CHAPTER VI
EFFECTS OF CHEMICAL REACTION AND HEAT RADIATION ON UNSTEADY MHD ELASTICO-VISCOUS FLOW PAST A POROUS PLATE EMBEDDED IN A POROUS MEDIUM

6.1 INTRODUCTION
The study of heat and mass transfer with chemical reaction and radiation is of considerable importance in chemical and hydrometallurgical industries. Many processes in engineering areas occur in high temperature and consequently heat radiation plays a very significant role. This study has many applications in industries such as drying, evaporation at the surface of water body, energy transfer in a wet cooling tower and flow in a desert cooler etc. Chemical reaction can be codified as either heterogeneous or homogeneous processes. The present trend in the field of chemical reaction analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field. In addition to this combined heat and mass transfer in fluid saturated porous media finds application in a variety of engineering processes such as petroleum technology for the flow of oil through porous rocks, in chemical engineering for the purification and filtration processes and in cases like drug permeation through human skin. Nield and Bejan (1998) have studied the convection in porous media. The effect on free convection currents on the oscillatory flow through a porous medium has been discussed by Hiremath and Patil (1993). Fluctuating heat and mass transfer on three-dimensional flow through a porous medium with variable permeability has been investigated by Sharma et al. (2007).

MHD is gaining renewed interest for the flow and heat transfer in porous media due to the effect of magnetic field on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. This type of flow

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finds application in many engineering problems such as MHD generators, plasma studies, nuclear reactors etc. A comprehensive review of the studies of unsteady hydromagnetic free convection flow of Newtonian fluid has been made by Helmy (1998). Chaudhury and Sharma (2006) have considered combined heat and mass transfer by laminar mixed convection flow from a vertical surface with induced magnetic field. They (2008) also have studied hydromagnetic unsteady mixed convection and mass transfer flow past a vertical porous plate immersed in a porous medium with Hall effect. El-Amin (2001) has analyzed magnetohydrodynamic free convection and mass transfer flow in micropolar fluid with constant suction. Cussler (1998) has presented an analysis of diffusion mass transfer in fluid systems. Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction was solved by Das et al. (1994). Muthucumarswamy and Ganesan (2001), Muthucumarswamy (2002) have studied first order homogeneous chemical reaction on flow past infinite vertical plate. Kandasamy et al. (2005) has examined effects of chemical reaction, heat and mass transfer along wedge with heat source and concentration in the presence of suction or injection. Sharma et al. (2006) have reported an analysis to describe the radiation effect with simultaneous thermal and mass diffusion in MHD mixed convection flow from a vertical surface with Ohmic heating. They (2006, 2007, 2008, 2009) also have done some significant works in this area.

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of more intricate nature. It has been observed that an energy flux can be generated not only by temperature gradients but also by concentration gradients. The energy flux caused by a concentration gradient is termed the diffusion-thermo (Dufour) effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the thermal diffusion (Soret) effect. These effects are considered as second-order phenomena and may become significant in areas such as hydrology, petrology, geosciences etc. Due to the importance of these effects many researchers have discussed and examined results of these effects of which the names are Eckert and Drake (1972), Dursunkaya and Worek (1992), Anghel et al. (2000), Postelnicu (2004), Alam and Rahman (2005), Choudhury and Dey (2010, 2012) are worth mentioning. Recently Sharma et al. (2012) have investigated
Soret and Dufour effects on unsteady MHD mixed convection flow past a radiative vertical porous plate embedded in a porous medium with chemical reaction.

The purpose of this investigation is to study the effects of chemical reaction and heat radiation on unsteady MHD visco-elastic flow past a porous plate embedded in a porous medium.

6.2 MATHEMATICAL FORMULATION

An unsteady two dimensional mixed convective flow of an incompressible and electrically conducting elasto-viscous fluid past an infinite vertical porous plate embedded in a porous medium is considered. The \( \bar{x} \)-axis is taken on the infinite plate, and parallel to the free stream velocity which is vertical and the \( \bar{y} \)-axis is taken normal to the plate. A uniform magnetic strength \( B_0 \) is applied transversely to the direction of the flow. Initially the plate and the fluid are at same temperature \( \bar{T}_\infty \) in a stationary condition with concentration level \( \bar{C}_\infty \) at all points. For \( \bar{t} > 0 \), the plate starts moving impulsively in its own plane with a velocity \( U_0 \), its temperature is raised to \( \bar{T}_w \) and the concentration level at the plate is raised to \( \bar{C}_w \). The flow configuration and coordinate system are shown in the figure 6.1. The fluid is assumed to have constant properties except that the influence of the density variations with temperature and concentration, which are considered only in the body force term. Under the above assumptions, the physical variables are functions of \( \bar{y} \) and \( \bar{t} \) only.

