

CHAPTER-VI

EFFECTS ON MHD BOUNDARY LAYER FLOW THROUGH A POROUS MEDIUM OVER A STRETCHING SHEET WITH HEAT GENERATION

1. INTRODUCTION

In recent years, a great deal of interest has been generated in the area of heat and mass transfer of the boundary layer flow over a stretching sheet, in view of its numerous and wide-ranging applications in various fields like polymer processing industry in particular in manufacturing process of artificial film and artificial fibers and in some applications of dilute polymer solution. Sakiadis [1, 2] was the first study of boundary layer problem assuming velocity of a boundary sheet as constant. This work is followed by the pioneering work of Tsou *et al.* [3] studied the flow and heat transfer developed by continuously moving surface both analytically and experimentally, in which the flow is caused by an elastic sheet moving in its own plane with a velocity varying linearly with the distance from a fixed point studied by Crane [4]. Chakrabarti and Gupta [5] studied the temperature distribution in this MHD boundary layer flow over a stretching sheet in the presence of suction. There are several extensions to this problem, which include consideration of more general stretching velocity and the study of heat transfer [6-12].

The applications of hydromagnetic incompressible viscous flow in science and engineering involving heat and mass transfer is of great importance to many areas of science and engineering. This frequently occurs in petro-chemical industry, power and cooling systems, chemical vapour deposition on surfaces, cooling of nuclear reactors, heat exchanger design, forest fire dynamics and geophysics as well as in magnetohydrodynamic power generation systems. Many analytical and numerical studies have been conducted to explain the various aspects of boundary layer flow with heat and mass transfer over flat surfaces using both Darcian and non-Darcian models for the porous medium drag effects. Raptis *et al.* [13] steady for the MHD asymmetric flow of an electrically conducting fluid past a semi-infinite stationary plate. Liu [14] analyzed the hydromagnetic fluid flow past a stretching sheet in presence of a uniform transverse magnetic field. Chen [15] investigated the fluid

flow and heat transfer on a stretching vertical sheet, and his work has been extended by Ishak *et al.* [16] to hydromagnetic flow and they found that as the magnetic field increases, the surface skin friction as well as the surface Nusselt number decreases.

The study of two-dimensional boundary layer flow, heat and mass transfer over a porous stretching surface is very important as it finds many practical applications in different areas. To be more specific, it may be pointed out that many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid and that in the process of drawing these strips, are sometimes stretched. Makinde [17] investigated the free convection flow with radiation and mass transfer past a moving vertical porous plate. Same author [18] studied the effects of mass transfer on MHD boundary layer flow past a vertical plate embedded in porous medium in presence of constant heat flux. The problem of magnetohydrodynamic natural convection about a vertical stretching sheet embedded in porous medium can be found in Pop and Postelnicu [19]. Chamkha [20] analyzed the steady hydromagnetic two-dimensional flow and heat transfer in a stationary electrically-conducting and heat-generating fluid driven by a continuously moving porous surface immersed in a fluid-saturated porous medium. It was shown the heat transfer characteristics can be enhanced by the porous medium. Abbas and Hayat [21] investigated the magnetohydrodynamics boundary layer flow in a porous space.

The radiative effects have important applications in physics and engineering particularly in space technology and high temperature processes by Mukhopadhyay and Layek [22]. Effects of radiation have been studied by Abdul Hakeem and Sathiyathan [23], Seddeek and Abdelmeguid [24], Mamun Molla and Anwar Hossain [25], Chen [26], Cortell [27], Sajid and Hayat [28] and Bataller [29]. Ouaf [30] obtained an exact solution of thermal radiation on magnetohydrodynamics flow over a stretching porous sheet.

The heat source/sink effects in thermal convection are significant where there many

exist a high temperature differences between the surface (e.g. space craft body) and the ambient fluid. Heat generation is also important in the context of exothermic or endothermic chemical reaction. The thermal radiation and heat generation effects on MHD convective flow is new dimension added to the study of stretching surface has important applications in physics and engineering particularly in space technology and high temperature processes such as it plays an important role in controlling the heat transfer process in polymer processing industry. Dulal Pal [31] has investigated the combined effects of non-uniform heat source/sink and thermal radiation on heat transfer over a stretching surface. Chamkha and Camille [32] studied the effect of heat generation and thermo phoresis on MHD flow with heat and mass transfer over a stretching sheet. Moalem [33] investigated steady state heat transfer within porous medium in presence of temperature depended and heat generation.

Chemical reactions usually accompany a large amount of exothermic and endothermic reactions. These characteristics can be easily seen in a lot of industrial processes. Recently, it has been realized that it is not always permissible to neglect the convection effects in porous constructed chemical reactors by Nield and Bejan [34]. The reaction produced in a porous medium was extraordinarily in common, such as the topic of PEM fuel cells modules and the polluted underground water because of discharging the toxic substance, etc. Afify and Ahmed [35] found the chemical reaction effect on free convective flow over a stretching sheet. Seddeek [36] proposed the effects of chemical reaction and heat generation/absorption on a boundary layer MHD flow over a heat surface.

However the interaction of radiation with mass transfer of an electrically conducting and diffusing fluid past a stretching surface has received little attention. Hence an attempt is made to investigate the radiation effects on a steady free convection flow over a vertical stretching sheet in presence of magnetic field, heat generation and chemical reaction. The

governing equations are transformed by using similarity transformation and the resultant dimensionless equations are solved numerically using the Runge-Kutta fourth order method with shooting technique. The effects of various governing parameters on the velocity, temperature, concentration, skin-friction coefficient, Nusselt number and Sherwood number are shown in figures and tables and analyzed in detail.

