The study of atmospheric flows has great significance in Geophysical fluid dynamics. The formation of stagnation zones near a mountain barrier and the occurrence of finite amplitude waves in the lee-side of it, have lead several authors to study the flow aspects of a nonhomogeneous fluids, in the last two decades. It is well established that when the fluid is nonhomogeneous the buoyancy force which is nonconservative supports internal gravity motion. Basically, in the nonhomogeneous case, nonuniformity in density has two effects (1) Inertial effect (2) Gravity effect. The ratio of a characteristic inertial force to the buoyancy force determines whether the inertial effect and the gravity effect are equally important or one of them dominates over the other. In terms of a characteristic velocity $U$ and a length $l$, this ratio reduces to $U^2/g l$ and is defined as the 'Freude number $F$'. In the extreme case ($F > 1$) when inertial effect dominates over the gravity effect, the totality of solutions for the motion of an incompressible density stratified fluid is completely equivalent to the totality of solutions for a homogeneous fluid, provided the flow is steady and the fluid is inviscid and non-diffusive. In slow
steady flows, the gravity effect dominates over the inertial effect. Such slow steady flow of an inviscid, non-diffusive density-stratified fluid exhibits an interesting and remarkable property. For example, consider a steady flow of an inviscid fluid with inertial terms so small compared to others that they can be neglected. The equations of motion are then

\[
\frac{\partial p}{\partial x} = 0, \quad \frac{\partial p}{\partial y} = 0, \quad -\frac{\partial p}{\partial z} - \rho g = 0
\]

The equation of incompressibility is

\[
u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} = 0
\]

Differentiating the equations of motion with respect to \(x\) and \(y\), it follows that

\[
\frac{\partial \rho}{\partial x} = 0 \quad \text{and} \quad \frac{\partial \rho}{\partial y} = 0
\]

Hence from the incompressibility equation we obtain

\( w = 0 \)

This shows that the stratification inhibits vertical motion and the motion can be thought of as the motion of several homogeneous layers piled on one another. If the motion is also two-dimensional, the continuity equation reduces to

\[
\frac{\partial u}{\partial x} = 0 \Rightarrow u = u(z)
\]

Thus if a cylinder is dragged slowly
and steadily through a stratified fluid in the horizontal direction, the fluid before and behind it will be dragged as if it is a solid. In other words, the fluid remains stationary relative to the cylinder. This phenomenon where stagnation zone occurs upstream and even downstream from a barrier or a moving body, is known as 'Blocking Phenomenon'. This phenomenon can also be observed in an inviscid homogeneous rotating fluid. This interesting feature was first predicted by Proudman (15) and confirmed experimentally by Taylor (20). Taylor (20) towed an obstacle slowly across the bottom of a rotating tank, and found that the fluid contained within the circumscribing cylinder gets trapped there and moves with the body as a unit. The flow about the circumscribing cylinder is much the same as if the entire pillar were an impermeable solid. The column of fluid trapped within the circumscribing cylinder is referred as the Taylor - Proudman column. The appearance of the blocked column of fluid in steady two-dimensional flow of an inviscid density-stratified fluid and in steady rotating flow of an inviscid homogeneous fluid, evidently shows that there exists an analogy between the two flows. In fact this analogy can be clearly established in the unsteady case, the details of which are discussed later.

2. BLOCKING IN STEADY FLOWS:

In dealing with the above flows, Long (11,12), Yih (23,24),
Fraenkel (7) etc., based their work on the steady non-linear
equations of motion, which reduce to a linear equation if
uniform undisturbed upstream conditions are assumed. The
assumption of a uniform undisturbed upstream flow lead to the
invalidity of the steady linear model for very small values of
the governing parameters (either the Rossby or the Internal
Froude number), owing to the occurrence of the upstream in-
fluence, and the existence of one-dimensional (or geostrophic)
flow both upstream and downstream. The emergence of a
stagnation zone in the stratified case for small internal
froude number can be clearly understood by considering a sim-
ple configuration of two-dimensional flow into a line sink
(Yih ) using steady linear model (Long's model). Under the
Yih's transformations the governing equation for the asso-
ciated stream function in non-dimensional form is
\[ \nabla^2 \psi + \nabla^2 \psi = F \eta \]
where \( F = \frac{U}{\sqrt{g \cdot f}} \) is the
Internal Froude number. The associated velocity is assumed
to be uniform for upstream.

