CHAPTER 5

Applications of MADM in Finance

An investor would like to build a balanced portfolio with stocks representing different sectors. Several researchers have attempted the portfolio selection problem by different methods. Many of these methods consider companies of different sectors together. However, it can be argued that the attributes affecting the company’s growth vary for different sectors. Therefore, it is advisable to attempt the problem sector wise. There are many options for the selection of a stock from a particular sector. In this chapter, a stock ranking method is proposed using MADM methods based on overall performance under a stochastic environment. Of many MADM methods, SAW, AHP, TOPSIS, and VIKOR are applied. Usually, Euclidean distances (2-norm) are considered in the implementation of TOPSIS and VIKOR methods. In this work, we generalized this norm to $p$-norm, where $p > 1$. The model is tested for 13 companies in the field of Information Technology Sector listed on National Stock Exchange of India and 13 criteria as performance indicators of a company. A MATLAB GUI system is developed and the results are obtained for several values of $p$ in case of TOPSIS and VIKOR methods besides other methods. We have reported this work for possible publication in [18].
23. Introduction

In 1947, V. Neumann and O. Morgenstern published their famous book, *Theory of Games and Economic Behavior*, to conceive a mathematical theory of economic and social organization based on game theory. This work opened the doors to Multiple Attribute Decision-making Methods (MADM) [34]. We propose a multiple criteria stock ranking model on overall performance under a stochastic environment. The proposed model displays the ability to provide better information on the overall performance of particular stock to investors and the ability to be used as supplementary information. Stock selection can be classified into two groups with single criterion and multiple criteria. For a single criterion on the expected returns, optimization methods are widely used, such as the constraint generation method (T. Nguyen and W. Andrew, 2012), linear programming and fuzzy compromise programming (A. Terol, M. Arenas et al., 2006) and algorithmic complexity theory (R. Giglio and D. Sergio, 2009) [31].

The proposed multiple criteria are dominated by the financial performance in terms of 13 attributes listed in Table (5.29.0.1). Various approaches in dealing with multiple criteria decision making in stock selection have been proposed. These are MADM. Most of the criteria for stock selection have inter-dependent/interactive characteristics. The aim of the proposed ranking framework is to reflect the overall corporate performance of the firms under consideration by significant financial and nonfinancial performance which could be classified into quantitative and qualitative criteria. The results indicate that the proposed method for stock selection is effective and provides
meaningful implications for investors and management teams to refer. The following five methods are commonly used:

(i) Simple Additive Weight method (SAW)
(ii) Analytic Hierarchy Process method (AHP)
(iii) Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)
(iv) Modified TOPSIS method
(v) VIskriterijumsko KOmpromisno Rangiranje -VIKOR.

Each decision problem can be broken up into three components, defined here. The goal of the problem is overall objective that drives the decision problem. The goal should be specific to the problem at hand, and should be something that can be examined properly by the decision makers. The alternatives are the different options that are being weighed in the decision. The criteria of a decision problem are the factors that are used to evaluate the alternatives with regard to the goal.

**Figure 23.1. Goal, Criteria, and Alternatives.**
Each alternative will be judged based on these criteria, to see how well they meet the goal of the problem. We can go further to create sub-criteria, when more differentiation is required. For instance, if we were to look at a goal of buying a new car for a family, we may want to consider safety as a criterion. There are many things that determine the overall safety of a car. So we may create sub-criteria such as car size, safety ratings, and number of airbags.

MADM methods are generally discrete, with a limited number of predetermined alternatives. MADM is an approach employed to solve problems involving selection from a finite number of alternatives. An MADM method specifies how attribute information is to be processed in order to arrive at a choice. MADM methods require both inter- and intra-attribute comparisons, and involve appropriate explicit tradeoffs. Each decision table (also called decision matrix) in MADM methods has four main parts, namely: (a) alternatives, (b) attributes, (c) weight or relative importance of each attribute, and (d) measures of performance of alternatives with respect to the attributes. The decision table is shown in the Table (5.23.0.1).

Table 5.23.0.1. Data Matrix in MADM Methods.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Attributes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$B_1$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>$a_{11}$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$a_{21}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$A_m$</td>
<td>$a_{m1}$</td>
</tr>
</tbody>
</table>
24. The SAW Method

The Simple Additive Weighting (SAW) method is probably the best known and widely used method for MADM. C. Churchman and R. Ackoff (1954) first utilized the SAW method to cope with a portfolio selection problem [34]. SAW can be considered as the most intuitive and easy way to deal with multiple criteria decision-making problems because the linear additive function can represent the preferences of decision makers. Here each attribute is given a weight, and the sum of all weights must be equal to 1. Each alternative is assessed with regard to every attribute. The composite performance score is given by

\[ P_i = \sum_{j=1}^{n} w_j a_{ij} \quad i = 1, 2, \ldots, m. \]  

(5.24.0.1)