Assuming that the Boussinesq and boundary layer approximations hold and using the Darcy-Forchheimer model, the basic equations, which govern the problem, are given by:

\[
\frac{\partial \bar{v}}{\partial \bar{y}} = 0
\]  (6.2.1)

\[
\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{k_0}{\rho} \left( \frac{\partial^3 \bar{u}}{\partial \bar{t} \partial \bar{y}^2} + \bar{v} \frac{\partial^3 \bar{u}}{\partial \bar{y}^3} \right) \\
\quad + g \beta (\bar{T} - \bar{T}_\infty) + g \beta^*(\bar{C} - \bar{C}_\infty) - \frac{\sigma B_0^2 \bar{u}}{\rho} - \frac{\nu}{K} \bar{u}
\]  (6.2.2)

\[
\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \alpha \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial \bar{y}} + \frac{Q_0}{\rho c_p} (\bar{T} - \bar{T}_\infty)
\]  (6.2.3)

\[
\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{C}}{\partial \bar{y}} = \frac{D_m k_r}{T_m} \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - k_r (\bar{C} - \bar{C}_\infty)
\]  (6.2.4)

89
The radiative heat flux term by using Rosseland approximation (1998, 2006) is given by

\[ q_r = -\frac{4\sigma^* \partial T^4}{3a_\rho \partial y^*} \]

where \( \bar{u} \) and \( \bar{v} \) are the Darcian velocity components in the \( \bar{x} \) and \( \bar{y} \)-directions respectively, \( \bar{\tau} \) is the time, \( \nu \) is the kinematic viscosity, \( g \) is the acceleration due to gravity, \( \rho \) is the density, \( \beta \) is the coefficient of volume expansion, \( \beta^* \) is the volumetric coefficient of expansion with concentration, \( K \) is the Darcy permeability, \( B_0 \) is the magnetic induction, \( \bar{T} \) and \( \bar{T}_\infty \) are the temperature of the fluid inside the thermal boundary layer and the fluid temperature in the free stream respectively, while \( \bar{C} \) and \( \bar{C}_\infty \) are the corresponding concentrations. Also, \( \sigma \) is the electric conductivity, \( \alpha \) is the thermal diffusivity, \( \sigma^* \) is the Stefan-Boltzmann constant, \( a_\rho \) is the mean absorption coefficient, \( D_m \) is the coefficient of mass diffusivity, \( c_p \) is the specific heat at constant pressure, \( T_m \) is the mean fluid temperature, \( k_T \) is the thermal diffusion ratio and \( c \) is the concentration susceptibility. The term \( Q_0(\bar{T} - \bar{T}_\infty) \) is assumed to be amount of heat generated or absorbed per unit volume \( Q_0 \) is a constant, which may take on either positive or negative values, \( q_r \) is the radiative heat flux in the \( \bar{y} \)-direction, \( k_r \) is chemical reaction parameter.

On integrating equation (6.2.1) we get

\[ \bar{v} = \text{constant} = -v_0, v_0 > 0 \]

Initially, at \( t = 0 \) fluid and the plate are at rest. Thus the no slip boundary conditions at the surface of the plate for the above problem for \( t > 0 \) are:

\[ \bar{u} = U_0(1 + \varepsilon e^{i\omega t}), \bar{v} = v(t), \bar{T} = T_w + \varepsilon e^{i\omega t}(T_w - T_\infty), \]

\[ y = 0: \bar{C} = C_w + \varepsilon e^{i\omega t}(C_w - C_\infty) \]

\[ y \to \infty: \bar{u} = 0, \bar{T} = T_\infty, \bar{C} = C_\infty \]

(6.2.5)