2. FORMATION OF THE PROBLEM

A steady two- dimensional free convection boundary layer flow over a vertical stretching surface in an incompressible, viscous and electrically conducting fluid in the presence of a transverse magnetic field, thermal radiation, heat generation and chemical reaction embedded in porous media is considered. The coordinate x is being taken along the stretching sheet and y is normal to the surfaced, two equal and opposite forces are applied along the x -axis, so that the sheet is stretched, keeping the origin fixed. The stretching surface is assumed to have the velocity of the form $U(x) = ax$ where a is a constant. The ambient temperature T_∞ , surface temperature T_w , the ambient concentration C_∞ and the surface concentration C_w are assumed to be constant. A uniform transverse magnetic field of strength B_0 is applied parallel to the y –axis and the chemical reaction is taking place in the flow. The viscous dissipation effect and Joule heat are neglected on account of the fluid is finitely conducting. It is assumed that the induced magnetic field, the external electric field and the electric field due to the polarization of charges are negligible. The density variation and the effects of the buoyancy are taken into account in the momentum equation (Boussinesq's approximation) and the concentration of species far from the wall is infinitesimally small and the viscous dissipation term in the energy equation is neglected (as the fluid velocity is very low). Under these assumptions, the governing boundary layer equations of momentum, energy and diffusion under Boussinesq approximations could be written as follows:

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2.1)$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{K^*} u \quad (2.2)$$

Energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + \frac{Q_0}{\rho C_p} (T - T_\infty) \quad (2.3)$$

Species equation

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - Kr^*(C - C_\infty) \quad (2.4)$$

where u, v are velocity components, T and C are temperature and concentration of fluid, ν is the kinematic viscosity, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, β^* is the volumetric concentration coefficient, B_0 is the magnetic induction, ρ is the fluid density, σ is the fluid electrical conductivity, K^* is the permeability coefficient of porous medium, k is the thermal conductivity, q_r is the radiative heat flux, C_p is the specific heat at constant pressure, Q_0 is the dimensional heat generation/absorption coefficient, D is the mass diffusivity and Kr^* is the chemical reaction parameter.

The boundary conditions for velocity, temperature and concentration fields are

$$\begin{aligned} u = U(x) = ax, \quad v = 0, \quad T = T_w, \quad C = C_w \quad \text{at } y = 0 \\ u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty \end{aligned} \quad (2.5)$$

Using the Rosseland approximation [37], the radiative heat flux

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \quad (2.6)$$

where σ^* is the Stefan-Boltzman constant and k^* is the absorption coefficient. The temperature differences within the flow are assumed to be sufficiently small such that T^4 may be expressed as a linear function of temperature, (T^4 may be expanded in a Taylor's series, Expanding T^4 about T_∞ and neglecting higher orders we get)

$$T^4 \equiv 4T_\infty^3 T - 3T_\infty^4 \quad (2.7)$$

Substituting (2.6) and (2.7) into (2.3) yield

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha(1+R) \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p} (T - T_\infty) \quad (2.8)$$

where $\alpha = \frac{k}{\rho C_p}$, $R = \frac{16\sigma^* T_\infty^3}{3k^* k}$ is the radiation parameter.

We introduce the similarity transformation as follows:

$$u = axf'(\eta), v = -\sqrt{av} f(\eta), \quad \eta = \sqrt{\frac{a}{v}} y, \quad \psi = \sqrt{av} xf(\eta)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (2.9)$$

where η is the similarity variable, a is the positive stretching velocity, ψ is the stream function f, θ and ϕ are dimensionless stream function, temperature and concentration respectively. The stream function ψ is defined by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (2.10)$$

with these changes of variables equation (2.1) is identically satisfied and equations (2.2), (2.8) and (2.4) are transformed to nonlinear ordinary differential equations as follows

$$f''' + ff'' - f'^2 + Gr\theta + Gc\phi - (M+K)f' = 0 \quad (2.11)$$

$$\frac{1}{\text{Pr}}(1+R)\theta'' + f\theta' + Q\theta = 0 \quad (2.12)$$

$$\phi'' + Scf\phi' - ScKr\phi = 0 \quad (2.13)$$

The corresponding boundary conditions take the form:

$$f = 0, f' = 1, \theta = 1, \phi = 1 \quad \text{at } \eta = 0$$

$$f' = 0, \theta = 0, \phi = 0 \quad \text{as } \eta \rightarrow \infty \quad (2.14)$$

Where primes denote differentiation with respect to η and $Gr = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2}$ is the

Grashof number, $Gc = \frac{g\beta^*(C_w - C_\infty)x^3}{\nu^2}$ is the modified Grashof number, $M = \frac{\sigma B_0^2}{\rho a}$ is

magnetic parameter, $K = \frac{\nu}{K^*a}$ is permeability parameter, $\text{Pr} = \frac{\nu}{\alpha}$ is the Prandtl number,

$R = \frac{16\sigma^*T_\infty^3}{3k^*k}$ is the radiation parameter, $Q = \frac{Q_0}{a\rho C_p}$ is the heat generation coefficient, $Sc = \frac{\nu}{D}$

is the Schmidt number and $Kr = \frac{Kr^*}{a}$.

The important physical quantities of our interest are the local skin friction C_f , the Nusselt

number Nu_x and the Sherwood number Sh_x defined by

$$C_f = \frac{\tau_w}{\rho U^2 / 2}, Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, Sh_x = \frac{xm_w}{k(C_w - C_\infty)} \quad (2.15)$$

Respectively, where the surface shear stress τ_w , the surface heat flux q_w and the mass flux

m_w are given by

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}, m_w = -k \left(\frac{\partial C}{\partial y} \right)_{y=0} \quad (2.16)$$

with μ and k being the dynamic viscosity and thermal conductivity, respectively.