It was argued that this method should be used only when the decision attributes can be expressed in identical units of measure. However, if the decision matrix is normalized, then it can be used for any type of attributes. In this case, the above equation will take the following form

\[ P_i = \sum_{j=1}^{n} w_j r_{ij}, \]  

(5.24.0.2)

where \( r_{ij} \) is the normalized preferred ratings of the \( i^{th} \) alternative with respect to the \( j^{th} \) criterion for all commensurable units. The normalized preferred ratings \( r_{ij} \) of the \( i^{th} \) alternative with respect to the \( j^{th} \) criterion can be obtained in the following way. For benefit criteria (profit), \( r_{ij} = \frac{a_{ij}}{a^*_j} \) and for non-benefit criteria (cost), \( r_{ij} = \frac{a'_{ij}}{a_{ij}} \), where \( a^*_j = \max_i a_{ij} \) and \( a'_{j} = \min_i a_{ij} \).
25. The AHP Method

The Analytic Hierarchy Process (AHP) is a structured technique for organizing and analyzing complex decisions, based on mathematics and psychology. It was developed by T. Saaty in the 1970s [8, 60]. The AHP is a decision support tool which can be used to solve complex decision problems. Satty has developed the quantification of pairwise comparisons of criteria in the scale of 1-9 as shown in the Table (5.25.0.1).

Consider \( n \) elements to be compared; say, \( A_1, A_2, \ldots, A_n \). Denote the relative priority or significance of \( A_i \) with respect to \( A_j \) by \( a_{ij} \) and form a matrix \( A = [a_{ij}] \) of order \( n \) such that

\[
a_{ij} = \begin{cases} 
\frac{1}{a_{ji}} & \text{for } i \neq j \\
1 & \text{for } i = j 
\end{cases}
\]

Thus the reciprocal matrix \( A \) has the form

\[
A = \begin{bmatrix}
1 & a_{12} & \cdots & a_{1n} \\
\frac{1}{a_{12}} & 1 & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{1}{a_{1n}} & \frac{1}{a_{2n}} & \cdots & 1
\end{bmatrix}
\]
Table 5.25.0.1. Fundamental Scale of Absolute Numbers.

<table>
<thead>
<tr>
<th>Intensity of Importance</th>
<th>Definition</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal Importance</td>
<td>Two activities contribute equally to the objective</td>
</tr>
<tr>
<td>2</td>
<td>Weak or slight</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Moderate importance</td>
<td>Experience and judgement slightly favour one activity over another</td>
</tr>
<tr>
<td>4</td>
<td>Moderate plus</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Strong importance</td>
<td>Experience and judgement strongly favour one activity over another</td>
</tr>
<tr>
<td>6</td>
<td>Strong plus</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Very strong or demonstrated importance</td>
<td>An activity is favoured very strongly over another; its dominance is demonstrated in practice</td>
</tr>
<tr>
<td>8</td>
<td>Very, very strong</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Extreme importance</td>
<td>The evidence favouring one activity over another is of the highest possible order of affirmation</td>
</tr>
<tr>
<td>Reciprocals of above</td>
<td></td>
<td>A reasonable assumption</td>
</tr>
</tbody>
</table>
Having recorded the quantified judgments on pairs \((A_i, A_j)\) as numerical entries \(a_{ij}\) in the matrix \(A\), the problem now is to assign to the \(n\) elements \(A_1, A_2, \ldots, A_n\) a set of numerical weights \(w_1, w_2, \ldots, w_n\) which allows diverse elements to be compared to one another in a rational way. In order to do so, the vaguely formulated problem must first be transformed into a precise mathematical one. In the ideal case the relation between weight \(w_i\) and \(a_{ij}\) is given by \(\frac{w_i}{w_j} = a_{ij}\). Thus the matrix \(A\) is

\[
A = \begin{bmatrix}
w_1 & w_1 & \cdots & w_1 \\
w_2 & w_2 & \cdots & w_2 \\
\vdots & \vdots & \ddots & \vdots \\
w_n & w_n & \cdots & w_n
\end{bmatrix}
\]

**Definition 5.25.1.** [60] If the elements are transitive, i.e., \(a_{ik} = a_{ij}a_{jk}\) for all \(i, j, k\), then \(A\) is called a consistent matrix.

Such a matrix might exist if the \(a_{ij}\)’s are calculated from exactly measured data. What this implies in our matrices is that for a consistent matrix there is some underlying standard scale. That is, each element has a set weight, which does not change when compared to another element. Here we note that \(AW = nW\), where \(W = \begin{bmatrix} w_1 & w_2 & \cdots & w_n \end{bmatrix}^T\). We can see that \(n\) is then eigenvalue and the weight matrix \(W\) is an eigenvector of \(A\). We know that the sum of the eigenvalues of matrix \(A\) is simply its trace. Since \(Tr(A) = n\) and \(n\) is the eigenvalue of \(A\), it is the largest or principal eigenvalue of \(A\). We denote it by \(\lambda_{\text{max}}\).