We introduce the following non-dimensional quantities:

\[ U = \frac{\bar{u}}{u_0}, \bar{v} = \frac{\bar{v}u_0^2}{\nu}, \bar{T} = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}, \bar{C} = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_w - \bar{C}_\infty}, \]

\[ Gr = \frac{g\beta(T_w - T_\infty)\nu}{u_0^2}, S = \frac{Q_0 \nu}{\rho c_p u_0^2}, r = \frac{k_r \nu}{u_0^2}, \]

\[ Gm = \frac{g\beta^*(C_w - C_\infty)\nu}{u_0^2}, Pr = \frac{\nu}{\alpha}, Sr = \frac{D_m k_T(T_w - T_\infty)}{T_m \nu(C_w - C_\infty)}. \]
\[ Da = \frac{K u_0^2}{\nu^2}, M = \frac{\sigma B_0^2 v}{\rho u_0^2}, R = \frac{k_r k^*}{4 \sigma_r T_o^3} \]

where \( \theta \) is the dimensionless temperature, \( \Phi \) is the dimensionless concentration, \( Gr \) is the thermal Grashof number, \( Gm \) is the solutal Grashof number, \( Pr \) is the Prandtl number, \( Sr \) is the Soret number, \( Da \) is the Darcy permeability parameter, \( M \) is the magnetic parameter, \( R \) is the radiation parameter, \( r \) is the chemical reaction parameter, \( S \) is the heat source parameter.

The non-dimensional form of equations (6.2.1) to (6.2.4) become

\[
\begin{align*}
\frac{\partial U}{\partial t} - v_0 \frac{\partial U}{\partial y} &= \frac{\partial^2 U}{\partial y^2} + k \left( \frac{\partial^3 U}{\partial t \partial y^2} - v_0 \frac{\partial^3 U}{\partial y^3} \right) + Gr \theta + Gm \Phi - MU - \frac{1}{Da} U \\
\frac{\partial \theta}{\partial t} - v_0 \frac{\partial \theta}{\partial y} &= \left( \frac{3R + 4}{3R Pr} \right) \frac{\partial^2 \theta}{\partial y^2} + S \theta \\
\frac{\partial \Phi}{\partial t} - v_0 \frac{\partial \Phi}{\partial y} &= Sr \frac{\partial^2 \theta}{\partial y^2} - r \Phi
\end{align*}
\]  

(6.2.6)

(6.2.7)

(6.2.8)

The corresponding boundary conditions for \( t > 0 \) are:

\[
y = 0: \quad U = 1 + \varepsilon e^{i \alpha t}, \theta = 1 + \varepsilon e^{i \alpha t}, \Phi = 1 + \varepsilon e^{i \alpha t} \quad \text{at} \quad y = 0
\]

\[
y \to \infty: \quad U = 0, \quad \theta = 0, \quad \Phi = 0 \quad \text{at} \quad y \to \infty
\]  

(6.2.9)

**6.3 METHOD OF SOLUTION**

For the solution of the equations (6.2.6)-(6.2.8), we expand \( U, \theta \) and \( \Phi \) in powers of \( \varepsilon \), where \( \varepsilon \) is a small constant quantity (\( \varepsilon \ll 1 \)).

\[
U = U_0(y) + \varepsilon e^{i \alpha t} U_1(y) + O(\varepsilon^2)
\]

\[
\theta = \theta_0(y) + \varepsilon e^{i \alpha t} \theta_1(y) + O(\varepsilon^2)
\]

\[
\Phi = \Phi_0(y) + \varepsilon e^{i \alpha t} \Phi_1(y) + O(\varepsilon^2)
\]  

(6.3.1)

Using (6.3.1) in equations (6.2.6) - (6.2.8) and equating the co-efficients of like powers of \( \varepsilon \), we obtain the following set of differential equations:

Zeroth-order equations:

\[
- v_0 \frac{dU_0}{dy} = \frac{d^2 U_0}{dy^2} + Gr \theta_0 + Gm \Phi_0 - MU_0 + k \frac{d^3 U_0}{dy^3} - \frac{1}{Da} U_0
\]  

(6.3.2)

\[
- v_0 \frac{d\theta_0}{dy} = \left( \frac{3R + 4}{3R Pr} \right) \frac{d^2 \theta_0}{dy^2} + S \theta_0
\]  

(6.3.3)

\[
- v_0 \frac{d\Phi_0}{dy} = Sr \frac{d^2 \theta_0}{dy^2} - r \Phi_0
\]  

