CHAPTER III

EXPERIMENTAL RESULTS PART II – USING WAVELET TRANSFORM

Wavelets are mathematical functions that were developed by scientists working in several different fields for the purpose of sorting data by frequency. In few of the respects wavelet is different than other transform coding. [25] Translated data can then be sorted at a resolution which matches its scale. Studying data at different levels allows for the development of a more complete picture. Both small features and large features are discernable because they are studied separately. [30]

3.1 Haar Transform

The Haar wavelet operates on data by calculating the sums and differences of adjacent elements. The Haar wavelet operates first on adjacent horizontal elements and then on adjacent vertical elements. The Haar transform is computed using

\[
\begin{pmatrix}
1 & 1 \\
\sqrt{2} & -1
\end{pmatrix}
\]

One nice feature of the Haar wavelet transform is that the transform is equal to its inverse. As each transform is computed the energy in the data in relocated to the top left hand corner. [31]

3.2 Interband Correlation

Spearman’s rank correlation coefficient does not require the assumption that the relationship between the variables is linear, nor does it require the variables to be measured on interval scales; it can be used for variables measured at the ordinal level. Spearman’s rho does assume that subsequent ranks indicate equidistant positions on the variable measured. In spearman’s rank correlation, the raw scores are converted to ranks, and the differences \(d_i\) between the ranks of each observation on the two variables is calculated. [32]
If there are no tied ranks, $\rho$ is given by:

$$
\rho = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}
$$

Where,

$$\begin{align*}
d_i &= x_i - y_i = \text{the difference between the ranks of corresponding values } X_i \text{ and } Y_i, \\
n &= \text{the number of values in each data set (same for both sets)}.
\end{align*}
$$

If tied ranks exist, classic Pearson’s correlation coefficient between ranks has to be used instead of this formula:

$$
\rho = \frac{n(\sum x_i y_i) - (\sum x_i)(\sum y_i)}{\sqrt{n(\sum x_i^2) - (\sum x_i)^2} \sqrt{n(\sum y_i^2) - (\sum y_i)^2}}
$$

Spearman rank correlation works by converting each variable to ranks. Thus, if you’re doing a Spearman rank correlation of blood pressure vs. body weight, the lightest person would get a rank of 1, second-lightest a rank of 2, etc. The lowest blood pressure would get a rank of 1, second lowest a rank of 2, etc. When two or more observations are equal, the average rank is used. For example, if two observations are tied for the second-highest rank, they would get a rank of 2.5 (the average of 2 and 3). Once the two variables are converted to ranks, a correlation analysis is done on the ranks. The correlation coefficient is calculated for the two columns of ranks, and the significance of this is tested in the same way as the correlation coefficient for a regular correlation. (This Spearman’s correlation coefficient is also called Spearman’s rho). The $\rho$-value from the correlation of ranks is the $\rho$-value of the Spearman rank correlation.

An alternative approach available for sufficiently large sample sizes is an approximation to Student’s t-distribution with degrees of freedom N-2. For sample sizes above about 20, the variable
\[
    t = \frac{\rho}{\sqrt{(1 - \rho^2)/(n - 2)}}
\]

\[
    \rho = \frac{t}{\sqrt{n - 2 + t^2}}
\]

has a Student’s \( t \)-distribution in the null case (zero correlation). In the non-null case (i.e. to test whether an observed \( \rho \) is significantly different from a theoretical value, or whether two observed \( \rho \)s differ significantly) tests are much less powerful, though the \( t \)-distribution can again be used. So we can make the efficient use of interband correlation before applying wavelets for better results for image compression. [33]

### 3.3 System Architecture and implementation

#### 3.3.1 System Architecture

The detailed explanation of the steps of architecture [34] is given below:

1. LANDSAT 7 Multispectral data which is 3-dimensional data (usually 512x512x7 Bands images) is read into the MATLAB using “multibandread” command. LANDSAT 7 multispectral images which are used for the research work are in .lan format.

2. Now using inbuilt MATLAB command known as “multibandread” which is a command to read band-interleaved data from binary file, we can read .lan images and display all The 7 bands of the .lan Multispectral image. While reading the .lan image we specify the name of the image, number of rows, number of columns and number of bands of that image. We also specify the format of the data to be read as along with the interleaved which could be BSQ, BIL or BIP.