Using non-dimensional variable in (2.9) we obtain

$$\frac{1}{2}C_f \text{Re}_x^{1/2} = f''(0), \quad Nu_x / \text{Re}_x^{1/2} = -\theta'(0), \quad Sh_x / \text{Re}_x^{1/2} = -\phi'(0) \quad (2.17)$$

where $\text{Re}_x = \frac{Ux}{\nu}$ is the local Reynolds number.

3. NUMERICAL SOLUTION OF THE PROBLEM

The nonlinear ordinary differential equations (2.11) – (2.13) subjects to the boundary conditions (2.14) are solved numerically using the Runge-Kutta fourth order technique along with shooting method. In this method, it is important to choose the appropriate finite value of the edge of boundary layer, $\eta \rightarrow \infty$, that is 6, which is in accordance with the standard practice in the boundary layer analysis. First of all, higher order non-linear differential equations (2.11) – (2.13) are converted into simultaneous linear differential equations and they are further transformed into initial value problem by applying the shooting method. The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The initial step size $h = \Delta\eta = 0.01$ is used to obtain the numerical solution with six decimal place accuracy as the criterion convergence.

4. RESULTS AND DISCUSSION

In order to get a physical insight into the problem, a representative set of numerical results are shown in Figures 1 – 9, which illustrates the influence of physical parameters viz., Grashof number Gr , modified Grashof number Gc , Magnetic field parameter M , Permeability of porous medium K , Prandtl number Pr , Radiation parameter R , Heat generation coefficient Q , Schmidt number Sc and chemical reaction parameter Kr on the velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$ profiles. Throughout the calculations, the parametric values are fixed to be $Pr = 0.71$, $M = 1.0$, $Gr = 1.0$, $Gc = 1.0$, $K = 1.0$, $R = 1.0$,

$$Q = 0.1, Sc = 0.6, Kr = 0.5.$$

Fig.1. shows the variation of the dimensionless velocity component f' for several sets of values of thermal Grashof number Gr . The thermal Grashof number Gr signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity as Gr increases.

The variation of the dimensionless velocity component f' for several sets of values of solutal Grashof number Gc is depicted in Fig.2. The solutal Grashof number Gc , defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases as modified Grashof number (Gc) increases.

For various values of the magnetic parameter M , the dimensionless velocity component f' is plotted in Fig.3 (a). It can be seen that as M increases, the velocity decreases. As M increases, the Lorentz force, which opposes the flow, also increases and leads to enhanced deceleration of the flow. This result qualitatively agrees with the expectations, since the magnetic field exerts a retarding force on the free convection flow. Figs. 3(b)-3(c) shows that the dimensionless temperature and concentration profiles for different values of magnetic parameter M . It is observed that the temperature and concentration increases with an increase in the magnetic parameter M .

The effect of the permeability parameter K on the velocity field is shown in Figure 4(a). An increase in K will therefore increase the resistance of the porous medium (as the permeability physically become less with increasing) which will tend to decelerate the flow and reduce the velocity. This behavior is evident from Figure 4(a). Figs. 4(b) - 4(c) shows that the dimensionless temperature and concentration profiles for different values of Permeability of porous medium K . It is observed that the temperature and concentration increases with an increase in the magnetic parameter M .

Fig.5 (a). Illustrates the dimensionless velocity component f' for different values of the Prandtl number Pr , the Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity. From Fig.5 (b), it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of Pr . Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

The effect of the Radiation parameter R on the dimensionless velocity component f' and dimensionless temperature are shown in Figs. 6(a) and 6(b) respectively. Fig.6 (a) shows that velocity component f' increases with an increase in the radiation parameter R . From Fig.6 (b) it is seen that the temperature increases as the radiation parameter R increases.

Figs. 7(a) and 7(b) depict the dimensionless velocity f' and temperature profiles for different values of the heat generation parameter Q . It is noticed that an increase in the heat generation parameter Q results in an increase in the dimensionless velocity f' and temperature within the boundary layer.

The influence of the Schmidt number Sc on the dimensionless velocity f' and concentration profiles are plotted in Figs. 8(a) and 8(b) respectively. The Schmidt number Sc embodies the ratio of the momentum to the mass diffusivity. The Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects

to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers. These behaviors are clear from Figs. 8(a) and 8(b).

The effects of the chemical reaction parameter Kr on dimensionless velocity component f' and concentration profiles are plotted in Figs. 9(a) and Fig. 9(b). As the chemical reaction parameter number increases the concentration and velocity profiles are decreases. These behaviors are clear from Figs. 9(b) and 9(a).

Tables 1 – 3 represents values of skin-friction coefficient $f''(0)$, Nusselt number $-\theta'(0)$ and Sherwood number $-\phi'(0)$ for various values of Grashof number (Gr), modified Grashof number (Gc), Magnetic field parameter (M), Permeability parameter (K), Prandtl number (Pr), radiation parameter (R), heat generation parameter (Q), Schmidt number (Sc) and chemical reaction parameter (Kr).

From Table 1, it is clear that, with increasing Gr and Gc , skin-friction coefficient, Nusselt number and Sherwood number increases, whereas with increasing M and K , skin-friction coefficient, Nusselt number and Sherwood number decreases.

From Table 2 it is seen that the values of $f''(0)$ are always negative. Physically, negative sign of $f''(0)$ implies that the stretching sheet exerts a drag force on the fluid that cause the movement of the fluid on the surface. The absolute values of skin friction coefficient and the Nusselt number increase with increasing of Prandtl number and heat generation parameter, whereas with increasing radiation parameter, skin friction coefficient and Nusselt number decrease.

From Table 3, the skin friction coefficient and Sherwood number increase as Schmidt number and chemical reaction parameter increases.