**Theorem 5.25.2.** [61] For a reciprocal matrix \(A = [a_{ij}]\), \(\lambda_{\text{max}} \geq n\). It is equal if and only if \(A\) is consistent.
With this theorem we can now tackle the idea of measuring inconsistency. We define the consistency index as follows.

**Definition 5.25.3.** [61] The *consistency index* (CI) is the value \( \frac{\lambda_{\text{max}} - n}{n-1} \).

The smaller the value of CI, the smaller is the deviation from the consistency. With this idea of a consistency index, we can extend further to look at what the CI would be for some completely random reciprocal matrix. Moreover, we can find the CI for a large number of these matrices, and determine an average random CI. We then have the following definition.

**Definition 5.25.4.** [61] The *random index* (RI) of size \( n \) is the average CI calculated from a large number of randomly generated reciprocal matrices.

By the help of computer, one can generate high number (more than 1,00,000) of randomly generated reciprocal matrices. The following definition give us a value of the inconsistency.

**Definition 5.25.5.** [61] *Consistency Ratio* (CR) of a reciprocal matrix \( A \) is

\[
CR = \frac{CI}{RI}.
\]

Usually, a CR of 0.1 or less is considered as acceptable, and it reflects an informed judgement attributable to the knowledge of the analyst regarding the problem under study. This consistency ratio tells us essentially how inconsistent our matrix is. When running through the AHP, if the value of CR is greater than 0.1, then the ratio matrix under consideration is inconsistent to give reliable results. In this case, there is not much of an underlying scale present, and the process does not work. In this situation, we will go back to the ratio matrix and try to reevaluate our comparisons, until we get a CR
that falls within our acceptable range. This approach to determining whether a given matrix falls within our tolerances for inconsistency was given by T. Satty. There is an approach that allows us to immediately know upon calculation of $\lambda_{max}$ developed by J. Alonso and M. Lamata whether a matrix falls within our tolerances. In this method, they have calculated the average value of $\lambda_{max}$ for a large number of $n \times n$ matrices, say $\bar{\lambda}_{max}$ [2]. They used 5,00,000 randomly generated matrices for each $3 \leq n \leq 13$. Note that, for a matrix of size 2, we will always have a consistent matrix and thus $\lambda_{max} = 2$.

By finding $\bar{\lambda}_{max}$ for the matrices of each size $n$, J. Alonso and M. Lamata were able to plot these values against the size of the matrix and find a least-square line to model the relationship. The curve that fits the data best is linear with equation

$$\bar{\lambda}_{max}(n) = 2.7699n - 4.3513.$$  \hspace{1cm} (5.25.5.1)

This line fits the data very well with a correlation coefficient of 0.99. The following calculations show the maximum acceptable value of $\lambda_{max}$ if we want CR to be less than 0.1.

First, we know that we can represent the RI for some matrix of size $n$ by

$$RI = \frac{\bar{\lambda}_{max} - n}{n - 1}. \hspace{1cm} (5.25.5.2)$$

Using the definition CR, which we want to be less than 0.1, we can then see that

$$CR = \frac{\lambda_{max} - n}{\bar{\lambda}_{max} - n} < 0.1.$$  

Solving for $\lambda_{max}$, we have $\lambda_{max} < n + 0.1(\bar{\lambda}_{max} - n)$. Thus, combining this result with Equation (5.25.5.1), we see that

$$\lambda_{max} < 1.17699n - 0.43513.$$  \hspace{1cm} (5.25.5.3)
Using this equation, we can determine the maximum permissible value of $\lambda_{max}$ for which CR to be less than 0.1. However, we are not interested in these standard scales, since they often do not tell us much about the real world problems we will be dealing with. Near consistency is essential for response to attributes because when it is used to compare attributes that are intangible, human judgment is approximate and mostly inconsistent. If with new information one is able to improve inconsistency to near-consistency, that could improve the validity of the priorities derived from the judgments. To derive priorities from an inconsistent matrix $A = [a_{ij}]$, it is necessary to obtain the principal right eigenvector $W$ to represent these priorities.

26. $p$-TOPSIS Method

The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is a multi-criteria decision analysis method, which was originally developed by C. Hwang and K. Yoon in 1981 with further developments by K. Yoon in 1987, and C. Hwang, Y. Lai and T. Liu in 1993 [71]. The basic concept of this method is that the selected alternative should have the shortest distance from the ideal solution and the farthest distance from the negative - ideal (anti - ideal) solution in some geometrical sense. The TOPSIS method assumes that each criterion has a tendency of monotonically increasing or decreasing utility. Any outcome, which is expressed in a non - numerical way, should be quantified through the appropriate scaling technique. Therefore, it is easy to define the positive ideal and negative ideal solutions. The Euclidean distance approach was proposed to evaluate the relative closeness of the alternatives to the ideal
solution. Thus, the preference order of the alternatives can be derived by a series of comparisons of these relative distances.

As an illustration, Figure(26.1) shows 5 alternatives, A, B, C, D and E, with a choice of two criteria; it also shows the ideal and anti-ideal points. It is obvious that, if we use the usual Euclidean distance \( p = 2 \) with equal weights, then the point C is the closest to the ideal and D is the longest.