(6.3.4)
subject to boundary conditions:
\[ y = 0: \quad U_0 = 1, \theta_0 = 1, \varphi_0 = 1 \]
\[ y \to \infty: \quad U_0 = 0, \theta_0 = 0, \varphi_0 = 0 \]  \hspace{1cm} (6.3.5)

First-order equations:
\[ i\omega U_1 - v_0 \frac{dU_1}{dy} = \frac{d^2U_1}{dy^2} + Gr\theta_1 + Gm\varphi_1 - MU_1 + k \left( \frac{d^3U_1}{dy^3} - i\omega \frac{d^2U_1}{dy^2} \right) - \frac{1}{Da} U_1 \]  \hspace{1cm} (6.3.6)
\[ i\omega \theta_1 - v_0 \frac{d\theta_1}{dy} = \left( \frac{3R + 4}{3RPr} \right) \frac{d^2\theta_1}{dy^2} + S\theta_1 \]  \hspace{1cm} (6.3.7)
\[ i\omega \varphi_1 - v_0 \frac{d\varphi_1}{dy} = Sr \frac{d^2\theta_1}{dy^2} - r\varphi_1 \]  \hspace{1cm} (6.3.8)

with relevant boundary conditions:
\[ y = 0: \quad U_1 = 1, \theta_1 = 1, \varphi_1 = 1 \]
\[ y \to \infty: \quad U_1 = 0, \theta_1 = 0, \varphi_1 = 0 \]  \hspace{1cm} (6.3.9)

The solutions of the equations (6.3.3), (6.3.4), (6.3.7) and (6.3.8) subject to the boundary conditions given in (6.3.5), (6.3.9) are as follows:
\[ \theta_0 = e^{-a_1y} \]  \hspace{1cm} (6.3.10)
\[ \varphi_0 = e^{-a_1y} \]  \hspace{1cm} (6.3.11)
\[ \theta_1 = e^{-a_3y} \]  \hspace{1cm} (6.3.12)
\[ \varphi_1 = e^{-a_3y} \]  \hspace{1cm} (6.3.13)

Again, to solve the equations (6.3.2) and (6.3.6), we use the multi-parameter perturbation technique and the velocity components are expanded in the power of viscoelastic parameter \( k \) as \( k \ll 1 \). Thus the expressions for velocity components are considered as
\[ U_0 = U_{00}(y) + kU_{01}(y) + O(k^2) \]
\[ U_1 = U_{10}(y) + kU_{11}(y) + O(k^2) \]  \hspace{1cm} (6.3.14)

Substituting (6.3.14) into the equations (6.3.2) and (6.3.6) and after equating the like powers of \( k \), we get the following set of ordinary differential equations:
\[ U_{00}'' + v_0U_{00} - \left( M + \frac{1}{Da} \right) U_{00} = -(Gr + Gm)e^{-a_1y} \]  \hspace{1cm} (6.3.15)
\[ U_{10}'' + v_0 U_{10}' - \left( M + i \omega + \frac{1}{D_R} \right) U_{10} = -(Gr + Gm)e^{-a_3 y} \]  \hfill (6.3.16)

\[ U_{01}'' + v_0 U_{01}' - \left( M + \frac{1}{D_R} \right) U_{01} = -U_{00}'' \]  \hfill (6.3.17)

\[ U_{11}'' + v_0 U_{11}' - \left( M + i \omega + \frac{1}{D_R} \right) U_{11} = U_{10}'' - i \omega U_{10}'' \]  \hfill (6.3.18)

The modified boundary conditions are:

\[ y = 0: \quad U_{00} = 1, \quad U_{01} = 0, \quad U_{10} = 0, \quad U_{11} = 0 \]

\[ y \to \infty: \quad U_{00} = 0, \quad U_{01} = 0, \quad U_{10} = 0, \quad U_{11} = 0 \]  \hfill (6.3.19)

The differential equations (6.3.15) to (6.3.18) are solved subject to the boundary condition (6.3.19). The solutions of the differential equations are obtained as

\[ U_{00} = A_1 e^{-a_2 y} + A_2 e^{-a_1 y} \]  \hfill (6.3.20)

\[ U_{01} = A_3 (e^{-a_2 y} - e^{-a_1 y}) \]  \hfill (6.3.21)

\[ U_{10} = A_5 e^{-a_4 y} - A_4 e^{-a_3 y} \]  \hfill (6.3.22)

\[ U_{11} = A_6 (e^{-a_3 y} - e^{-a_4 y}) \]  \hfill (6.3.23)

