M/2) M/2} \right]}
\]

Response time
\[
\Delta \bar{t} = \int \frac{h/\omega}{w^2} \, dt
\]

\[
\Delta \bar{t} = \frac{4}{81 \sqrt{3}} \, I
\]

where I is given by equation (10.39)

10.2.5 Infinitely long rectangular plate

The differential equation is
\[
\frac{\partial^2 b}{\partial z^2} = \frac{h}{\left[ \frac{2h}{M^3} \{ \tanh(M/2) - (M/2) \} - \frac{\Psi h^3}{4\mu C^2} \right] \left[ \frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \tanh(M/2) M/2} \right]}
\]

(10.54)
When equation (10.54) is solved with the boundary conditions
\[ \bar{p} (+ \frac{b}{2}) = 0 \]

The expression for the pressure distribution in dimensionless form is

\[ \bar{p} = - \frac{h^3 \bar{p}}{\mu h a b} = \frac{1}{2(a/b)} \left[ \frac{1}{4} - \frac{2}{b^2} \right]. \]

\[
\begin{align*}
&\frac{1}{\left[ \frac{2}{M^3} \{\tanh \left( \frac{M}{2} \right) - \left( \frac{M}{2} \right) \} - \frac{\phi}{C^2} \right] \left[ \frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \tanh(M/2)} \right]} \\
&= (10.55)
\end{align*}
\]

The load carrying capacity is

\[ W = \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \bar{p} \, dx \, dz \]

and in dimensionless form

\[ \bar{W} = - \frac{W h^3}{\mu h a b^2} = \left[ \frac{1}{12(a/b)} \right]. \]
For constant load $W$ the film thickness and the time relation can be obtained by integrating equation (10.56) which yields

$$\Delta t = \int \frac{t}{t_0} \frac{W h_0 dt}{u a^2 b^2}$$

$$= \frac{1}{12(a/b)} I$$

(10.57)

where $I$ is given by equation (10.39)

10.2.6 Complete cone

The differential equation is

$$\frac{1}{x} \frac{d}{dx} \left( x \frac{dh}{dx} \right) = \frac{h^3}{\alpha M^3 \left\{ \tanh \left( \frac{M}{2} \right) - \left( \frac{M}{2} \right) \right\} - \Phi h^3 \left( \frac{M}{2} \right) } \left[ \frac{\Phi_0 + \Phi_1 + 1}{\phi_0 + \phi_1 + \frac{\tanh(M/2)}{M/2}} \right]$$

(10.58)
When equation (10.58) is solved with the boundary conditions

$$\begin{align*}
\phi &= 0 \quad \text{at } x = a \csc \omega \\
\frac{\partial \phi}{\partial x} &= 0 \quad \text{at } x = 0,
\end{align*}$$

The expression for the pressure distribution in dimensionless form is

$$\bar{p} = -\frac{bh^3}{\mu \pi a^2 \csc \omega} = \frac{\csc \omega}{4\pi} [1 - \frac{x^2 \sin^2 \omega}{a^2}].$$

$$\frac{1}{\left[ \frac{2}{M^3} \left\{ \tanh \left( \frac{M}{2} \right) - \left( \frac{M}{2} \right) \right\} - \frac{\Psi}{C^2} \right] \left[ \frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \frac{\tanh(M/2)}{M/2}} \right]}$$

(10.59)

The dimensionless load-carrying capacity is obtained from

$$\bar{W} = 2\pi \int_{0}^{a} \bar{p} x \, dx$$

$$\therefore \quad \bar{W} = -\frac{bh^3}{\mu \pi a^2 \csc \omega} = \frac{\csc \omega}{8\pi}.$$
Response time

$$\Delta \bar{t} = \int_1^2 \frac{\Delta^2 \omega \csc \omega}{\mu \pi a^4 \sec \omega} \, dt$$

$$= \frac{\csc \omega}{8\pi} \bar{I} \bar{I}$$

where \( \bar{I} \) is given by equation (10.39)

10.3 RESULTS AND DISCUSSION

For all geometries considered the general formula for dimensionless load carrying capacity is

$$\bar{W} = -\frac{W_h}{\mu h A^2} = SJ$$

where \( S \) is the shape factor, \( A \) is the characteristic area, and \( J \) is the parameter characterizing the hydromagnetic effects and is defined as

$$J = \frac{1}{\left[ \frac{2}{M^3} \{ \tanh \left( \frac{M}{2} \right) - \left( \frac{M}{2} \right) \} - \frac{\Psi}{C^2} \right] \left( \frac{\Phi_0 + \Phi_1 + 1}{\Phi_0 + \Phi_1 + \tanh(M/2)} \right) \frac{1}{M/2}}$$

The shape factors for different geometries are given in Table 10.1.
The time-height relation for all the geometries considered has the general formula.

\[ \Delta \frac{t}{\bar{t}} = \frac{2W\epsilon_o}{\Delta A} \Delta t = IS \]

where \( I \) is given by equation (10.39). Thus for non-porous conducting plates the results for squeeze film analysed by Shukla and Prasad (1965) are recovered when \( \Psi = 0 \). For non-magnetic porous squeeze films the results of Prakash and Vij (1973) are obtained in the limiting case when we take \( m \to 0 \). The results of Patel and Gupta (1979) are obtained when \( \phi_0 \) and \( \phi_1 \) are taken as zero.