As an example to put overall standings of some of NSE/BSE listed IT companies, relevant decision matrix relating the different financial and non-financial attributes are first created, then they are normalized, and then the weighted normal decision matrix is created. Positive ideal and negative ideal solutions are created. Separation measures are calculated and then relative closeness to the ideal solution is calculated so that relative order of their performance can be demonstrated. Here we propose \( p \)-norm besides Euclidean norm for the calculation of separation measures. Hence we name this method
as p-TOPSIS method. We incorporate them in the following steps and implement in the MATLAB program [[34], P. 69].

1. **Creation of Decision Matrix:** Create an evaluation matrix $A = [a_{ij}]_{n \times n}$ consisting of $n$ alternatives and $n$ attributes (criteria), with the intersection of each alternative and attributes given as $a_{ij}$.

2. **Construction of the Normalized Decision Matrix:** To transform the various attribute dimensions into non-dimensional attributes, which allows comparison across the attributes. The decision matrix $A = [a_{ij}]_{n \times n}$ is then normalized to form the matrix $R = [r_{ij}]_{n \times n}$ using the normalization method

   $$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{k=1}^{n} a_{kj}^2}}.$$ 

3. **Construction of the Weighted Normalized Decision Matrix:** Let $w_j$ be the weight given to the criterion $j$ and $w_j = W_j / \sum W_j$ so that $\sum w_j = 1$. Calculate the weighted normalized decision matrix

   $$T = [t_{ij}]_{n \times n} = [w_j r_{ij}]_{n \times n}.$$

4. **Determine Ideal and Negative-Ideal Solutions:** Let

   $$J_+ = \{ j = 1, 2, \ldots, n \mid j \text{ associated with the criteria having a +ve impact} \}$$

   $$J_\text{-} = \{ j = 1, 2, \ldots, n \mid j \text{ associated with the criteria having a -ve impact} \}$$

   We determine the worst alternative $A_w$ and the best alternative $A_b$ as under:

   $$A_w = \{ \langle \max (t_{ij} | i = 1, 2, \ldots, n) \mid j \in J_\text{-} \rangle , \langle \min (t_{ij} | i = 1, 2, \ldots, n) \mid j \in J_+ \rangle \}$$

   $$\equiv \{ t_{wj} | j = 1, 2, \ldots, n \} ;$$
\[ A_b = \{ \langle \min (t_{ij} \mid i = 1, 2, \ldots, n) \mid j \in J_- \rangle, \langle \max (t_{ij} \mid i = 1, 2, \ldots, n) \mid j \in J_+ \rangle \} \]
\[ \equiv \{ t_{bj} \mid j = 1, 2, \ldots, n \}. \]

Both sets of positive and negative - ideal solutions consist of \( n \) number of elements which are evaluation factors.

5. Calculation of Separation Measures: The \( l^2 \) distance (Euclidean distance) approach is used to calculate the separation of each alternative from the positive ideal and negative ideal solution measures as follows:

\[ d_{iw} = \sqrt{\sum_{j=1}^{n} (t_{ij} - t_{wj})^2} \quad (1 \leq i \leq n); \]
and

\[ d_{ib} = \sqrt{\sum_{j=1}^{n} (t_{ij} - t_{bj})^2} \quad (1 \leq i \leq n). \]

Generalizing the above separation measures to \( l^p \) norm with \( 1 \leq p \leq \infty \), we have

\[ d_{iw}^p = \left\{ \sum_{j=1}^{n} |t_{ij} - t_{wj}|^p \right\}^{\frac{1}{p}} \quad (1 \leq i \leq n); \quad (5.26.0.1) \]
and

\[ d_{ib}^p = \left\{ \sum_{j=1}^{n} |t_{ij} - t_{bj}|^p \right\}^{\frac{1}{p}} \quad (1 \leq i \leq n). \quad (5.26.0.2) \]

Also, for \( p = \infty \), the \( l_\infty \) norm is

\[ d_{iw}^\infty = \max \{ |t_{ij} - t_{wj}| : 1 \leq j \leq n \} \quad (1 \leq i \leq n); \]
and

\[ d_{ib}^\infty = \max \{ |t_{ij} - t_{bj}| : 1 \leq j \leq n \} \quad (1 \leq i \leq n). \]
6. **Calculation of Relative Closeness to the Ideal Solution:** The relative closeness to the ideal solution, which will be used for the ranking of options, is calculated as in formula

\[ s_{iw}^p = \frac{d_{iw}}{d_{iw} + d_{ib}} \quad (1 \leq i \leq n). \]

Note that \(0 \leq s_{iw}^p \leq 1\). Rank the alternatives according to the value of \(s_{iw}^p\).