### 6.4 RESULTS AND DISCUSSION

The velocity, temperature and concentration of the fluid are given by

\[ U = (A_1 e^{-a_2 y} + A_2 e^{-a_1 y} ) + k A_3 (e^{-a_2 y} - e^{-a_1 y}) \]

\[ + \alpha e^{i \omega t} \left( (A_5 e^{-a_4 y} - A_4 e^{-a_3 y}) + k A_6 (e^{-a_3 y} - e^{-a_4 y}) \right) \]  \hfill (6.4.1)

\[ \theta = e^{-a_1 y} + \alpha e^{i \omega t} e^{-a_3 y} \]  \hfill (6.4.2)

\[ \varphi = e^{-a_1 y} + \alpha e^{i \omega t} e^{-a_3 y} \]  \hfill (6.4.3)

The non dimensional shearing stress at the plate is given by

\[ \sigma = \left( \frac{\partial U}{\partial y} \right)_{y=0} + k \left( \frac{\partial^2 U}{\partial t \partial y} - v_0 \frac{\partial^2 U}{\partial y^2} \right)_{y=0} \]

\[ = B_1 + k B_2 + \alpha e^{i \omega t} (B_3 + k B_4 - k i \omega (B_3 + k B_4) + v_0 (B_7 + k B_0)) \]

\[ + \alpha v_0 (B_5 + k B_6) \]

where the constants are:

\[ \alpha_1 = \frac{v_0 + \sqrt{v_0^2 - 4 (\frac{3R + 4}{3RPr}) S}}{2 \left( \frac{3R + 4}{3RPr} \right)} \]

\[ \alpha_2 = \frac{v_0 + \sqrt{v_0^2 + 4 (M + \frac{1}{D_R})}}{2} \]
\[
\alpha_3 = \frac{v_0 + \sqrt{v_0^2 + 4 \left( \frac{3R + 4}{3RPr} \right) (S - i\omega)}}{2 \left( \frac{3R + 4}{3RPr} \right)}, \\
\alpha_4 = \frac{v_0 + \sqrt{v_0^2 + 4 \left( M + i\omega + \frac{1}{Da} \right)}}{2}, \\
A_1 = 1 + \frac{Gr + Gm}{\alpha_1^2 - v_0 \alpha_1 - \left( M + \frac{1}{Da} \right)}, \\
A_2 = -\frac{Gr + Gm}{\alpha_2^2 - v_0 \alpha_2 - \left( M + \frac{1}{Da} \right)}, \\
A_3 = \frac{A_2 \alpha_1^2}{\alpha_1^2 - v_0 \alpha_1 - \left( M + \frac{1}{Da} \right)}, \\
A_4 = \frac{A_3 \alpha_1^2}{\alpha_1^2 - v_0 \alpha_1 - \left( M + \frac{1}{Da} \right)}, \\
A_5 = 1 + \frac{Gr + Gm}{\alpha_2^2 - v_0 \alpha_2 - \left( M + i\omega + \frac{1}{Da} \right)}, \\
A_6 = \frac{A_4 \alpha_2^2}{\alpha_2^2 - v_0 \alpha_2 - \left( M + i\omega + \frac{1}{Da} \right)}, \\
B_1 = -(A_1 \alpha_2 + A_2 \alpha_1), \\
B_3 = A_4 \alpha_3 - A_5 \alpha_4, \\
B_5 = A_1 \alpha_3^2 + A_2 \alpha_1^2, \\
B_7 = A_5 \alpha_4^2 + A_4 \alpha_3^2, \\
B_2 = A_3 (\alpha_1 - \alpha_2), \\
B_4 = A_6 (\alpha_4 - \alpha_3), \\
B_6 = A_3 (\alpha_2^2 - \alpha_1^2), \\
B_8 = A_6 (\alpha_3^2 - \alpha_2^2)
\]

The object of this study is to investigate the effects of chemical reaction and heat radiation on unsteady MHD visco-elastic flow past a porous plate embedded in a porous medium. The elastico-viscous effect is exhibited through the non-dimensional parameter \( k \). The non-zero values of the parameter \( k \) characterize the elastico-viscous fluid and \( k = 0 \) represents the Newtonian fluid flow phenomenon. The fluid velocity and the shearing stress at the plate are illustrated graphically for various values of flow parameters involved in the solution.

