CHAPTER 1

INTRODUCTION
CHAPTER - I

INTRODUCTION

1.1. INTRODUCTION :

Performance assessment is essential in managerial applications and to develop control methods to plug inefficiency. Data envelopment analysis is a tool for efficiency gauging developed on a sound theoretical platform. Its roots can be traced in the works of Koopman (1951) and Farrell (1957), while the seminal contribution of the later, author proposed graphical illustration to empirical evaluation of technical, allocative and cost efficiency. Technical efficiency (TE) estimation enquires proportional reductions of inputs such that input mix is unaltered, the reason why it is called ‘technical efficiency’. Assessment of cost efficiency requires factor minimal cost and as a by product, factors of production leading to this can be obtained. The ratio of factor minimal cost to the observed cost results in a measure called Farrell’s cost efficiency measure.

The cost efficiency (CE) measure is obtained under very restrictive assumptions :

i) the inputs employed and outputs produced are homogeneous across the decision making units;

ii) the producers are price takers in input markets. Consequently no producer is able to influence the price;

iii) input prices are fixed and
iv) common input prices prevail all over the decision making units.

Never-the-less the producer of every decision making unit attained equilibrium so that the ratio of marginal products are equated to the relative prices of the concerned inputs.

All the assumptions placed above require that the input markets are perfectly competitive. If one or more of the above conditions are violated, the conditions of perfect competition are not completely met with.

Input prices vary across decision making units which normally happened due to geographical location, local governments regulations and due to incomplete perfect completion prevails. In such circumstances, Farrell cost efficiency measure acquires limitations.

To assess Farrell cost efficiency, Fare et al. (1985) formulated a linear programming problem which yields factor minimal cost. Two decision making units which employ the same inputs to produce the same outputs in levels, but face prices of one DMU are twice the prices of the other which possess the same Farrell cost efficiency (FCE).

Tone (2002) calls this deficiency as a ‘strange case’ of Farrell cost efficiency measure.

A decision making unit with a smaller total observed cost, but employs the same inputs to produce outputs at levels may emerge with a
poorer Farrell cost efficiency. Due to these deficiencies, the CE measure of Farrell needs to be corrected.

The factor prices as required to be known and fixed by FCE measure are so in a very short run. If input prices vary from one DMU to another, price inefficiency pervades input markets. Failure to operate at minimal set of prices leads to market inefficiency. In multi-inputs and multi-outputs settings operation at input prices higher than minimal prices leads to price inefficiency, alternatively called as market inefficiency by Camanho, A.S. and R.S Dyson (2008).

1.2 THEORY AND METHOD:

Two approaches are thought of to estimate market efficiency of a decision making unit.

(1) PRICE EFFICIENCY:

Define \( p_{i}^{\text{min}} = \min_{1 \leq j \leq n} p_{ij} \), \( i = 1, 2, \ldots, m \)

where \( p_{ij} \) is price of \( i^{th} \) input of \( j^{th} \) DMU

For DMU \( o \), the following Linear Programming Problem may be solved:

\[
Q(u, p^{\text{min}}) = \max_{x} \sum_{i=1}^{m} p_{i}x_{i}
\]

subject to
\[
\sum_{j=1}^{N} \lambda_j x_{ij} \leq x_i, \quad i=1,2,...,m
\]

\[
\sum_{j=1}^{N} \lambda_j u_{rj} \geq u_{r0}, \quad r=1,2,...,s
\]

\[
\lambda_j \geq 0
\]

Price Efficiency of DMU \(_0\) : \[
\frac{Q(u_0, p_{min})}{Q(u_0, p_0)}
\]

where \(Q(u_0, p_0)\) is factor minimal cost.

(2) **FARRELL COST EFFICIENCY** :

Farrell Cost Efficiency assessment requires to solve the following Linear Programming Problem :

\[
Q(u_0, p_0) = \min_{x} \sum_{i=1}^{m} p_i x_i
\]

subject to

\[
\sum_{j=1}^{N} \lambda_j x_{ij} \leq x_i, \quad i=1,2,...,m
\]

\[
\sum_{j=1}^{N} \lambda_j u_{rj} \geq u_{r0}, \quad r=1,2,...,s
\]

\[
\lambda_j \geq 0, \quad j=1,2,...,m
\]
Farrell Cost Efficiency: \[ \frac{Q(u_0, p_0)}{\sum_{i} p_i x_{i0}} \]

(3) ECONOMIC EFFICIENCY:

Departure of observed cost from 'minimum price' factor minimal cost leads to 'economic inefficiency'\[ \text{Economic Efficiency} : \frac{Q(u_0, p_{\text{min}})}{\sum_{i=1} m p_i x_{i0}} \]

Economic efficiency can be decomposed into the product of price and Farrell cost efficiency.

\[ \frac{Q(u_0, p_{\text{min}})}{\sum_{i=1} m p_i x_{i0}} = \frac{Q(u_0, p_{\text{min}})}{Q(u_0, p_0)} \times \frac{Q(u_0, p_0)}{\sum_{i=1} m p_i x_{i0}} \]