27. **Modified TOPSIS Method**

In the TOPSIS method, the normalized decision matrix \(R\) is weighted by multiplying each column of the matrix by its associated attribute weight. The overall performance of an alternative is then determined by its Euclidean distance to \(A_w\) and \(A_b\). However, this distance is interrelated with the attribute weights, and should be incorporated in the distance measurement. This is because all alternatives are compared with \(A_w\) and \(A_b\), rather than directly among themselves. H. Deng et al. (2000) presented the weighted Euclidean distances, rather than creating a weighted decision matrix [19]. In this process, the positive ideal solution \(R^+\) and the negative ideal solution \(R^-\), which are not dependent on the weighted decision matrix, are defined as [[71], P. 35]:

\[ R^+ = \{\langle \max (r_{ij} | i = 1, 2, \ldots, n) \rangle_j \in J_-\}, \langle \min (r_{ij} | i = 1, 2, \ldots, n) \rangle_j \in J_+\} \equiv \{R^+_j | j = 1, 2, \ldots, n\}; \]

and

\[ R^- = \{\langle \min (r_{ij} | i = 1, 2, \ldots, n) \rangle_j \in J_-\}, \langle \max (r_{ij} | i = 1, 2, \ldots, n) \rangle_j \in J_+\} \equiv \{R^-_j | j = 1, 2, \ldots, n\}. \]
The weighted Euclidean distances are calculated as

\[
D_i^+ = \left\{ \sum_{j=1}^{n} w_j (r_{ij} - R_{ij}^+)^2 \right\}^{\frac{1}{2}} \quad (1 \leq i \leq n); \tag{5.27.0.1}
\]

and

\[
D_i^- = \left\{ \sum_{j=1}^{n} w_j (r_{ij} - R_{ij}^-)^2 \right\}^{\frac{1}{2}} \quad (1 \leq i \leq n). \tag{5.27.0.2}
\]

The relative closeness of a particular alternative to the ideal solution, \( P_i \), can be expressed as

\[
P_i = \frac{D_i^-}{D_i^+ + D_i^-}. \tag{5.27.0.3}
\]

A set of alternatives is made in the descending order, according to the value of \( P_i \) indicating the most preferred and least preferred feasible solutions.

28. \( p \)-VIKOR Method

VIKOR method was developed for multicriteria optimization of complex systems. It was originally developed by Serafim Opricovic to solve decision problems with conflicting and noncommensurable (different units) criteria in his Ph.D. thesis in 1979, and an application was published in 1980. The idea of compromise solution was introduced in MCDM by P. Yu in 1973 [74]. The name VIKOR appeared in 1990 from Serbian: VIseKriterijumska Optimizacija I Kompromisno Resenje, that means: Multicriteria Optimization and Compromise Solution.

Let \( f_j^* = \max_i a_{ij} \) and \( f_j' = \min_i a_{ij} \) if the \( j^{th} \) function is benefit. Determine the best \( f_j^* \) and the worst \( f_j' \) values of all criterion functions, \( j = 1, 2, \ldots n \). If the \( j^{th} \) function is cost, then \( f_j^* = \min_i a_{ij} \) and \( f_j' = \max_i a_{ij} \). The multicriteria
measure for compromise ranking is developed from the $l^p$-metric used as an aggregating function in a compromise programming method [[34], P.73].

\[
l_{p,i} = \left\{ \sum_{j=1}^{n} \left| w_j (f^*_j - a_{ij})/(f^*_j - f'_j) \right|^p \right\}^{\frac{1}{p}} \quad (1 \leq p < \infty; \ 1 \leq i \leq n);
\]

\[
l_{\infty,i} = \max \left\{ \left| w_j (f^*_j - a_{ij})/(f^*_j - f'_j) \right| : 1 \leq j \leq n \right\} \quad (1 \leq i \leq n).
\]

The compromise ranking algorithm $p$-VIKOR has the following steps:

**Step 1:** The first step is to determine the objective, and to identify the pertinent evaluation attributes. Determine the best $f^*_j$ and the worst $f'_j$ values of all criterion functions, $j = 1, 2, \ldots n$.

**Step 2:** Compute the values $S_i$ and $R_i$, $i = 1, 2, \ldots n$, by the relations

\[
S_i = l_{p,i} \quad \text{and} \quad R_i = l_{\infty,i}, \quad i = 1, 2, \ldots n.
\]

**Step 3:** Compute the value $Q_i$, $i = 1, 2, \ldots n$ by the relation

\[
Q_i = v \frac{S_i - S^*}{S' - S^*} + (1 - v) \frac{R_i - R^*}{R' - R^*},
\]

where $S^* = \min_i S_i$, $S' = \max_i S_i$, $R^* = \min_i R_i$, $R' = \max_i R_i$. Here $v$ is introduced as weight of the strategy of the majority of attributes. Usually, the value of $v$ is taken as 0.5. However, $v$ can take value from 0 to 1.

**Step 4:** Rank the alternatives, sorting by the values of $S, R$ and $Q$ in decreasing order. The results are three ranking lists. The compromise ranking list for a given $v$ is obtained by ranking with $Q_i$ measures. The best alternative is the one with the minimum value of $Q_i$.