Figures 6.2 to 6.9 demonstrate the patterns of fluid velocity against the displacement \( y \) for various values of visco-elastic parameters. These graphs interpret that speed diminishes as the fluid moves far away from the plate. Also, it is visualized from these figures that the fluid accelerates rapidly in the neighbourhood of the plate in case of both Newtonian and visco-elastic fluids. Again, during the growth of visco-elasticity a rising trend is observed in the velocity at every point of the fluid flow region in comparison with the simple Newtonian fluid.

The influence of the thermal Grashof number on the velocity is presented in Figure 6.2. Thermal Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is
observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Here, the positive values of Gr correspond to cooling of the plate. Also, as Gr increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity.

Figure 6.3 represents typical fluid velocity in the boundary layer for various values of the solutal Grashof number Gm, while all other parameters are kept at some fixed values. The solutal Grashof number Gm defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value.

The effects of suction parameter and Hartmann number in the boundary layer of both Newtonian and visco-elastic fluids have been shown in Figures 6.4 and 6.5. From Figure 6.4, it can be observed that the fluid velocity enhances rapidly and then diminishes gradually with the increasing values of suction parameter. The application of transverse magnetic field generates a resistive force called Lorentz force which increases the viscosity of the fluid flow and hence a retarding trend is observed in the speed of the fluid flow. Both the visco-elastic fluid and Newtonian fluid experience a rapid acceleration and then follow a decreasing trend during the rise of magnetic parameter.

The significance of Prandtl number cannot be ignored in heat transfer flow problems as it helps to study the simultaneous effects of momentum and thermal diffusion in fluid flow. The effects of Prandtl number on both visco-elastic fluid and Newtonian fluid are analyzed in Figure 6.6. The rising value of Prandtl number raises the thickness of the fluid and hence the fluid experiences a decelerating trend. This physical phenomenon is observed in Newtonian as well as visco-elastic fluid.

For different values of the radiation parameter R the fluid velocity is depicted in Figure 6.7. The radiation parameter R defines the relative contribution of conduction heat transfer to thermal radiation transfer. It is observed that an increase in the radiation parameter R results in a decrease in the velocity within the boundary layer. The same trend can be noticed for both elastico-viscous and simple fluid flow.
The effect of heat source parameter $S$ on visco-elastic fluid and Newtonian fluid are presented in Figure 6.8. The increasing values of the heat source parameter diminish the fluid velocity in both Newtonian and non-Newtonian fluid flow phenomenon.

Figure 6.9 illustrates that the fluid velocity $U$ increases near the plate and it decreases away from the plate with an increase in Darcy number $Da$. Darcy number is the measurement of the porosity of the medium. As the porosity of the medium increases, the value of $Da$ increases. For large porosity of the medium, fluid gets more space to flow as a consequence its velocity decreases for both non-Newtonian and simple fluid mechanism.

After knowing the velocity field, the study of shearing stress for the concerned flow is very important from practical point of view. Figures 6.10-6.15 depict the effect of viscous drag on Newtonian and visco-elastic fluid flow. The analysis is given for both flow past an externally heated plate ($Gr < 0$) and externally cooled plate ($Gr > 0$).

Figure 6.10, illustrates the nature of shearing stress for both types of fluids against solutal Grashof number ($Gm$). During the flow past a cooled surface as well as heated surface, the shearing stress shows an increasing trend for both Newtonian and non-Newtonian fluids.

Figure 6.11, exhibits the nature of shearing stress for the concerned flow against Prandtl number ($Pr$). During the flow past a cooled surface, the shearing stress produced at the surface will subdue its magnitude but a reverse behaviour is observed for the flow past a heated plate.

The effects of Lorentz force on shearing stress are illustrated in Figure 6.12. The shearing stress experienced by the Newtonian fluid ($k = 0$) and non-Newtonian fluid flow mechanism ($k = 0.1, 0.2$) experience a declined trend for $Gr > 0$.

Figure 6.13 represents the influence of suction parameter $v_0$ on viscous drag of the fluid flow. This figure also follows the same trend as Figure 6.11. Most interesting part is that the elastico-viscous as well as simple fluid flow mechanism follows the same trend.

On the other hand, Figures 6.14 and 6.15 show the effect of radiation parameter and heat source parameter on viscous drag. In both the cases, the viscous drag formed by both Newtonian and non-Newtonian fluid will experience a diminishing trend for
externally heated plate but an opposite behaviour is observed for flow past a cooling surface.

