CHAPTER 9
NON-LINEAR ANALYSIS OF SMART COMPOSITE PLATES

9.1 GENERAL

Behaviour of laminated composite plates with piezoelectric elements in the geometrically non-linear domain has not attracted much attention of researchers. Though the behaviour is similar to that of ordinary laminated composite plates, the effect of voltage applied to the actuators can be different in the non-linear analysis from that in the linear analysis. Results on such numerical studies are not available in literature. To fill this gap, the behaviour of smart plates under static and dynamic loading is studied by performing an analysis including Von-Karman non-linear effects.

9.2 STATIC ANALYSIS

Static behaviour of laminated composite plates with integrated piezoelectric elements in the non-linear domain is studied in the same manner as that of laminated composite plates, incorporating the effect of voltage applied to the piezoelectric elements. The voltage applied to the piezoelectric elements, serving as actuators, reduces the deflection and stresses. But, the load that is acting on the structure being large, low values of voltages will not have any influence on the response, especially in the case of thick plates. Hence, higher voltages are applied assuming that the constitutive relation for the piezoelectric element layer is linear in that domain also. Since linear load incrementing method is used for the analysis, the total voltage is also considered to be applied in an incremental fashion, the number of increments being the same as the number of load steps. The increment is kept the same for all load steps.
9.2.1 Numerical Results

Results of numerical studies carried out using the 4-noded element based on higher-order shear deformation theory, including the Von-Karman non-linear effects are given below:

Example 9.1

4-layer symmetric cross-ply laminate with surface-bonded piezoelectric elements (PZT/0/90/90/0/PZT), simply supported on all edges and subjected to a non-dimensional uniform load of $Q = 350$, is considered as first example. The geometry and properties of the materials of the plate are same as those given in Example 5.2. The load deflection behaviour for different voltages applied to the actuators is presented in Figs. 9.1 and 9.2 for $b/h = 10$ and 100 respectively. All the three curves coincide in the case of plates with $b/h = 10$, indicating the necessity of a very high voltage to be applied for any reduction in deflection. But, in the case of plates with $b/h = 100$, there is a reduction of 1.5% at a voltage of 200 volts and a reduction of 4.9% at a voltage of 500 volts, for a non-dimensional load of $Q = 350$.

Since the effect of low values of voltage on the load-deflection behaviour of thick plates is found to be negligible, further studies are carried out only on thin ($b/h = 100$) plates.

Example 9.2

To study the effect of voltage in the non-linear analysis, 4-layer thin ($b/h = 100$) square cross-ply and angle-ply laminates with surface-bonded piezoelectric elements, with all the edges simply supported, are analysed. Both symmetric and anti-symmetric arrangements are considered. The properties used are same as in Example 9.1. The plate is subjected to a uniform non-dimensional load, $Q = 100$. The load-deflection behaviour with $V = 0$ and $V = 500$ volts for symmetric cross-ply laminates is presented in Fig. 9.3 along with linear
solutions. The variation of percentage reduction in deflection with increase in load under an applied voltage of 500 volts for the various plates is presented in Figs. 9.4 and 9.5. It is seen that the reduction in deflection is constant in linear analysis, whereas the reduction in deflection decreases with increase in load in non-linear analysis. The rate of decrease is more in anti-symmetric arrangements, both for cross-ply and angle-ply laminates.

Figure 9.1 Load-deflection behaviour of smart cross-ply laminate (b/h = 10)
Figure 9.2 Load-deflection behaviour of smart cross-ply laminate (b/h = 100)

Figure 9.3 Effect of voltage on load-deflection behaviour of symmetric cross-ply laminate
Figure 9.4 Variation of percentage reduction in deflection (cross-ply)

Figure 9.5 Variation of percentage reduction in deflection (angle-ply)
Example 9.3

To study the effect of fibre orientation angle on the non-linear static behaviour, square, thin \((b/h = 100)\), 4-layer simply supported symmetric angle-ply laminates with surface bonded piezoelectric elements \((\text{PZT} / \alpha / \text{PZT})\) are analysed by varying the fibre orientation angle. Non-dimensional uniform load of \(Q = 350\) is applied to the plate in steps of 1. Equal and opposite voltage of 500 volts is applied to the actuators. The percentage reduction in deflection for different values of \(\alpha\) is given in Fig. 9.6. The percentage reduction in central deflection is found to decrease with increase in fibre orientation angle and load. It is also seen that the variation in the reduction of central deflection for different values of fibre orientation angle is negligible in the linear range (for lower values of load). As the value of \(\alpha\) increases, a larger voltage is required to keep the percentage reduction in deflection the same.