The dimensionless load carrying capacity for conducting surfaces of various geometrical shapes are tabulated in Table 10.2 for the fixed values of \( M = 10 \), \( m = 0.6 \), \( \kappa h^2 = 2.5 \times 10^{-2} \), \( a/b = 2 \), \( \bar{w} = \pi/4 \) and different values of \( (\phi_0 + \phi_1) \) and \( \Psi \). It is easily observed that for all geometrical shapes as \( (\phi_0 + \phi_1) \) increases, \( \bar{W} \) increases for fixed values of \( \Psi \). Also \( \bar{W} \) decreases with increase of \( \Psi \) for fixed values of \( (\phi_0 + \phi_1) \). The effect of conductivity on the pressure distribution \( \bar{p} \), load carrying capacity \( \bar{W} \) and the response time \( \Delta \bar{t} \) comes through the factor

\[ \frac{\phi_0 + \phi_1 + \frac{\tanh (M/2)}{M/2}}{\phi_0 + \phi_1 + 1} \]
which for large $M$ becomes \[ \frac{\phi_o + \phi_1}{\phi_o + \phi_1 + 1} \]

because $\tanh M \sim 1$, $2/M \sim 0$. Since both of these functions are increasing functions of $\phi_o + \phi_1$, it may be observed from the mathematical analysis also that as $\phi_o + \phi_1$ increases, the pressure, load carrying capacity and the response time increases. Increase of $\phi_o + \phi_1$, suggests the increase of, the plate conductivities $\sigma_o$, $\sigma_1$, and plate thickness $h_o'$ and $h_1'$ as seen from equation (10.13) and (10.14). Hence as the plate conductivities and plate thicknesses increases the lubricant pressure, load carrying capacity and response time increase. Also it is seen for both small as well as large values of $M$ that the bearing performance suffers when the plates are taken to be electrically conducting, in comparison to the hydromagnetic case when the plates are considered to be nonconducting. This can physically be explained by fringing phenomena which occurs when the plates are electrically conducting.
# TABLE 10.1

The shape factors for different geometries

<table>
<thead>
<tr>
<th>Bearing Geometry</th>
<th>Characteristic Area $A$</th>
<th>Shape Factor $S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular</td>
<td>$\pi a^2$</td>
<td>$\frac{1}{8\pi}$</td>
</tr>
<tr>
<td>Annular</td>
<td>$\pi(a^2-b^2)$</td>
<td>$\frac{1}{8\pi} \left[ \frac{(a/b)^2 + 1}{(a/b)^2 - 1} - \frac{1}{\ln(a/b)} \right]$</td>
</tr>
<tr>
<td>Elliptical</td>
<td>$\pi ab$</td>
<td>$\frac{1}{4\pi} \frac{(a/b)}{(a/b)^2 + 1}$</td>
</tr>
<tr>
<td>Triangular</td>
<td>$3\sqrt{3} a^2$</td>
<td>$4/81. \sqrt{3}$</td>
</tr>
<tr>
<td>Infinitely Long</td>
<td>ab</td>
<td>$\frac{1}{12(a/b)}$</td>
</tr>
<tr>
<td>rectangular</td>
<td>Complete cone</td>
<td>$\pi a^2 \csc \omega$</td>
</tr>
</tbody>
</table>
Table 10.2  Values of \( \bar{W} \) for \( M = 10, m = 0.6, \quad \frac{K}{h} = 2.5 \times 10^{-2}, \quad \frac{a}{b} = 2, \quad \bar{W} = \frac{\pi}{4} \)

1. Circular Plate

<table>
<thead>
<tr>
<th>( \psi )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi + \phi )</td>
<td>0.00</td>
<td>0.995</td>
<td>2.985</td>
<td>3.648</td>
<td>3.980</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>0.0528</td>
<td>1.586</td>
<td>1.938</td>
<td>2.114</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.184</td>
<td>0.552</td>
<td>0.674</td>
<td>0.735</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>0.101</td>
<td>0.304</td>
<td>0.321</td>
<td>0.405</td>
</tr>
<tr>
<td></td>
<td>0.50</td>
<td>0.022</td>
<td>0.066</td>
<td>0.081</td>
<td>0.088</td>
</tr>
</tbody>
</table>

2. Annular Plate

<table>
<thead>
<tr>
<th>( \psi )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi + \phi )</td>
<td>0.00</td>
<td>0.223</td>
<td>0.669</td>
<td>0.817</td>
<td>0.892</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>0.118</td>
<td>0.355</td>
<td>0.434</td>
<td>0.474</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.041</td>
<td>0.124</td>
<td>0.151</td>
<td>0.165</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>0.023</td>
<td>0.068</td>
<td>0.083</td>
<td>0.090</td>
</tr>
<tr>
<td></td>
<td>0.50</td>
<td>0.004</td>
<td>0.015</td>
<td>0.018</td>
<td>0.020</td>
</tr>
</tbody>
</table>
Table 10.2 (contd.)