Defining the cartesian system \( O(\xi, \eta) \) in the non-dimen-
sional form such that the line sink is situated in the hori-
Zontal plane \( \eta = 0 \) at the origin, in view of the symmetry of
the flow the boundary conditions reduce to
\[
\begin{align*}
\psi &= 0 \quad \text{at} \quad \eta = 0 \\
\psi &= 1 \quad \text{at} \quad \eta = 1 \quad \text{and} \quad \xi = 0, \quad 0 < \eta < 1
\end{align*}
\]
The upstream condition is
\[ \psi = \eta \quad \text{at} \quad \xi = -\infty. \]

The general solution satisfying these conditions is
\[ \psi = \eta + \sum_{n=1}^{\infty} \frac{2}{n} \exp \left( \frac{x^2 - y^2}{2} \right)^{1/2} \sin n \pi \eta \]

The above solution is valid as long as \( F > \frac{1}{3} \). If \( F < \frac{1}{3} \)

the general solution consists of persistent terms and no
unique solution can be obtained in this case. This indeter-
mimacy is due to the development of back flow which violates
the imposed upstream condition. As \( F \) decreases the flow
lines gradually sink and at \( F \approx 0.35 \) there is an eddy
growth which gradually increases in size and extends to far
upstream, causing a backflow. This back flow ultimately
divides the region into flow region and an essentially stag-
nation zone. This formation of the stagnation zone is
observed by Debler (4).

The above analysis has been extended to the flow past
a barrier by Long (12), Yih (25), Drasin and Moore (6),
Miles (13), Krishna (10), Segur (18) and Janowits (8).
Long's solution is appropriate for very low barriers even for
small \( F \) and becomes invalid for barriers of finite height.
Assuming uniform undisturbed conditions far upstream, Drasin
and Moore (6) obtained a solution for the flow over a ver-
tical barrier in a channel, using Yih's inverse method.
The solution for $F < 1.5$ and finite barriers show very complicated pattern which has no physical relevance. Miles (13) solved the problem in half-space using elliptic coordinate transformation. He noted that the back flow occurs for $F, > 1.75$ and the solution obtained rules out the possibility of eddies in contrast to the experimental observation. Thus ignoring the possibility of blocking and insisting on a formal solution with prescribed upstream conditions, Drasin and Moore may have pushed the limit of validity of their solution too far, and Miles, in ruling that the appearance of eddies invalidates the solution may be too much on the safer side. Krishna (10) discussed the flow past a cylinder in an unbounded medium using elliptic coordinate transformation. He has found that the indeterminacy arises in the case of finite cylinder whereas a unique solution can be obtained for the flow over thin barriers. Segur (15) has obtained a criterion which delimits the range of application of Long's steady state model. In all the above theoretical models, the predicted flow provides a reasonable description of the real flow for small $F$ and sufficiently small obstacles, but for large $F$ or large obstacles the predicted flow gives rise to elongated eddies upstream apart from containing complex lee-wave patterns which bear no resemblance to the observations. In fact the experiments of Long (12), Debler (4) and Yih (24) all indicate that at very small $F$ the stagnation
some occur upstream and even downstream from a barrier or a moving body. Taking into account these theoretical and experimental works the following observations can be made. The phenomenon of blocking is not synonymous with the presence or absence of upstream influence of an obstacle. If the stream is supercritical ($F > \frac{1}{\xi}$), then low barriers will have no upstream influence and therefore no blocking. If the barrier is high the original upstream condition may be effected but still no blocking. If the stream is subcritical ($F < \frac{1}{\xi}$), any barrier will have an upstream influence although blocking may not occur unless it is moderately high. Also it can be shown that the shape of the boundary has a lot to do with the occurrence or absence of blocking. It has been established by Yih, Dobler and O'neill (26) and Rao and Nathal (16) that it is possible to prevent stagnation region at low internal freude numbers by suitably modifying the shape of the boundary.