EE = PE \times FCE

Where EE = Economic Efficiency

PE = Price Efficiency

FCE = Farrell Cost Efficiency
The Farrell cost efficiency can further be decomposed into the product of technical and allocative efficiency. The former efficiency (input) evaluates and interior DMU, comparing it with a composite DMU viewed as a linear combination of extremely efficient decision making units.

\[ P_1x_1 + P_2x_2 = Q(u_o, p_o) \]

Figure (1.2.1)

In the diagram furnished above a unit input isoquant is plotted \((X_i = \frac{x_1}{u}, X_2 = \frac{x_2}{u})\). The DMU that operates at P is inefficient. It can reduce its inputs radially in the direction of origin, remain efficient reaching Q.

\((X_1^Q, X_2^Q)\) are efficient inputs.

Some input mix is noticed both at P and Q, thus the technique of production remains to be the same.
\[ F_1(u, x_1, x_2) = \frac{OR}{OP} \text{ measures input technical efficiency of DMU P.} \]

\[ 0 \leq F_1(u,x) \leq 1 \]

where \( F_1 \) is the Farrell input technical efficiency of DMU whose input and output mixes are respectively \( x \) and \( u \). To attain input cost efficiency DMU P should operate at S and by doing so it should change input mix.

Failure to operate at cost minimizing inputs lead to input allocative efficiency, which for DMU P is,

\[ \text{IAE} = \frac{\text{Cost at S}}{\text{cost at Q}} = \frac{\sum_{i=1}^{M} p_i x_i^Q}{\sum_{i=1}^{M} p_i x_i^Q} \]

where \( p_0 \) is input vector of DMU P, known and fixed. But cost at S is same as cost at R.

\[ \text{IAE} = \frac{OR}{OQ} \]

Deviation of observed cost from factor minimal cost leads to Farrell Cost Efficiency.

\[ \text{FCE} = \frac{\sum_{i=1}^{M} p_i x_i^P}{\sum_{i=1}^{M} p_i x_i^P} \]
\[ FCE \quad \frac{Q(u_0, p_0)}{\sum_{i=1}^{n} p_i^0 x_i^Q} \quad \frac{p_i^0 x_i^Q}{p_i^0 x_i^P} \]

Since \( x_i^Q = \lambda^{\text{min}} x_i^P \)

\[
\sum_{i=1}^{m} p_i^0 \lambda^{\text{min}} x_i^P = \lambda^{\text{min}} \quad \frac{OQ}{OP}
\]

\( \lambda^{\text{min}} \) measures input technical efficiency.

\[
\lambda^{\text{min}} = \text{Max} \quad \lambda
\]

subject to

\[
\sum_{j=1}^{n} \lambda_j x_{ij} \leq \lambda x_{s0}
\]

\[
\sum_{j=1}^{n} \lambda_j u_{rj} \geq u_{r0}
\]

\( \lambda_j \geq 0, j = 1, 2, \ldots, m \)

\[ FCE \quad \lambda^{\text{min}} \cdot \frac{Q(u_0, p_0)}{\sum_{i=1}^{m} p_i^0 x_i^Q} = \lambda^{\text{min}} (\text{IAE}) \]

\[ FCE \quad \text{ITE} \times \text{IAE} \]

Where \( \text{ITE} = \text{Input Technical Efficiency} \)

\( \text{IAE} = \text{Input Allocative Efficiency} \)
In the analysis of costs of inputs when not full perfect competition prevails, since DMUs fail to procure their inputs at minimum prices which may be possible to some DMUs, lead to identification of two additional efficiencies, viz., economic and market efficiency. The method described above provides a means to estimate these efficiencies where the former efficiency is the product of market (price) efficiency and Farrell cost efficiency. Selection of minimum prices proposed above may likely lead to the non-existence of a DMU that attains 100% market and/or economic efficiency. In the light of this realization the following method is proposed:

1) Evaluate the factor minimal cost of jth DMU using the price vectors of other DMUs as support prices.

\[ \frac{x_2}{u} = x_2 \]

Figure (1.2.2)

In the above diagram there are price lines one accounting the prices of A and another representing the isocost line of some DMU.
Let the support prices at \( S \) and \( T \) be denoted by,

\[
\begin{pmatrix}
S \\
(p_1, p_2)
\end{pmatrix}
\quad \text{and} \quad
\begin{pmatrix}
T \\
(p_1, p_2)
\end{pmatrix}
\]

respectively.