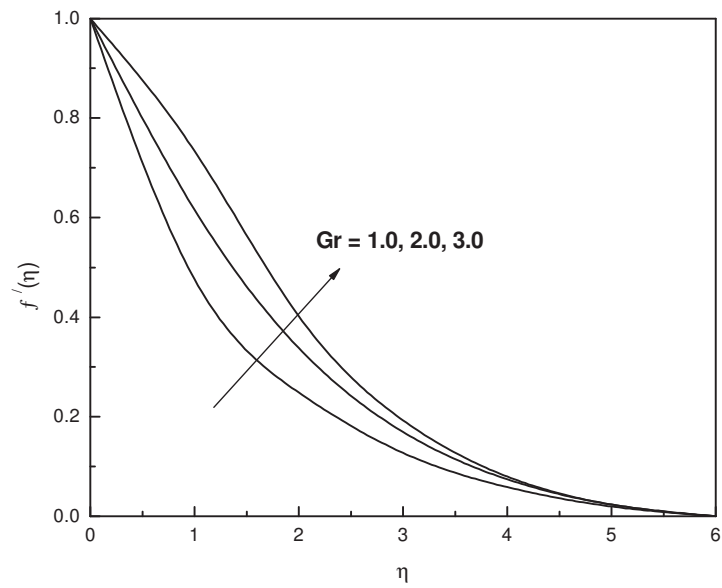


Fig.1 Velocity profiles for various values of Grashof number (Gr)

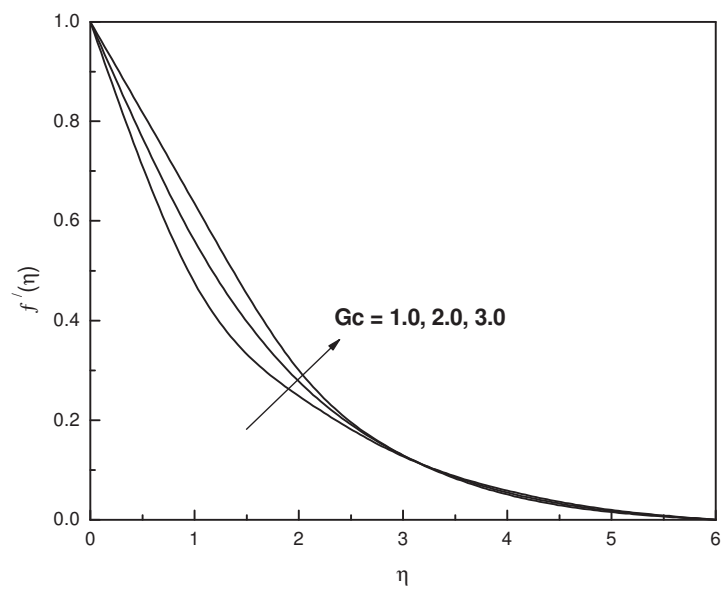


Fig.2 Velocity profiles for various values of modified Grashof number (Gc)

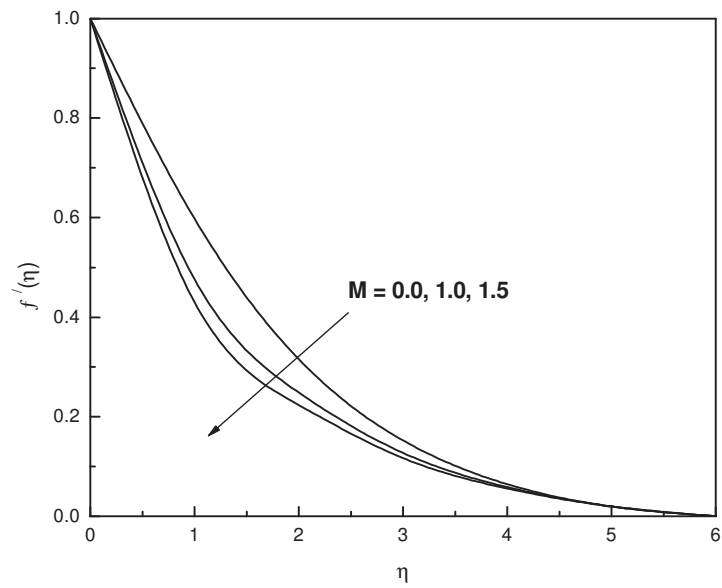


Fig.3 (a) Velocity profiles for various values of Magnetic field parameter (M)

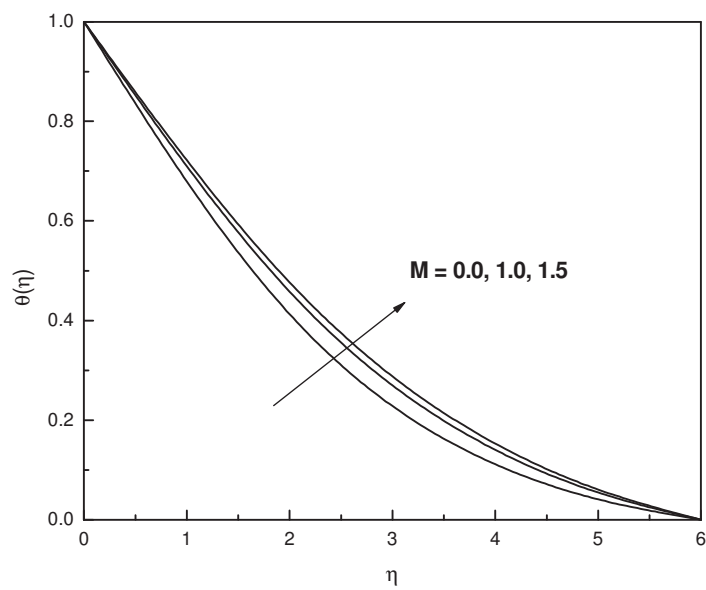


Fig.3 (b) Temperature profiles for various values of Magnetic field parameter (M)