3. After the initial analysis of Multispectral image we use Spearman’s Correlation Ratio methodology to find the inter-band correlation in the given image. This would help us in discarding the uncorrelated bands and selecting the highly correlated bands with respect to our domain of interest. We get a matrix which is
a 7x7 matrix in which the correlation ratio of a band to itself is 1 which with other bands it could be from -1 to +1. The value -1 indicates that there is no correlation

Figure 41: Proposed architecture for compression using wavelet
between the two bands and +1 means two bands are highly correlated. For our research we have selected the bands whose correlation ratios are above 0.8 with respect to the band of interest. The correlation matrix also helps to decide whether our assumption about the selection of bands for fusion is correct or not.

4. Once we know the bands which contain that information, the next step is to apply wavelet compression using ‘HAAR’ filter. After selecting the wavelet transform we need to select the level of decomposition i.e. from Level 1 to Level 5. The output of this stage is image and its various decomposed images at Approximation, Horizontal details, Vertical details and Diagonal Details. The embedded image coding using zerotress can also be applied to wavelet coefficients [35] or the approximation [36] of coefficients can be done.

5. Finally to calculate the Error that would exist in the reconstructed image after applying compression technique to the fused image we have calculated various Error Metrics like Mean Absolute Error (MAE), Mean Square Error (MSE), Root Mean Squared Error (RMSE) and Peak Signal to Noise Ratio (PSNR) for all the images.

### 3.3.2 Implementation

**Input Data: [20]**

This section gives the insight about the implementation of Compression of Multispectral Remote Sensing Images. The platform used for implementation is MATLAB. MATLAB is a great tool for simulation and data analysis; it has many inbuilt toolboxes and easy visualization. MATLAB is an interpreter by default although there is a possibility of compile. There are two modes command line mode and script. There is a help feature which allows user to select the pre-defined functions and toolboxes.
The user can select amongst the images mentioned below:

1. artha_s.lan : 1.793 MB (Martha Island)
   Lines: 512  Number of bands: 7
   Columns: 512  Band Format: BIL
   Geographic Information: UTM zone 19N
   Pixel size=30 meter including the thermal band which is usually 60 meter resolution. The thermal band was resampled to create the .lan file.

2. njisl_s.lan : 1.793 MB (New Jersey)
   Lines: 512  Number of bands: 7
   Columns: 512  Band Format: BIL
   Geographic Information: UTM zone 18N
   Pixel size=30 meter including the thermal band which is usually 60 meter resolution. The thermal band was resampled to create the .lan file.

3. phila_s.lan : 1.793 MB (Philadelphia)
   Lines: 512  Number of bands: 7
   Columns: 512  Band Format: BIL
   Geographic Information: UTM zone 18N
   Pixel size=30 meter including the thermal band which is usually 60 meter resolution. The thermal band was resampled to create the .lan file.

4. bostos.lan : 1.793 MB (Boston)
   Lines: 512  Number of bands: 7
   Columns: 512  Band Format: BIL
Geographic Information: UTM zone 19N
Pixel size=30 meter including the thermal band which is usually 60 meter resolution. The thermal band was resampled to create the .lan file.

5. orlea_s.lan : 1.793 MB (New Orleans)
   Lines: 512
   Number of bands: 7
   Columns: 512
   Band Format: BIL
   Geographic Information: UTM zone 19N
   Pixel size=30 meter including the thermal band which is usually 60 meter resolution. The thermal band was resampled to create the .lan file.

**Bands of Multispectral Image**

The next stage is to display all the bands of the input Multispectral image orlea_s.lan) along with their Histograms. A program is written in MATLAB that makes the use of *multibandread* function to read .lan multispectral images. While reading the .lan image we specify the name of the image, number of rows, number of columns and number of bands of that image. We also specify the format of the data to be read along with interleave, that could be BSQ, BIL or BIP. A Histogram is plotted for every band using ‘*imhist*’ as shown in Figure 42.
Inter-band Correlation

The Interband correlation for the given Multispectral image provides us with the insight about the degree of correlation that exists between the various bands. Spearman’s Correlation ratio provides us the degree of correlation that exists between the bands. The degree of correlation could be between -1 to +1, where value near to -1 indicates that there is no correlation between the two bands and value nearer to +1 indicates that the bands are highly correlated. [32]

Procedure for using Spearman’s Correlation

- State the null hypothesis i.e. “There is no relationship between the two sets of data.”
- Rank both sets of data from the highest to the lowest. Make sure to check for tied ranks.
- Subtract the two sets of ranks to get the difference \( d \).
- Square the values of \( d \).
- Add the squared values of \( d \) to get \( \Sigma d^2 \).
- Use the formula \( Rs = 1 - \frac{6 \Sigma d_i^2}{(n^3 - n)} \) where \( n \) is the number of ranks you have.
- If the \( Rs \) value….
  - Is -1, there is a perfect negative correlation.
  - Falls between -1 and -0.5, there is a strong negative correlation.