6.5 CONCLUSION:

The problem of unsteady, MHD elastico-viscous flow past a porous plate embedded in a porous medium with chemical reaction and heat radiation has been studied. The velocity, temperature and concentration fields are obtained analytically. Graphical results are presented and discussed for various physical parametric values. The main findings are as follows:

- The fluid flow is accelerated and then retarded slowly during the enhancement of visco-elastic parameter.
- The fluid is accelerated with the increasing values of visco elastic parameter in comparison with the Newtonian fluid.
- The viscous drag formed by both Newtonian and non-Newtonian fluid will experience a increasing trend during the growth of solutal Grashof number.
- An accelerating trend in viscous drag is observed in case of flow past an externally heated plate for both Newtonian and non-Newtonian fluid.
- A decelerating trend in shearing stress is observed in case of flow past an externally cooled plate for both Newtonian and non-Newtonian fluid.
- The rate of heat transfer and the rate of mass transfer will not differ significantly during the various values of visco-elastic parameter.
Figure 6.1: Geometry of the problem.

Figure 6.2: Variation of transient velocity $U$ versus $y$ for $Pr=3$, $Sr=2$, $Da=0.5$, $Gm=6$, $v_0=0.5$, $R=0.4$, $M=1$, $S=1$.

Figure 6.3: Variation of transient velocity $U$ versus $y$ for $Pr=3$, $Sr=2$, $Da=0.5$, $Gr=12$, $v_0=0.5$, $R=0.4$, $M=1$, $S=1$. 
Figure 6.4: Variation of transient velocity $U$ versus $y$ for $Gr=12$, $Pr=3$, $Sr=2$, $Da=0.5$, $Gm=6$, $R=0.4$, $M=1$, $S=1$.

Figure 6.5: Variation of transient velocity $U$ versus $y$ for $Gr=12$, $Pr=3$, $Sr=2$, $Da=0.5$, $Gm=6$, $v_0=0.5$, $R=0.4$, $S=1$.

Figure 6.6: Variation of transient velocity $U$ versus $y$ for $Gr=12$, $Sr=2$, $Da=0.5$, $Gm=6$, $v_0=0.5$, $R=0.4$, $M=1$, $S=1$. 
Figure 6.7: Variation of transient velocity $U$ versus $y$ for $Gr=12$, $Pr=3$, $Sr=2$, $Da=0.5$, $Gm=6$, $v_0=0.5$, $M=1$, $S=1$.

Figure 6.8: Variation of transient velocity $U$ versus $y$ for $Gr=12$, $Pr=3$, $Sr=2$, $Da=0.5$, $Gm=6$, $v_0=0.5$, $R=0.4$, $M=1$.

Figure 6.9: Variation of transient velocity $U$ versus $y$ for $Gr=12$, $Pr=3$, $Sr=2$, $Gm=6$, $v_0=0.5$, $R=0.4$, $M=1$, $S=1$. 
Figure 6.10: Shearing stress versus Gm for Pr=3, Sr=2, Da=0.5, \(v_0=0.5\), R=0.4, M=1, S=1.

Figure 6.11: Shearing stress versus Pr for Gm=6, Sr=2, Da=0.5, \(v_0=0.5\), R=0.4, M=1, S=1.

Figure 6.12: Shearing stress versus M for Gm=6, Pr=3, Sr=2, Da=0.5, \(v_0=0.5\). R=0.4, M=1, S=1.
Figure 6.13: Shearing stress versus $v_0$ for $\text{Gm}=6$, $\text{Pr}=3$, $\text{Sr}=2$, $\text{Da}=0.5$, $\text{R}=0.4$, $\text{M}=1$, $\text{S}=1$.

Figure 6.14: Shearing stress versus $R$ for $\text{Gm}=6$, $\text{Pr}=3$, $\text{Sr}=2$, $\text{Da}=0.5$, $v_0=0.5$, $\text{M}=1$, $\text{S}=1$.

Figure 6.15: Shearing stress versus $S$ for $\text{Gm}=6$, $\text{Pr}=3$, $\text{Sr}=2$, $\text{Da}=0.5$, $\text{R}=0.4$, $v_0=0.5$, $\text{M}=1$, $\text{S}=1$. 

102