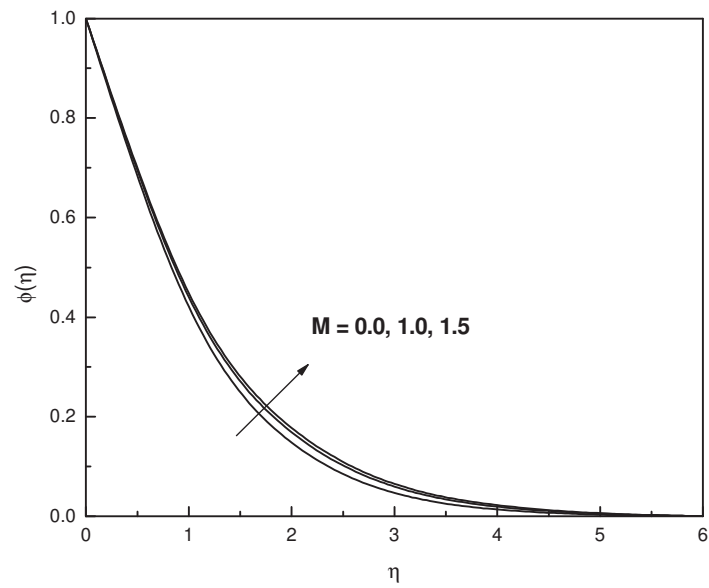


Fig.3(c) Concentration profiles for various values of Magnetic field parameter (M)

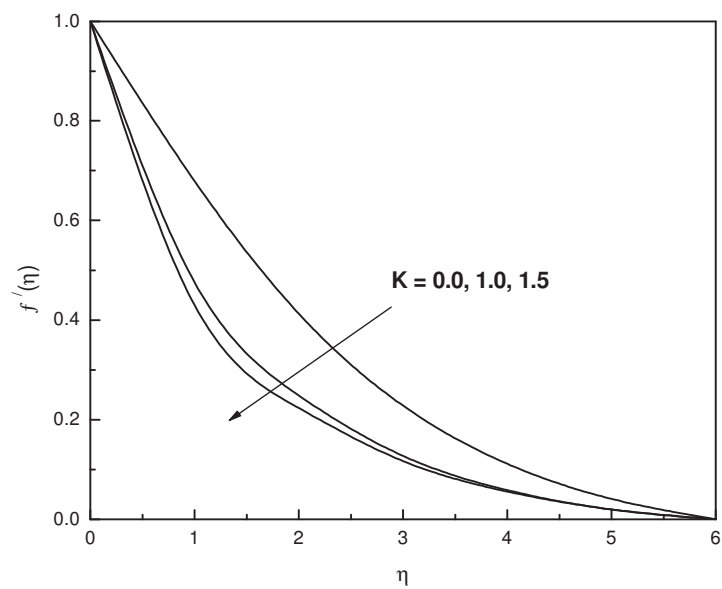


Fig.4 (a) Velocity profiles for various values of Permeability of porous medium (K)

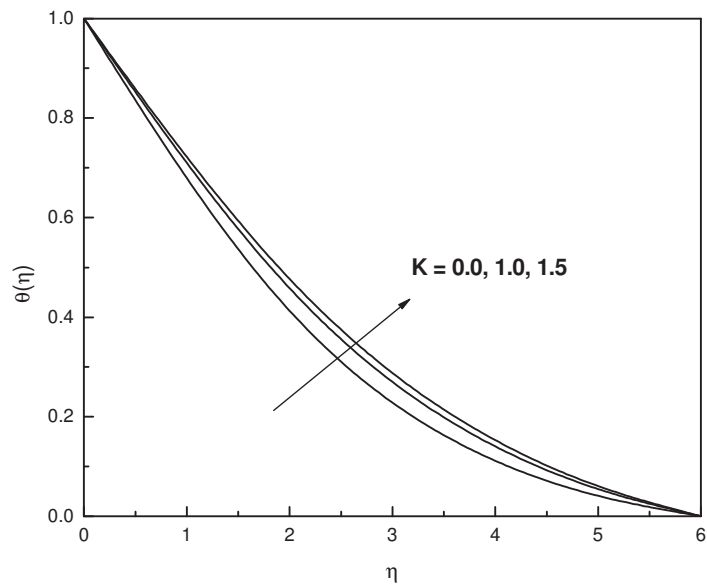


Fig.4 (b) Temperature profiles for various values of Permeability of porous medium (K)

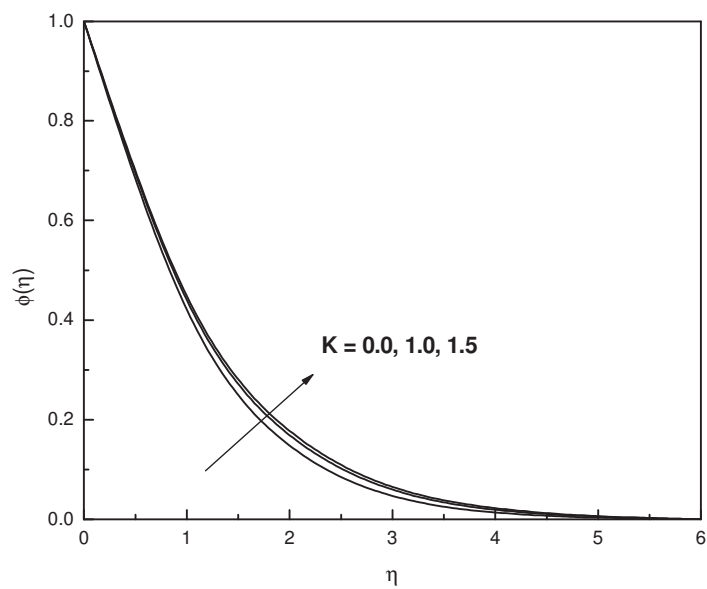


Fig.4(c) Concentration profiles for various values of Permeability of porous medium (K)