---

**Figure 42**: Band 1 to Band 7 of Multispectral image of New Orleans City *(orlea_s.lan)*, along with their respective histogram.
- Falls between -0.5 and 0, there is a weak negative correlation.
- Is 0, there is no correlation
- Falls between 0 and 0.5, there is a weak positive correlation.
- Falls between 0.5 and 0.75, there is a positive correlation
- Falls between 0.75 and 1, there is a strong positive correlation
- Is 1, there is a perfect positive correlation between the 2 sets of data.

If the Rs value is 0, state that null hypothesis is accepted. Otherwise, say it is rejected.

**Table 9: Interband Correlation Ratio of New Orleans City (orlea_s.lan)**

*Multispectral image using Spearman's Correlation Ratio*

<table>
<thead>
<tr>
<th>Band</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1</td>
<td>0.8638</td>
<td>0.8386</td>
<td>0.3621</td>
<td>0.3198</td>
<td>0.2301</td>
</tr>
<tr>
<td>2</td>
<td>0.8638</td>
<td></td>
<td>1</td>
<td>0.8850</td>
<td>0.4580</td>
<td>0.3985</td>
<td>0.2403</td>
</tr>
<tr>
<td>3</td>
<td>0.8386</td>
<td>0.8850</td>
<td></td>
<td>1</td>
<td>0.5390</td>
<td>0.4736</td>
<td>0.2614</td>
</tr>
<tr>
<td>4</td>
<td>0.3621</td>
<td>0.4580</td>
<td>0.5390</td>
<td></td>
<td>1</td>
<td>0.6218</td>
<td>0.1512</td>
</tr>
<tr>
<td>5</td>
<td>0.3198</td>
<td>0.3985</td>
<td>0.4736</td>
<td>0.6218</td>
<td></td>
<td>1</td>
<td>0.2580</td>
</tr>
<tr>
<td>6</td>
<td>0.2301</td>
<td>0.2403</td>
<td>0.2614</td>
<td>0.1512</td>
<td>0.2580</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>0.4436</td>
<td>0.4857</td>
<td>0.5562</td>
<td>0.4830</td>
<td>0.8339</td>
<td>0.3016</td>
<td></td>
</tr>
</tbody>
</table>

**Selecting Domain of Interest and Discrete Wavelet Transform**

Selecting the domain of interest may vary from a person to person, depending upon what information he requires to analyze from the Multispectral Image. Every band of a Multispectral image has certain information content. Given below in Table 10 is the description of various bands of Landsat 7 Multispectral Image. For example if we want to analyze forest classification from the given Multispectral image from the Table 10 we can analyze the Forest classification information is present in Band 9, and with the help of the Table 4 which shows the interband correlation we can know which bands are correlated with Band 4 and the bands which are least correlated so that we can discard them during the Fusion process.
Table 10: Description & Classification of Bands of LANDSAT 7 Multispectral image.

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Identification</th>
<th>Bands</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Costal water mapping</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Soil/vegetation discrimination</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Forest classification</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Man-made feature identification</td>
<td>1 2 3</td>
</tr>
<tr>
<td>5</td>
<td>Vegetation discrimination &amp; health monitoring</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>Plant species identification</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>Soil moisture monitoring</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>Vegetation monitoring</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>Water body discrimination</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>Vegetation moisture content monitoring</td>
<td>5 7</td>
</tr>
<tr>
<td>11</td>
<td>Surface temperature</td>
<td>6</td>
</tr>
<tr>
<td>12</td>
<td>Vegetation stress monitoring</td>
<td>6</td>
</tr>
<tr>
<td>13</td>
<td>Cloud differentiation</td>
<td>6</td>
</tr>
<tr>
<td>14</td>
<td>Volcanic monitoring</td>
<td>6</td>
</tr>
<tr>
<td>15</td>
<td>Mineral and rock discrimination</td>
<td>7</td>
</tr>
</tbody>
</table>

For the implementation process we have used the Identification of Man-Made features from the New Orleans City (NY.lan) Multispectral image. So from Table 10 we can see that the spectra for Man-made Feature identification are present in Band 1, Band 2 and Band 3. Further analyzing Table 9 of Interband Correlation of New York City image we can see that Correlation ratio between Band 1, Band 2 and Band 3 is high so these bands, but the correlation with Band 6 and Band 7 is also high, but for our research work we take only those bands whose correlation ratio is above 0.80, hence we do not consider Band 6 and Band 7 for our further analysis.