![Figure 9.6 Percentage reduction in deflection with fibre orientation angle](image)

Figure 9.6 Percentage reduction in deflection with fibre orientation angle
Example 9.4

The effect of number of layers in non-linear static response is studied by considering simply supported thin \((b/h = 100)\) cross-ply and angle-ply laminates with anti-symmetric arrangement and surface-bonded piezoelectric elements. The number of layers in substrate (base structure) is increased from 2 to 8 by keeping the overall depth constant. The external load and the voltage applied are same as those in Example 9.3. The percentage reduction in deflection with number of layers is presented in Figs. 9.7 and 9.8. It can be seen that the percentage reduction in deflection decreases with increase in load and increases with increase in number of layers, the increase being negligible beyond 6 layers. It is also noted that the behaviour of 2-layer plate is different from that of multi-layer laminates, as has been observed in earlier chapters.

\[\text{Figure 9.7 Percentage reduction in deflection with number of layers (cross-ply)}\]
9.3 DYNAMIC ANALYSIS

The same formulation used for the static analysis has been extended to study the behaviour of laminated composite plates with piezoelectric elements under dynamic loading. The effect of voltage applied to the actuator in the non-linear domain is studied by considering square simply supported 4-layer thin (b/h = 100) symmetric cross-ply and angle-ply laminates. Both undamped and damped conditions are considered. In the case of a damped system a damping ratio of 0.05 is considered. The geometry and properties of the material of the plate are same as in Example 6.1. The thickness of the piezoelectric layer is assumed to be 1% of the thickness of the substrate. A uniform non-dimensional

Figure 9.8 Percentage reduction in deflection with number of layers (angle-ply)
load, \( Q = 200 \), is assumed to be suddenly applied. As the analysis is done by linear load incrementing method, the voltage to the actuator is also applied in steps. Equal and opposite voltage of 200V is applied in 50 steps to the actuator with \( \pm 4V \) in each step. The displacement response for the cross-ply laminates under undamped and damped conditions are presented in Figs. 9.9 and 9.10 respectively. The displacement response of damped angle-ply laminates with piezoelectric elements is shown in Fig. 9.11.

Figure 9.9 Displacement response of an undamped 4-layer cross-ply laminate with piezoelectric elements
Figure 9.10 Displacement response of a damped 4-layer cross-ply laminate with piezoelectric elements

Figure 9.11 Displacement response of a damped 4-layer angle-ply laminate with piezoelectric elements
The results do not show the anticipated reduction in amplitude of vibration. The response curves with non-zero gains do not show a consistent and regular behaviour. This may be due to many reasons. A finer mesh division or smaller load steps may be necessary for better results, which is computationally not practicable. A higher-order element may give better results, which has not been tried because of time constraints.

9.4 VIBRATION CONTROL

To study the effect of large displacements on vibration control, 4-layer symmetric cross-ply and angle-ply laminates with piezoelectric elements are analysed. The top layer is assumed to be the sensor and the bottom layer the actuator. A uniform non-dimensional load, \( Q = 200 \), is suddenly applied. The geometry of the plate and the properties of the material are the same as those in dynamic analysis in section 9.2. A damping ratio of 0.05 and a gain of 4000 volts per ampere are assumed. The constitutive relation for the piezoelectric material is assumed to be linear in the non-linear domain also. The charge developed in the sensor layer under mechanical deformation is evaluated as given in Eq. (6.3) incorporating the non-linear strain components also. The displacement history for the plates with and without gain is depicted in Figs. 9.12 and 9.13. These figures also show that only a small reduction in amplitude of vibration is achieved.
Figure 9.12 Displacement response of a damped 4-layer cross-ply laminate with piezoelectric elements under different gains.
9.5 DISCUSSION

Based on the studies conducted the following conclusions are drawn:

1. The effect of low values of voltage on static response of thick plates in the non-linear domain is practically negligible.

2. Even in the case of thin plates, the percentage reduction in response at low values of voltages is very small.
3. The percentage reduction in central deflection decreases with increase in fibre orientation angle and load.

4. The percentage reduction in central deflection decreases with increase in number of layers in substrate. The rate of reduction in deflection also decreases with increase in load and number of layers.

5. Vibration control of smart plates with large displacements is not explained properly by the present study. Further studies are required in this case, including experimental verification.