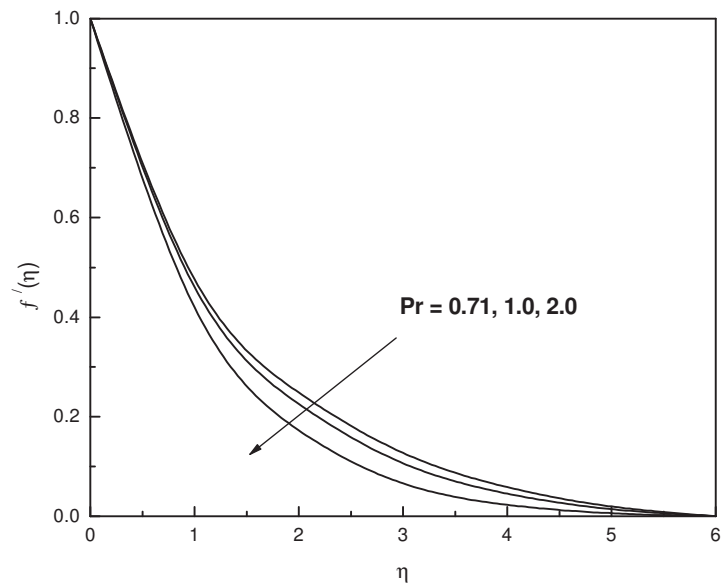


Fig.5 (a) Velocity profiles for various values of Prandtl number (Pr)

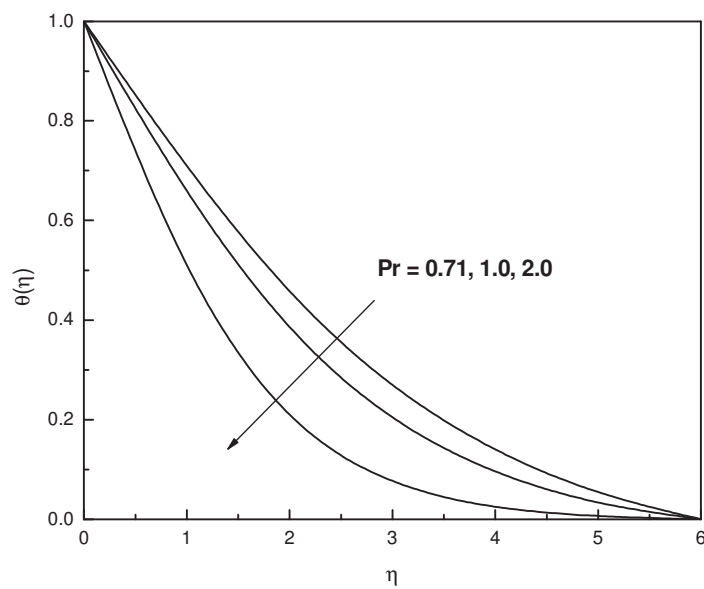


Fig.5 (b) temperature profiles for various values of Prandtl number (Pr)

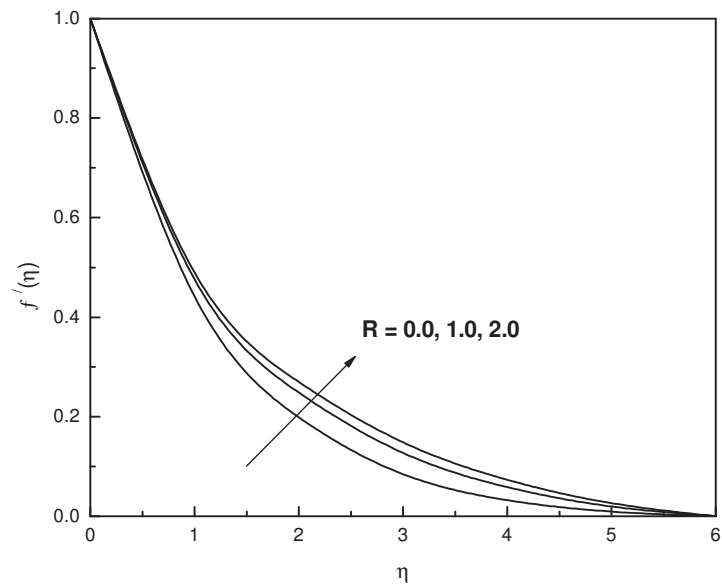


Fig.6 (a) Velocity profiles for various values of Radiation parameter (R)

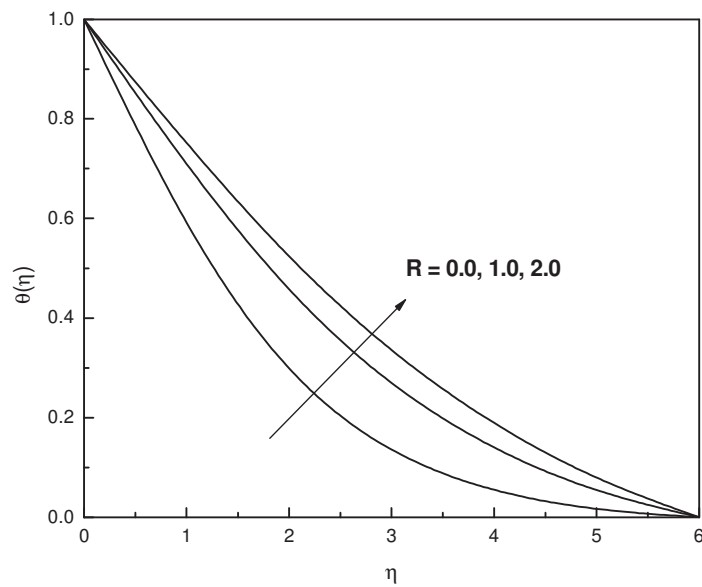


Fig.6 (b) temperature profiles for various values of Radiation parameter (R)