**Step 5:** For given attribute weights, propose a compromise solution, alternative $A_k$, which is the best ranked by measure $Q$, if the following two conditions are satisfied:
(i) Acceptable advantage: \( Q(A_k) - Q(A_1) \geq \frac{1}{n-1} \), where \( A_1 \) is the second best alternative in the ranking by \( Q \).

(ii) Acceptable stability in decision making: Alternative \( A_k \) must also be the best ranked by \( S \) or/and \( R \). This compromise solution is stable within a decision making process, which could be: voting by majority rule (when \( v > 0.5 \) is needed) or by consensus (when \( v \approx 0.5 \)) or with veto (when \( v < 0.5 \)).

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives \( A_k \) and \( A_1 \) if only condition (ii) is not satisfied.
- Alternatives \( A_k, A_1, \ldots, A_p \) if condition (i) is not satisfied; \( A_p \) is determined by the relation \( Q(A_p) - Q(A_1) < \frac{1}{n-1} \).

VIKOR is a helpful tool in multicriteria decision making, particularly in a situation where the decision maker is not able, or does not know, to express his/her preference at the beginning of system design.

Weights of the criteria are affected as much from characteristics of the criteria as from subjective point of view of the decision-maker. Such subjective weighting of the criteria is usually shaped by the decision-maker experience, knowledge and perception of the problem. For this reason, various subjective weighting methods have been developed. Such subjective weighting lead to doubt about reliability of the results. To overcome such problems, objective weighting methods are used [29]. Let \( A = [a_{ij}] \) is \( n \times n \) judgement matrix.
1. **Sum Method:** The sum method is as follows

\[ W_i = \frac{1}{n} \sum_{j=1}^{n} a_{ij} \quad (1 \leq i \leq n). \]

2. **Geometric Mean Method:** Geometric Mean Method is as follows

\[ W_i = \left( \prod_{j=1}^{n} a_{ij} \right)^{\frac{1}{n}} \quad (1 \leq i \leq n). \]

3. **Eigenvector Method:** It consists in taking weights as the components of the eigenvector of the matrix A. We have seen in AHP method that \( AW = \lambda_{max} W \). Note that \( \lambda_{max} \) is the largest eigenvalue of A.

4. **Least Square Method:** Construct generalized deviations function

\[ f(w_1, w_2, \ldots, w_n) = \sum_{i,j} \left( a_{ij} - \frac{w_i}{w_j} \right)^2. \]

Obviously, the reasonable weight vector \( W = (w_1, w_2, \ldots, w_n)^T \) should be induced by minimizing \( f(w_1, w_2, \ldots, w_n) \). This is rather difficult to solve because the objective function is nonlinear and usually nonconvex, moreover, no unique solution exists and the solutions are not easily computable. If the error is \( a_{ij} w_j - w_i \), the expression is linear. We can not only use the sum of squares of error as objective function, but also use the sum of error absolute value and the error absolute value as objective function. Three methods are given as follows.
Method 1: Using sum of squares of error as objective function

The model is

\[
\text{Minimize } \sum_{i=1}^{n} \sum_{j=1}^{n} (a_{ij}w_j - w_i)^2,
\]

subject to \( \sum_{i=1}^{n} w_i = 1, \ w_i \geq 0 \). Thus, we can construct Lagrange function as

\[
L = \sum_{i=1}^{n} \sum_{j=1}^{n} (a_{ij}w_j - w_i)^2 + \lambda \left( \sum_{i=1}^{n} w_i - 1 \right),
\]

where \( \lambda \) is a Lagrangian multiplier.

\[
\frac{\partial L}{\partial w_i} = -2(a_{i1}w_1 - w_i) - 2(a_{i2}w_2 - w_i) - \cdots - 2(a_{in}w_n - w_i) \\
+ 2a_{11}(a_{11}w_1 - w_1) + 2a_{21}(a_{21}w_1 - w_2) + \cdots + 2a_{ni}(a_{ni}w_i - w_n) + \lambda \\
= -2(a_{i1} + a_{1i})w_1 - 2(a_{i2} + a_{2i})w_2 - \cdots - \\
+ [2(n - 1) + 2 \sum_{j=1}^{n} a_{ji}^2]w_i - \cdots - 2(a_{in} + a_{ni})w_n + \lambda.
\]

On equating \( \frac{\partial L}{\partial w_i} \) to be equal to zero with the constraint \( \sum_{i=1}^{n} w_i = 1 \) we have a system of \( n + 1 \) linear equations. By solving we can obtain \( w_1, w_2, \ldots, w_n \).

Method 2: Minmax method

Using maximum error absolute value as objective function, the model is

\[
\text{Minimize } \max_{1 \leq i, j \leq n} |a_{ij}w_j - w_i|, \quad (5.28.0.3)
\]

subject to \( \sum_{i=1}^{n} w_i = 1, w_i \geq 0 \).
This is a linear programming method. \( w_i \) can be found by simplex method.