For cost inefficient DMU \( A \) we solve two linear programming problems:

The factor minimal cost of \( A \) is found using the prices (fixed and known) at \( S \) and \( T \). Consequently, we find \( Q(u_o, p^s) \), \( Q(u_o, p^T) \) and evaluate the cost efficiency of DMU \( A \).

\[
FCE \ (u_o, p^s) = \frac{Q(u_o, p^s)}{\sum p_i x_i}
\]

\[
FCE \ (u_o, p^T) = \frac{Q(u_o, p^T)}{\sum p_i x_i}
\]

More generally, for each DMU we solve as many linear programming problems as there are sets of distinct price vectors. If price vector changes from DMU to DMU one solves \( n \) linear programming problems:

\[
Q \ (u^k, p^j) = \min \sum_{i=1}^{m} p_i^j x_{ik}
\]

subject to

\[
\sum_{j=1}^{n} \Delta j x_{ij} \leq x_{ik} \quad , \quad i = 1, 2, \ldots, m
\]

\[
\sum_{j=1}^{n} \Delta j u_{rj} \geq u_{rk} \quad , \quad r = 1, 2, \ldots, s
\]

\[
\lambda_j \geq 0
\]
\( Q(u^k, p^j) \) is the factor minimal cost of \( k \) the DMU evaluated with respect to \( p^j \) as support price vector.

\[
\frac{Q(u^k, p^j)}{\sum_{j=1}^{m} p_i X_{ikj}} \quad k, j = 1, 2, \ldots, n
\]

For \( k \)th DMU we evaluate minimum of the above ratio.

\[
\min_{1 \leq j \leq n} \frac{Q(u^k, p^j)}{\sum_{j=1}^{m} p_i X_{ikj}}
\]

The price vector that minimizes the above ratio is minimal price vector of DMU \( K \) and to evaluate economic and hence market (price) efficiency the minimal price vector shall be used for DMU \( K \).

Thus, it is likely that minimal price vector vary from DMU to DMU or between subsets of DMUs.

It is of interest to see how close the economic and market efficiency in the two approaches.

However, both the methods provide efficiency measures free from the deficiencies addressed above. These methods are useful wherever, input prices vary across decision making units and inputs applied and outputs produced are of same quality across DMUs.

The quality of inputs employed and outputs produced may vary from DMU to DMU or over sets of DMUs leading to failure of perfect competition, yet in another dimension. Richard Tone (2004) suggested a
method to compute cost efficiencies constructing a value frontier in input
value space ignoring the production possibility set (PPS) in quantity space.

1) Define \( X_{ij} = p_{ij} x_{ij} \)

where \( p_{ij} \) and \( x_{ij} \) are price and input of the \( i^{th} \) factor of \( j^{th} \) DMU.

2) To compute cost efficiency using value frontier we solve the following linear programming problem.

\[
\begin{align*}
\text{Min} & \quad \sum_{i=1}^{m} X_{ik} \\
\text{subject to} & \\
\sum_{j=1}^{n} \lambda_j X_{ij} & \leq X_{ik}, \quad i = 1, 2, \ldots, m \\
\sum_{j=1}^{n} \lambda_j u_{rj} & \geq u_{r0}, \quad r = 1, 2, \ldots, s \\
\lambda_j & \geq 0, \quad j = 1, 2, \ldots, n
\end{align*}
\]

3) A measure of cost efficiency in this direction is as follows:

\[
\frac{\sum_{i=1}^{m} X_{ik}}{\prod_{i=1}^{m} p_{ik} x_{ik}}
\]

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1.3 **OBJECTIVES OF THE PRESENT STUDY:**

The present study aims at evaluating the cost efficiency of 77 Indian Commercial Banks employing a wide variety of inputs in order to produce a spectrum of outputs. Inclusion of too many inputs and outputs in data envelopment analysis (DEA) leads to a large proportion of efficient decision making units. Choice of inputs and outputs in DEA depends on the view points of analyst in terms his objectives and availability of data.

1.4 **DATA COLLECTION:**

Employees are the key resource to generate commercial banks outputs. They share major portion of banks operating costs, are viewed as input. Selection of inputs and outputs in DEA is not a straight forward issue, remained unsolved.

The approaches often used are the production and intermediation approach, in the former approach banks are viewed as providers of services to customers, in the later approach banks are considered financial intermediaries between savers and investors. The approaches help model building but to address certain objectives, the empirical studies often emphasize particular issues of concern to the organizations so that the listed inputs and outputs are based on the analysts view points and objectives.
Deposits produce advances, investments and assets as such it can be viewed as an input. There are studies which listed deposits as an output because they are obtained by 'resource consuming activity'.

However, advances are considered to be an output, undisputedly. Investments of a commercial bank includes Govt. approved and unapproved securities. The underlying model to measure efficiency of commercial banks is selected performing sensitivity analysis.

The data are secondary, collected from the Reserve Bank of India bulletins 2008.

Total wages/Number of employees is proxy for wage rate and there is no way of observing quality differences of employees within or between banks.

Rate of return on fixed assets is chosen as 'asset price'. Since 'deposits' are sum of demand deposits, saving deposits and term deposits, their interest components are averaged to assign price for deposits. 'Cost of funds' is chosen as price of NPA.
1.5 CHAPTER SCHEME:

The thesis is divided into five chapters.

Chapter -I Introduction
Chapter -II Review of literature
Chapter -III Theory and Methodology
Chapter -IV Empirical Investigation
Chapter -V Summary and Conclusions

APPENDIX

BIBLIOGRAPHY