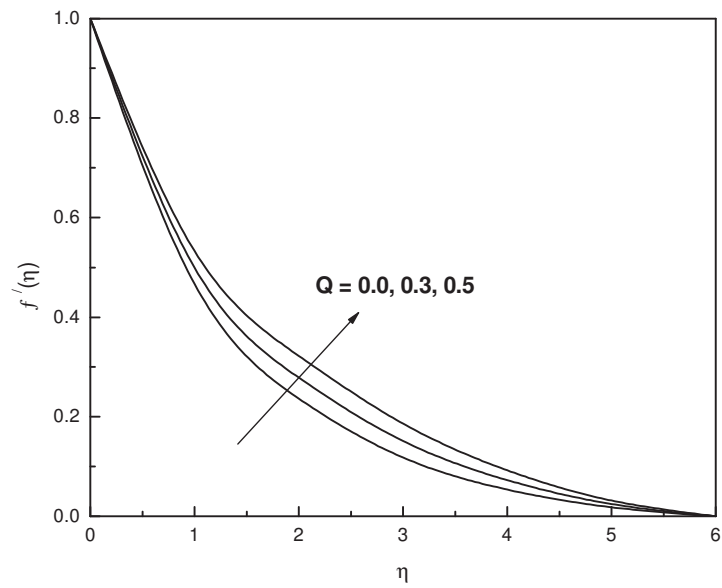


Fig.7 (a) Velocity profiles for various values of Heat generation parameter (Q)

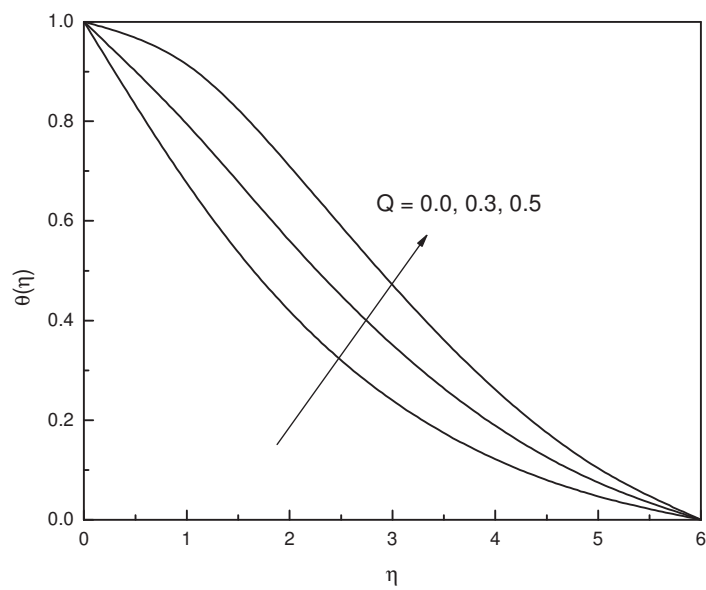


Fig.7 (b) Temperature profiles for various values of Heat generation parameter (Q)

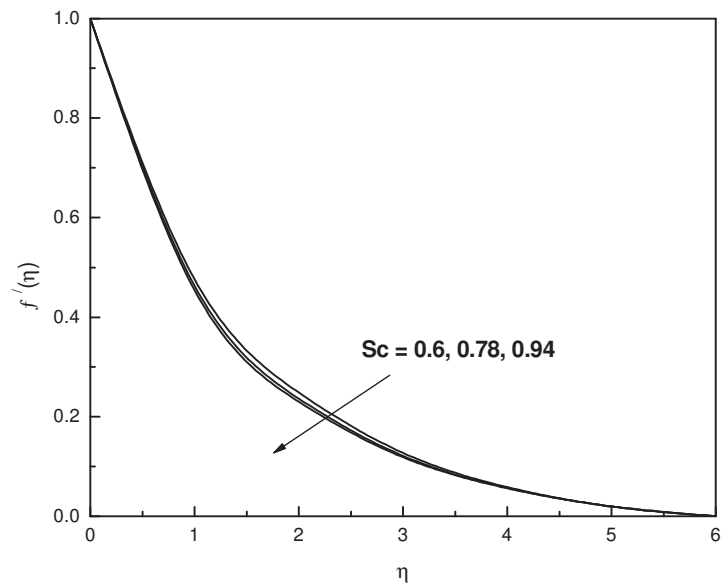


Fig.8 (a) Velocity profiles for various values of Schmidt number (Sc)

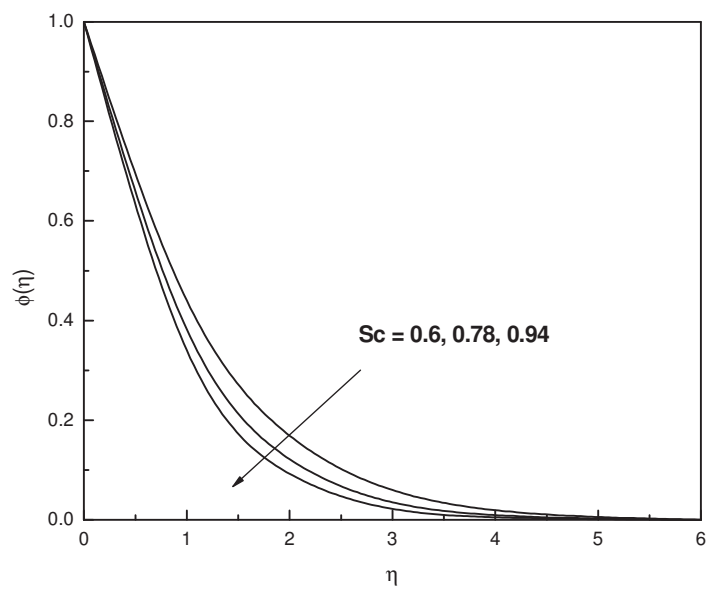


Fig.8 (b) Concentration profiles for various values of Schmidt number (Sc)