Method 3: Absolute Deviation Method

Using the sum of error absolute value as objective function, the model is

\[
\text{Minimize } \sum_{i=1}^{n} \sum_{j=1}^{n} |a_{ij}w_j - w_i|, \tag{5.28.0.4}
\]

subject to \( \sum_{i=1}^{n} w_i = 1, w_i \geq 0 \).

Let

\[
u_{ij} = \begin{cases} 
  a_{ij}w_j - w_i, & \text{if } a_{ij}w_j > w_i \\
  0, & \text{if } a_{ij}w_j \leq w_i
\end{cases}
\]

and

\[
v_{ij} = \begin{cases} 
  0, & \text{if } a_{ij}w_j > w_i \\
  -a_{ij}w_j + w_i, & \text{if } a_{ij}w_j \leq w_i
\end{cases}
\]

By the definition of \( u_{ij} \) and \( v_{ij} \) we have \( u_{ij} - v_{ij} = a_{ij}w_j - w_i \) and also 
\( |a_{ij}w_j - w_i| = u_{ij} + v_{ij} \). The model (5.28.0.4) is translated into

\[
\text{Minimize } \sum_{i=1}^{n} \sum_{j=1}^{n} (u_{ij} + v_{ij}),
\]

subject to,

\[
\sum_{i=1}^{n} w_i = 1, u_{ij} - v_{ij} = a_{ij}w_j - w_i, w_i \geq 0, u_{ij}, v_{ij} \geq 0 \ (1 \leq i, j \leq n).
\]

This is also a linear programming method. \( w_i \) can be found by simplex method.

5. Critic Method: In this method, both standard deviation of the criterion and its correlation between other criteria are included in the weighting process[21, 41]. Let \( \sigma_j \) be the standard deviation of the \( j^{th} \) criteria and \( r_{jk} \)
be the correlation coefficient between the $j^{th}$ and $k^{th}$ criteria. Let we define $C_j$ as

$$C_j = \sigma_j \sum_{k=1}^{n} (1 - r_{jk}).$$

In this regard, the weight $w_j$ of the $j^{th}$ criteria is

$$w_j = \frac{C_j}{\sum C_k} \quad (1 \leq j \leq n).$$

29. Ranking of Stocks by Applying MADM Methods

There are many different kinds of investment instruments available in the financial system such as stocks, bonds, options, swaps, futures contracts. But the stocks deserve a particular place in financial markets among investment instruments. Investors are mostly striving for finding out the modest method to construct the best performing investment portfolio by selecting best alternative stocks in an efficient way in order to have favorable future earnings and investment positions. Effective selection process of stocks for portfolio investments is one of the most complex process in competitive capital markets. Another prominent issue on the stock selection is to understand the dynamics of the financial markets. Therefore, the stock selection problem becomes an important research field in finance. Efficient market hypothesis demonstrates that investors can access all the data related to investment. The proposed model has been constructed for the stock selection of the stocks related to IT sector. We have collected the data as listed in the Table (5.29.0.1). The aim of this study is to analyze the stock selection problem by ranking the performance of different parameters. We have calculated the weights by using eigenvector method and critic method.
We have considered 13 parameters which affect the overall health of the company over the years. By considering many affecting parameters, the ranking allow us to consider the stocks which are the safest, most stable, and least risky investments. Stocks with high Safety ranks are often associated with large, financially sound companies. We considered 13 IT companies to be ranked.

**Table 5.29.0.1. Attributes.**

<table>
<thead>
<tr>
<th>No.</th>
<th>Name of Attribute</th>
<th>How Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Total Income</td>
<td>Average Yearly Return for 5 years</td>
</tr>
<tr>
<td>2.</td>
<td>Net Profit</td>
<td>Average Yearly Return for 5 years</td>
</tr>
<tr>
<td>3.</td>
<td>Net worth</td>
<td>Average Yearly Return for 5 years</td>
</tr>
<tr>
<td>4.</td>
<td>Return on Net worth</td>
<td>Average of 5 years</td>
</tr>
<tr>
<td>5.</td>
<td>Stock Price</td>
<td>Average Yearly Return for 5 years</td>
</tr>
<tr>
<td>6.</td>
<td>Promoter Holding</td>
<td>Latest Data</td>
</tr>
<tr>
<td>7.</td>
<td>FII + DII Holding</td>
<td>Latest Data</td>
</tr>
<tr>
<td>8.</td>
<td>Beta of a stock</td>
<td>Latest Data</td>
</tr>
<tr>
<td>9.</td>
<td>Operating Profit Margin</td>
<td>Average Yearly Return for 5 years</td>
</tr>
<tr>
<td>10.</td>
<td>Net Profit Margin</td>
<td>Average Yearly Return for 5 years</td>
</tr>
<tr>
<td>11.</td>
<td>Dividend Payout Ratio</td>
<td>Average of 5 years</td>
</tr>
<tr>
<td>12.</td>
<td>Bonus in Equity</td>
<td>Latest Data</td>
</tr>
<tr>
<td>13.</td>
<td>Reliability</td>
<td>Subjective</td>
</tr>
</tbody>
</table>
Table 5.29.0.2. Final Ranking.