Now since the domain of interest is selected, and we know which bands to analyze, we apply 2-dimensional Wavelet Transform to those bands (in our case it would be Band 1, Band 2 and Band 3). We use Haar Wavelet and the decomposition
can be up to 5 Levels. Figure 43 shows the 2-dimensional DWT Decomposition, when a Domain specific band is given as an input its is decomposed into Approximation, Horizontal Details, Vertical Details and Diagonal details depending upon the level of decomposition. In Figure 44 only single Level Decomposition diagram is show.

A MATLAB program is written using Wavelet Toolbox which is an in-built toolbox in MATLAB which is a command line interface program, which prompts for the Input band, followed by the type of Wavelet family which the user would want to use for analysis and finally the level of decomposition [37] one would require to do. The Wavelet toolbox functions used for our implementation are given below:

- **wavedec2**: wavedec2 is a two-dimensional wavelet analysis function.
  \[ [C, S] = \text{wavedec2}(X, N,’wname’) \]
  returns the wavelet decomposition of the matrix X at level N, using the wavelet named in string ‘wname’ (Haar). Outputs are the decomposition vector C and the corresponding bookkeeping matrix S.

- **dwt2**: dwt2 is a single-level discrete 2-D wavelet transform
  \[ [cA,cH,cV,cD] = \text{dwt2}(X,’wname’) \]
  computes the approximation coefficients matrix cA and details coefficients matrices cH, cV, and cD (horizontal, vertical, and diagonal, respectively), obtained by wavelet decomposition of the input matrix X. The ‘wname’ string contains the wavelet name.

- **wrcoef2**: Reconstruct single branch from 2-D wavelet coefficients
  \[ X = \text{wrcoef2}(’type’,C,S,’wname’,N) \]
  computes the matrix of reconstructed coefficients of level N, based on the wavelet decomposition structure \([C,S]\).

- **idwt2**: Single-level inverse discrete 2-D wavelet transform
  \[ X = \text{idwt2}(cA,cH,cV,cD,’wname’) \]
  uses the wavelet ‘wname’ to compute the single-level reconstructed approximation coefficients matrix X, based on
approximation matrix \( cA \) and details matrices \( cH, cV, \) and \( cD \) (horizontal, vertical, and diagonal, respectively).

![Block Diagram of Two-Dimensional Discrete Wavelet Transform Decomposition](image)

**Figure 43**: Block Diagram of Two-Dimensional Discrete Wavelet Transform Decomposition

### 3.4 Results

After the decomposition different bands of input image to varying decomposition levels yielded the following output as shown in figures. The bands are shown decomposed at level 1, 2 and 3 in figures 44, 45 and 46 respectively. In each decomposition level the coefficients are used from the coefficients obtained in the previous decomposition level.
Figure 44: Level 1 decomposition of Band 1 using Haar Wavelet

Figure 45: Level 2 decomposition of Band 1 using Haar Wavelet
Figure 46: Level 3 decomposition of Band 1 using Haar Wavelet
The above mentioned scheme can be used for the compression of medical images [38] and even can be exploited to be applicable for 4 dimensional images [39] also. If we do not make the use of KLT or interband correlation then we need to apply integer wavelet transform on 3D images [40] as well as for the four dimensional images [41].

As shown in the figures 44, 45 and 46, we can analyze Band 1 for level 4 and level 5 decomposition and other domain specific bands in similar fashion.

ERRPERF (T, P, M) uses T and P, which are target and prediction vectors respectively, and returns the value for M, which is one of several error related performance metrics. T and P can be row or column vectors of the same size. M can be one of the following performance metrics:

- MAE (Mean Absolute Error)
- MSE (Mean Square Error)
- RMSE (Root Mean Squared Error)
- PSNR (Peak Signal to Noise Ratio)

To compute the relevant performance metric, the function uses recursion to first compute one or more error vectors. The function can therefore secondarily be used to compute these error vectors. M can be one of the following:
Calculations of Error Metrics:

The various error metrics have been calculated along with the plot of its graph for the values obtained as shown in Tables 11, 12, 13, 14.