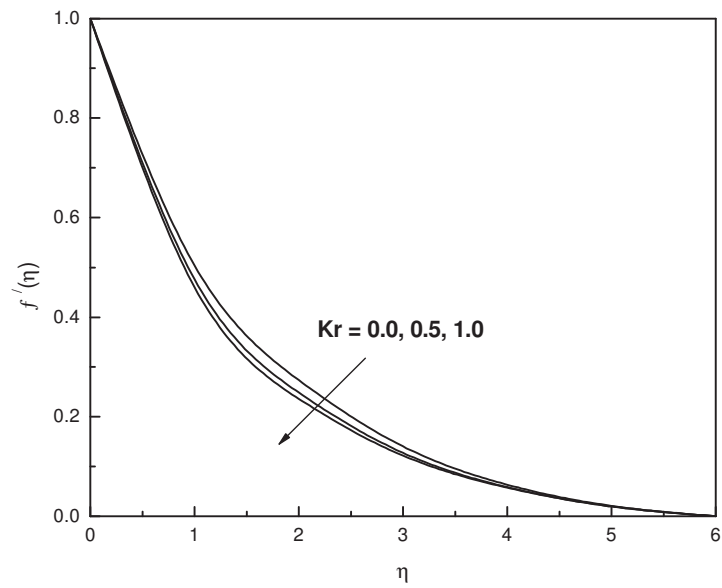


Fig.9 (a) Velocity profiles for various values of Chemical reaction parameter (Kr)

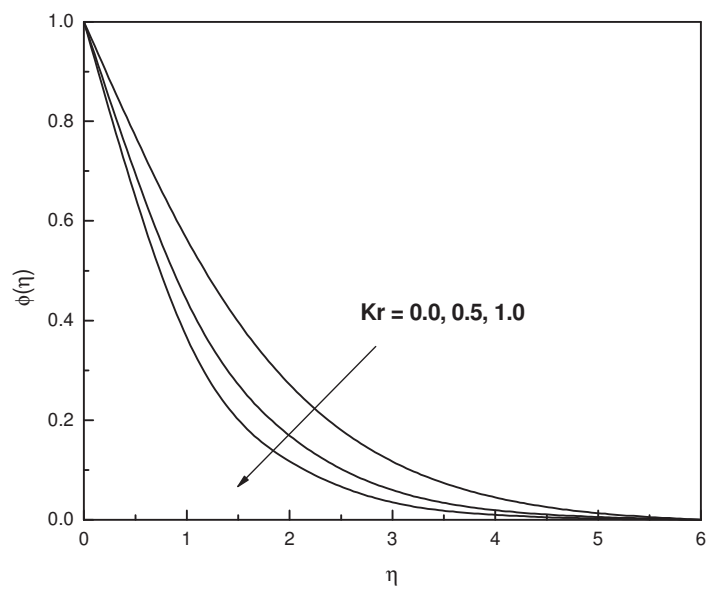


Fig.9 (b) Concentration profiles for various values of Chemical reaction parameter (Kr)

Table 1: Variation of $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ at the plate surface with Gr , Gc , M , K for $Pr = 0.71$, $R = 1.0$, $Q = 0.1$, $Sc = 0.6$, $Kr = 0.5$.

Gr	Gc	M	K	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
1.0	1.0	1.0	1.0	- 0.846647	0.288536	0.72423
2.0	1.0	1.0	1.0	- 0.409379	0.327708	0.750954
3.0	1.0	1.0	1.0	0.0039198	0.358302	0.773292
1.0	2.0	1.0	1.0	- 0.482778	0.310872	0.741598
1.0	3.0	1.0	1.0	- 0.130463	0.330521	0.757314
1.0	1.0	0.5	1.0	- 0.649613	0.304023	0.7352
1.0	1.0	1.5	1.0	- 1.02639	0.275128	0.714704
1.0	1.0	1.0	0.5	- 0.649613	0.304023	0.7352
1.0	1.0	1.0	1.5	- 1.02639	0.275128	0.714704

Table 2: Variation of $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ at the plate surface with Pr , R , Q for $Gr = 1.0$, $Gc = 1.0$, $M = 1.0$, $K = 1.0$, $Sc = 0.6$, $Kr = 0.5$.

Pr	R	Q	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.71	1.0	0.1	- 0.846647	0.288536	0.72423
1.0	1.0	0.1	- 0.86317	0.341957	0.721382
2.0	1.0	0.1	- 0.910845	0.515367	0.71384
0.71	2.0	0.1	- 0.832487	0.245252	0.726762
0.71	3.0	0.1	- 0.825302	0.22418	0.72808
0.71	1.0	0.5	- 0.779911	0.00249327	0.734225
0.71	1.0	1.0	- 1.05328	0.739466	0.672011

Table 3: Variation of $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ at the plate surface with Sc , Kr for $Gr = 1.0$, $Gc = 1.0$, $M = 1.0$, $K = 1.0$, $Pr = 0.71$, $R = 1.0$, $Q = 0.1$, $Sc = 0.6$, $Kr = 0.5$.

Sc	Kr	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.6	0.5	- 0.846647	0.288536	0.72423
0.78	0.5	- 0.865114	0.284725	0.832316
0.94	0.5	- 0.878789	0.282199	0.918849
0.6	1.0	- 0.871969	0.284027	0.910109
0.6	2.0	- 0.905202	0.278958	1.19777

5. REFERENCES

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