<table>
<thead>
<tr>
<th>Name of Method</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Good</td>
</tr>
<tr>
<td></td>
<td>1  2  3</td>
</tr>
<tr>
<td>SAW</td>
<td>2  6  1</td>
</tr>
<tr>
<td>TOPSIS</td>
<td>2  6  7</td>
</tr>
<tr>
<td>$p$ - TOPSIS($p = 4.5$)</td>
<td>6  7  2</td>
</tr>
<tr>
<td>$p$ - TOPSIS($p = 16$)</td>
<td>6  1  7</td>
</tr>
<tr>
<td>Modified Topsis</td>
<td>6  7  2</td>
</tr>
<tr>
<td>AHP</td>
<td>1  7  2</td>
</tr>
<tr>
<td>$p$-VIKOR($p = 1$)</td>
<td>6  2  8</td>
</tr>
<tr>
<td>$p$-VIKOR($p = 2$)</td>
<td>6  8  2</td>
</tr>
<tr>
<td>$p$-VIKOR($p = 1.5$)</td>
<td>6  8  2</td>
</tr>
</tbody>
</table>

Note that the results for only few values of $p$ are displayed in the Table (5.29.0.2), but other values of $p$ can also be tried for in case of $p$-TOPSIS and $p$-VIKOR method. Though the actual ordering of the better performing companies and inferior performing companies differ for different methods, it can be observed that they remain same with some exceptions. As an example, 2\textsuperscript{nd} company is selected by all the methods. Similarly, 6\textsuperscript{th} company is not selected by only one method, namely, AHP. 4\textsuperscript{th} company is selected as an exception by VIKOR method that can be neglected. Similar comments apply to the rejection of inferior companies. For example, 13\textsuperscript{th} and 5\textsuperscript{th} companies are declared as inferior companies by all the methods. As far as effects of varying $p$ are considered, the $p$-TOPSIS method exhibits the uniform ranking for $p$ in
the range $[1, 16]$, exhibiting stability of ranking over considerably large value of $p$. However, larger values of $p$ results into absurdity. Similarly, $p$-VIKOR method also gives satisfactory result for $p$ in the interval $[1, 2]$ only [18]. Selected screen shots show the actual execution process. There are several tables in the GUI. The upper table titled “Company Information” contains the collected data which remains fixed. The column titles indicate the attributes. The attributes considered along with their abbreviations are: Total Income (TI), Net Profit (NP), Net Worth (NW), Return on Net worth (RON), Stock Price (SP), Promoter Holding (PH), FII+DII Holding (FII), Operating Profit Margin (OPM), Net Profit Margin (NPM), Dividend Payout Ratio (DPR). The user has to just select the method among the methods listed in the table “Select the Method”. If the method requires the value of $p$, then a separate box will be displayed asking the user to enter the value. After this, when the user clicks the button “Decision Maker”, the results are displayed in the table titled “Output” and the four better and four inferior companies are displayed at proper place.
Figure 29.1. Ranking by SAW Method.

Figure 29.2. Ranking by TOPSIS Method.
29. Ranking of Stocks by Applying MADM Methods

**Figure 29.3.** Ranking by $p$-TOPSIS Method ($p = 4.5$).

**Figure 29.4.** Ranking by $p$-TOPSIS Method ($p = 16$).
Figure 29.5. Ranking by Modified TOPSIS Method.

Figure 29.6. Ranking by AHP Method.
Figure 29.7. Ranking by $p$-VIKOR Method ($p = 2$).

Figure 29.8. Ranking by $p$-VIKOR Method ($p = 1.5$).
Collective performance assessment of n entities in the context of chosen n attributes is an important aspect in decision making in finance, engineering, social sciences, management etc. There are several methods, called multiple attribute decision making methods, developed for this purpose. In the present note by replacing Euclidean norm by the $p$-norms, $1 < p < \infty$ in $\mathbb{R}^n$, we have modified two of these methods TOPSIS method and VIKOR method into $p$-TOPSIS and $p$-VIKOR methods respectively. We analyze performance of 13 companies of IT sector listed on Indian stock exchanges considering 13 financial and non-financial attributes using each of the MADM methods, namely, SAW, AHP, $p$-TOPSIS, $p$-VIKOR (with different values of $p$).

We concentrate on ranking of first four best performing companies and last four worst performing companies to assess stability of these methods for different values of $p$. It is found that for values of $p$ in the interval $[1, 16]$, this ranking by and large remains same in $p$-TOPSIS method. In $p$-VIKOR method, more or less same conclusion obtained for values of $p$ in the interval $[1, 2]$. This exhibits stability of these methods with respect to $p$. We have also considered comparative ranking of these firms with other MADM methods. By and large for first four positions and for last four positions, the same firms are permuted respectively.