3. BLOCKING IN UNSTEADY FLOWS

The disadvantages of the above steady state theory under the assumption of a uniform undisturbed upstream flow are the need to justify the neglect of upstream waves, which are a possible solution and the lack of any terms describing the Jet formation (one dimensional flows). Rayleigh (17) showed that the artifice of eliminating the upstream waves become unnecessary if we formulate an initial value problem. This would also enable one to allow upstream influence to occur
which is essential to discuss the phenomenon of blocking. In transient analysis the governing equation for the pressure or the stream function is established under the Stokes or Oseen type linearisation. As already pointed out, the analysis of homogeneous rotating fluid and stratified non-rotating fluids lead to the same mathematical formulation and very similar solutions under certain circumstances. This analogy is discussed for general unsteady motions by Trüestrum (21). A technique is developed by her to study the flows in either rotating or stratified fluids by means of initial value formulation using Oseen type linearised equations. The governing equation in non-dimensional form for two-dimensional flow of an inviscid, incompressible, non-diffusive density stratified fluid moving with a uniform velocity \( U \) initially, along \( x \)-direction is

\[
\left( \frac{1}{\partial t} + \frac{\partial}{\partial x} \right)^2 \nabla^2 \psi + F^2 \frac{\partial^2 \psi}{\partial x^2} = 0 \tag{1.1}
\]

where \( \psi \) is the stream function.

A small perturbation is introduced on the plane \( x = 0 \) of the form

\[
\psi(0, s, t) = \frac{n_2}{k} \sin ks H(t).
\]

The general solution in the upstreamside (\( x < 0 \)) in the limit \( t \to \) is

\[
\psi = \frac{n_2}{k} \sin ks \exp \left( 1 - \frac{F^2}{2} \right)^{1/2} k x, \text{ for } F > 1,
\]

\[
= \frac{n_2}{k} \sin ks, \text{ for } F < 1,
\]
i.e., for \( F > 1 \), the perturbation decays exponentially and has an irrotational character where as for \( F < 1 \), the solution is independent of \( x \) and described blocked flow extending to upstream infinity. In the downstream side

\[
\psi = \frac{\sin kx}{k} \left[ u_0 - \frac{\omega_0}{(1-\beta_1^2)^{1/2}} \left[ 1 - \exp \left( - (1-\beta_1^2)^{1/2} kx \right) \right] \right]
\]

for \( \beta_1 > 1 \),

\[
= \frac{\sin kx}{k} \left[ u_0 - \frac{\int_1 + \beta_1 \cos (R_1^{-2} - 1)^{1/2} k x}{(1 + \beta_1)} \right]
\]

\[
- \frac{\omega_0 \sin (R_1^{-2} - 1)^{1/2} k x}{(R_1^{-2} - 1)^{1/2}}
\]

\[
+ \frac{U k \sigma_0}{\beta (1 + \beta_1)} \left[ 1 - \cos (R_1^{-2} - 1)^{1/2} k x \right]
\]

for \( \beta_1 < 1 \).

The ultimate flow thus exhibits waves in the downstream as well as causing a blocked flow for small \( F \).

Now for an inviscid, homogeneous fluid rotating with an angular velocity \( \Omega \) about an axis \( ox \) and moving with a uniform axial velocity \( U \) along \( ox \), the governing equation in non-dimensional form is

\[
\left( \frac{\partial}{\partial t} + \frac{\partial}{\partial x} \right)^2 D^2 \psi + R_0^{-2} \frac{\partial^2 \psi}{\partial x^2} = 0
\]

(1.2)

where \( D^2 \) is the cylindrical operator \( \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial x^2} \) and
$R_o$ is the Rossby number. The equation (1.2) is completely analogous to the equation (1.1) in the stratified case. A small axisymmetric perturbation is caused initially on the plane $x = 0$ as

$$\psi(r, 0, t) = k^{-1} \omega_0 r J_1(k r) N(t).$$

The general solution in the upstream side ($x < 0$) in the limit $t \to \infty$ is

$$\psi = k^{-1} \omega_0 r J_1(k r) \exp\left(1 - R_o^{-2}\right)^{1/2} k x, \text{ for } R_o > 1,$$

$$= k^{-1} \omega_0 r J_1(k r), \quad \text{for } R_o < 1.$$

Thus the perturbation decays exponentially as $x \to -\infty$ and has an irrotational character for $R_o > 1$. For $R_o < 1$, the solution describes a one-dimensional geostrophic flow similar to the stratified case. In the downstream side for $R_o > 1$ the ultimate solution will have an irrotational character, though it contains one-dimensional flow terms, independent of $x$. For $R_o < 1$, waves appear apart from the one-dimensional flow terms. Thus in both rotating (or stratified) fluids parts of the solutions contain irrotational like terms for high Rossby number (or Froude number) plus wave-like terms for smaller Rossby (or Froude) number. In addition, for smaller values of parameter the solution consists of one-dimensional or geostrophic flow which causes the blocked flow.
Trustrum (2) also analysed the flow into a line sink using the above linearised equation (1.1). She confirmed that for \( F < 1/\alpha \), the solution contains one-dimensional flow terms which are responsible for blocking.