Table 11: Mean Absolute Error for decomposition level 1

<table>
<thead>
<tr>
<th>Band No.</th>
<th>Mean Absolute Error</th>
<th>MAE</th>
<th>MARE</th>
<th>MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAND-1</td>
<td></td>
<td>0.6291</td>
<td>0.0065</td>
<td>0.6511</td>
</tr>
<tr>
<td>BAND-2</td>
<td></td>
<td>0.6591</td>
<td>0.0083</td>
<td>0.8290</td>
</tr>
<tr>
<td>BAND-3</td>
<td></td>
<td>0.9444</td>
<td>0.0125</td>
<td>1.2479</td>
</tr>
<tr>
<td>BAND-4</td>
<td></td>
<td>0.6128</td>
<td>0.0090</td>
<td>0.8995</td>
</tr>
<tr>
<td>BAND-5</td>
<td></td>
<td>1.1595</td>
<td>0.0140</td>
<td>1.4035</td>
</tr>
<tr>
<td>BAND-6</td>
<td></td>
<td>1.1447</td>
<td>0.0207</td>
<td>2.0702</td>
</tr>
<tr>
<td>BAND-7</td>
<td>3.8084e-014</td>
<td>2.5826e-016</td>
<td>2.5826e-014</td>
<td></td>
</tr>
</tbody>
</table>

Figure 47: Mean absolute error
Table 12: Mean Square Error

<table>
<thead>
<tr>
<th>Band No.</th>
<th>MEAN SQUARE ERROR</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSE</td>
<td>MSRE</td>
<td>MSPE</td>
</tr>
<tr>
<td>BAND-1</td>
<td>0.7678</td>
<td>8.4180e-005</td>
<td>0.8418</td>
</tr>
<tr>
<td>BAND-2</td>
<td>0.8537</td>
<td>1.3941e-004</td>
<td>1.3941</td>
</tr>
<tr>
<td>BAND-3</td>
<td>1.7074</td>
<td>3.1650e-004</td>
<td>3.1650</td>
</tr>
<tr>
<td>BAND-4</td>
<td>0.7221</td>
<td>1.8125e-004</td>
<td>1.8125</td>
</tr>
<tr>
<td>BAND-5</td>
<td>2.5923</td>
<td>5.0130e-004</td>
<td>5.0130</td>
</tr>
<tr>
<td>BAND-6</td>
<td>2.4986</td>
<td>0.0011</td>
<td>10.7850</td>
</tr>
<tr>
<td>BAND-7</td>
<td>1.9483e-027</td>
<td>8.8970e-032</td>
<td>8.8970e-028</td>
</tr>
</tbody>
</table>

Figure 48: Mean Square Error
**Table 13: Root Mean Square Error**

<table>
<thead>
<tr>
<th>Band No.</th>
<th>ROOT MEAN SQUARE ERROR</th>
<th>RMSE</th>
<th>RMSRE</th>
<th>RMSPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAND-1</td>
<td></td>
<td>0.8762</td>
<td>0.0092</td>
<td>0.9175</td>
</tr>
<tr>
<td>BAND-2</td>
<td></td>
<td>0.9240</td>
<td>0.0118</td>
<td>1.1807</td>
</tr>
<tr>
<td>BAND-3</td>
<td></td>
<td>1.3067</td>
<td>0.0178</td>
<td>1.7790</td>
</tr>
<tr>
<td>BAND-4</td>
<td></td>
<td>0.8498</td>
<td>0.0135</td>
<td>1.3463</td>
</tr>
<tr>
<td>BAND-5</td>
<td></td>
<td>1.6100</td>
<td>0.0225</td>
<td>2.2390</td>
</tr>
<tr>
<td>BAND-6</td>
<td></td>
<td>1.5807</td>
<td>0.0328</td>
<td>3.2841</td>
</tr>
<tr>
<td>BAND-7</td>
<td></td>
<td>4.4140e-014</td>
<td>2.9828e-016</td>
<td>2.9828e-014</td>
</tr>
</tbody>
</table>

**Figure 49: Root Mean Square Error**
### Table 14: Peak Signal-to-Noise Ratio

<table>
<thead>
<tr>
<th>Band No.</th>
<th>PSNR1</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAND-1</td>
<td>49.2785</td>
</tr>
<tr>
<td>BAND-2</td>
<td>48.8177</td>
</tr>
<tr>
<td>BAND-3</td>
<td>45.8075</td>
</tr>
<tr>
<td>BAND-4</td>
<td>50.1351</td>
</tr>
<tr>
<td>BAND-5</td>
<td>43.4505</td>
</tr>
<tr>
<td>BAND-6</td>
<td>44.1539</td>
</tr>
<tr>
<td>BAND-7</td>
<td>51.1208</td>
</tr>
</tbody>
</table>

**Figure 50:** Peak Signal-to-Noise Ratio for all the 7 bands.