3. Elliptical Plate

<table>
<thead>
<tr>
<th>Ψ</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.796</td>
<td>2.388</td>
<td>2.919</td>
<td>3.185</td>
<td>3.344</td>
<td>3.450</td>
</tr>
<tr>
<td>0.01</td>
<td>0.423</td>
<td>1.269</td>
<td>1.551</td>
<td>1.692</td>
<td>1.776</td>
<td>1.833</td>
</tr>
<tr>
<td>0.05</td>
<td>0.147</td>
<td>0.441</td>
<td>0.539</td>
<td>0.588</td>
<td>0.618</td>
<td>0.637</td>
</tr>
<tr>
<td>0.10</td>
<td>0.081</td>
<td>0.243</td>
<td>0.297</td>
<td>0.342</td>
<td>0.340</td>
<td>0.351</td>
</tr>
<tr>
<td>0.50</td>
<td>0.018</td>
<td>0.053</td>
<td>0.065</td>
<td>0.071</td>
<td>0.0741</td>
<td>0.076</td>
</tr>
</tbody>
</table>

4 Triangular Plate

<table>
<thead>
<tr>
<th>Ψ</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.713</td>
<td>2.138</td>
<td>2.613</td>
<td>2.851</td>
<td>3.344</td>
<td>3.450</td>
</tr>
<tr>
<td>0.01</td>
<td>0.379</td>
<td>1.136</td>
<td>1.388</td>
<td>1.515</td>
<td>1.776</td>
<td>1.833</td>
</tr>
<tr>
<td>0.05</td>
<td>0.132</td>
<td>0.395</td>
<td>0.483</td>
<td>0.527</td>
<td>0.618</td>
<td>0.638</td>
</tr>
<tr>
<td>0.10</td>
<td>0.073</td>
<td>0.218</td>
<td>0.266</td>
<td>0.290</td>
<td>0.340</td>
<td>0.351</td>
</tr>
<tr>
<td>0.50</td>
<td>0.016</td>
<td>0.047</td>
<td>0.058</td>
<td>0.063</td>
<td>0.074</td>
<td>0.076</td>
</tr>
</tbody>
</table>
Table 10.2  (contd.)

5. Infinite Long Rectangular Plate

<table>
<thead>
<tr>
<th>Ψ</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.042</td>
<td>3.125</td>
<td>3.819</td>
<td>4.166</td>
<td>4.375</td>
<td>4.514</td>
</tr>
<tr>
<td>0.01</td>
<td>0.553</td>
<td>1.660</td>
<td>2.029</td>
<td>2.214</td>
<td>2.324</td>
<td>2.398</td>
</tr>
<tr>
<td>0.05</td>
<td>0.192</td>
<td>0.577</td>
<td>0.706</td>
<td>0.770</td>
<td>0.808</td>
<td>0.834</td>
</tr>
<tr>
<td>0.10</td>
<td>0.106</td>
<td>0.042</td>
<td>0.389</td>
<td>0.424</td>
<td>0.445</td>
<td>0.459</td>
</tr>
<tr>
<td>0.50</td>
<td>0.023</td>
<td>0.529</td>
<td>0.085</td>
<td>0.092</td>
<td>0.097</td>
<td>0.100</td>
</tr>
</tbody>
</table>

6 Complete Cone

<table>
<thead>
<tr>
<th>Ψ</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.407</td>
<td>4.222</td>
<td>5.160</td>
<td>5.630</td>
<td>5.911</td>
<td>6.099</td>
</tr>
<tr>
<td>0.01</td>
<td>0.748</td>
<td>2.243</td>
<td>2.742</td>
<td>2.990</td>
<td>3.140</td>
<td>3.240</td>
</tr>
<tr>
<td>0.05</td>
<td>0.260</td>
<td>0.780</td>
<td>0.954</td>
<td>0.040</td>
<td>1.092</td>
<td>1.127</td>
</tr>
<tr>
<td>0.10</td>
<td>0.143</td>
<td>0.249</td>
<td>0.525</td>
<td>0.573</td>
<td>0.602</td>
<td>0.621</td>
</tr>
<tr>
<td>0.50</td>
<td>0.031</td>
<td>0.094</td>
<td>0.114</td>
<td>0.125</td>
<td>0.131</td>
<td>0.135</td>
</tr>
</tbody>
</table>