It should be noted that the Oseen solution discussed by Trustrum (2) in the limit \( t \to \infty \) has one unknown constant in the upstream side and three in the downstream side. So, in dealing with inviscid flow past a barrier, the inviscid boundary condition is sufficient to determine the upstream flow but is insufficient to determine the downstream flow. This difficulty was circumvented by Trustrum (2) while studying the flow over a thin vertical barrier by allowing upstream influence to occur. Following the suggestions made by Stewartson (19) that the appropriate boundary conditions to describe a physically reasonable flow are inviscid conditions on the upstream side and the viscous conditions on the downstream side, a unique solution has been obtained by Trustrum which predicted blocked flow for \( F^{-1} = 2.25, 3.6 \) and 4.5 etc., Also the predicted flow agreed fairly with the descriptions of Davis (5) experimental observations.

Bretherton (3) and Krishna and Sharma (10) have discussed the inviscid, incompressible, density stratified flow over a cylinder of infinite span in an unbounded medium. Using Stokes linearised equations an initial value investigation has been made to study the flow in different regions. In the
limit $t \to 0$ they have observed that the tangential planes to the cylinder behave as singular surfaces, separating the fluid into two regions such that the fluid interior is blocked by the cylinder and outside it moves with a horizontal velocity.

The unsteady flow of rotating or stratified fluids in wavy tubes and channel has been studied by Balan et al. (1) using the Oseen type linearised equations. The solutions consist of a steady part and an unsteady part which can be represented as a uniformly convergent infinite series. It is observed that the unsteady part remains oscillating even after infinite time and there exists inertial waves propagating in both positive and negative directions of the flow. In each case, the range of the Rossby or Froude number for which there is no wave propagating in the negative direction of the flow is given. Also it is found that no solution is possible for a certain infinite set of critical numbers and when the Rossby or Froude number is close to the critical values the back flow region occurs.

4. PREVENTION OF BLOCKING

The formation of stagnation zones in a flow region for certain low values of governing parameters can be prevented by different methods. Yih (25), Rao (9), Rao and Rathna (16) and Bathaiah (2) et al. have given out different methods to prevent this blocking. While discussing the two-dimensional flow pattern into a line sink for $F < 1/a$, Yih (25)
suggested that by raising the bottom suitably near the sink the stagnation of the fluid can be prevented and hence a unique solution, valid for all \( F \), can be obtained. This in accordance with the statement made earlier, the shape of the boundary can be suitably modified depending on the smallness of \( F \) in order to prevent the stagnation zones. Rao (9) gave the method of introducing a fictitious sink distribution in order to obtain a solution valid for all \( F \). Rao and Rathna (16) discussed the steady, two-dimensional stratified flow in a wavy channel and concluded that this blocking can be prevented by suitably defining the boundary of the channel (considering large periodic deformations).

It is obvious that the atmospheric flows are affected to a notable extent by the earth's weak magnetic field and hence the interplay of the Lorentz force and the buoyancy force is an intriguing aspect of the atmospheric motions. When a uniform transverse magnetic field is imposed on a two-dimensional flow of an inviscid, density stratified fluid (or axisymmetric flow of an inviscid, homogeneous rotating fluid), it can be shown that the amplitude of the lee waves occurring in the downstream side decay exponentially at far distances of the body with amplitude decreasing with an increase in the magnetic parameter. Also the one-dimensional or geostrophic flow which is responsible for the jet formation in the non-magnetic case does not appear here. Instead there arise an infinite number of exponentially
decaying modes both upstream and downstream which would reduce to the one-dimensional terms in the limit $k \to 0$. Some of these features have been studied by Hathaih (2) in the stratified case and Prasada Rao and Krishna (14) in the rotating case. In the following chapter we aim at studying the phenomenon of blocking with respect to a two-dimensional inviscid flow in a wavy channel under the influence of a uniform transverse magnetic field. To start with we consider the flow of a stratified fluid and later analyze the flow of a homogeneous fluid.