CHAPTER V
AN ANALYSIS OF MIXED CONVECTIVE ELASTICO- VISCIOUS FLUID
PAST A VERTICAL POROUS PLATE IN PRESENCE OF INDUCED
MAGNETIC FIELD AND CHEMICAL REACTION

5.1 INTRODUCTION

Many natural phenomena and technological problems are susceptible to MHD visco-elastic fluid flow analysis. Such flows play important roles in chemical engineering, turbo-machinery, aerospace technology, polymer industries, paper industries etc.

The convection problem in porous medium has important application in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices. The phenomena of mass transfer are also significant in theory of stellar structure and observable effects are detectable at least on the solar surface. The thermal physics of hydromagnetic problems with mass transfer is of interest in power engineering and metallurgy. The basis for these models was the early experimental work of Raptis and Kafousias (1982) on Magnetohydrodynamics free convective flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux. Bejan and Khair (1985) have reported a pioneering work on heat and mass transfer in a porous medium. Acharya et al. (2000) have investigated the magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux. MHD effects on heat and mass transfer in flow of a viscous fluid with induced magnetic field have been discussed by Singh and Singh (2000). Postelincus (2004) has analyzed influence of a magnetic field on heat and mass transfer by a natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Singh et al. (2007) have noticed MHD free convection mass transfer flow past a flat plate. In light of these facts, a number of

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problems have been studied by Choudhury and Dey (2010, 2012), Choudhury and Das (2010, 2013), Choudhury et al. (2004, 2013, 2013) etc by considering visco-elastic fluid flow phenomenon. The analysis of heat and mass transfer in MHD visco-elastic fluid flows forms an integral part of the research activities in interdisciplinary fields.

The objective of the study is to investigate the MHD mixed convective flow with heat source and chemical reaction of a visco-elastic fluid over a vertical porous plate in presence of induced magnetic field. The velocity field and the shearing stress at the plate are obtained and illustrated graphically to observe the visco-elastic effects in combination with other flow parameters.

5.2 MATHEMATICAL FORMULATION

Consider the steady flow of a visco-elastic electrically conducting fluid past a vertical porous plate with constant heat flux and chemical reaction in presence of a heat source. A uniform magnetic field is assumed to be applied transversely to the direction of the free stream taking into account the induced magnetic field and mass transfer. Our investigation is restricted to the following assumptions:

- All fluid properties except the density in the buoyancy force term are constant.
- The plate is subjected to constant suction.
- The plate is electrically non conducting.

We introduce a coordinate system ($\bar{x}, \bar{y}, \bar{z}$) with $\bar{x}$-axis vertically upwards along the plate, $\bar{y}$-axis perpendicular to it and directed into the fluid region and $\bar{z}$-axis along the width of the plate. Let $\bar{q} = (\bar{u}, \bar{v}, 0)$ be the fluid velocity and $(\bar{B}_x, \bar{B}_y, 0)$ be the components of magnetic induction vector at a point $(\bar{x}, \bar{y}, \bar{z})$ in the fluid.

With the foregoing assumption, Boussinesq approximation and under usual boundary layer approximations, the governing equations are:

Equation of Continuity:

$$\frac{d\bar{v}}{d\bar{y}} = 0$$  \hspace{1cm} (5.2.1)

which is satisfied with $\bar{v} = -\bar{v}_0$, a constant

Momentum equation:

$$-\nu_0 \frac{d\bar{u}}{d\bar{y}} = \nu \frac{d^2\bar{u}}{d\bar{y}^2} + \frac{k_s}{\rho} \frac{d^2\bar{u}}{d\bar{y}^3} + g\beta(\bar{T} - \bar{T}_0) + g\beta(\bar{C} - \bar{C}_0) + \frac{\eta\sigma B_0}{\rho} \frac{d\bar{B}_x}{d\bar{y}}$$  \hspace{1cm} (5.2.2)
Energy equation:

\[-\nu_0 \frac{d\tilde{T}}{d\tilde{y}} = \frac{k}{\rho C_p} \frac{d^2 \tilde{T}}{d\tilde{y}^2} + \frac{\nu}{C_p} \left( \frac{d\tilde{u}}{d\tilde{y}} \right)^2 + \frac{k_0 \nu_0}{\rho C_p} \frac{d\tilde{u}}{d\tilde{y}} \frac{d^2 \tilde{u}}{d\tilde{y}^2} + \frac{\sigma \eta^2}{\rho C_p} \left( \frac{d\tilde{b}_x}{d\tilde{y}} \right)^2 + \frac{Q(\tilde{T}_{\infty} - \tilde{T})}{\rho C_p} \]  

(5.2.3)

Magnetic induction equation:

\[\eta \frac{d^2 \tilde{b}_x}{d\tilde{y}^2} + B_0 \frac{d\tilde{u}}{d\tilde{y}} + \nu_0 \frac{d\tilde{b}_x}{d\tilde{y}} = 0 \]  

(5.2.4)

Species continuity equation:

\[-\nu_0 \frac{d\tilde{C}}{d\tilde{y}} = D \frac{d^2 \tilde{C}}{d\tilde{y}^2} - \tilde{R}(\tilde{C} - \tilde{C}_{\infty}) \]  

(5.2.5)

where \(k_0\) is the visco-elastic parameter, \(\sigma\) is the electrical conductivity, \(k\) is the thermal conductivity, \(C_p\) is the specific heat at constant pressure, \(\eta\) is the magnetic diffusivity, \(\rho\) is the density of the fluid, \(Q\) is the heat source parameter, \(\tilde{C}\) is the species concentration, \(D\) is the coefficient of chemical molecular diffusivity and the other symbols have their usual meanings.

The relevant boundary conditions are:

\[\tilde{y} = 0: \tilde{u} = 0, \frac{d\tilde{T}}{d\tilde{y}} = -\frac{\tilde{q}}{K}, \tilde{b}_x = 0, \tilde{C} = \tilde{C}_{\infty}\]

\[\tilde{y} \to \infty: \tilde{u} = \tilde{U}, \tilde{T} \to \tilde{T}_{\infty}, \tilde{b}_x = \tilde{H}, \tilde{C} \to \tilde{C}_{\infty}\]  

(5.2.6)

We introduce the following non-dimensional quantities:

\[y = \frac{\tilde{y} v_0}{\nu}, u = \frac{\tilde{u}}{v_0}, \theta = \frac{K v_0 (\tilde{T} - \tilde{T}_{\infty})}{\tilde{q}}, U = \frac{\tilde{U}}{v_0}, b_x = \frac{\tilde{b}_x}{B_0}, \]

\[K = \frac{K v_0}{\nu}, Ec = \frac{K v_0^3}{\tilde{q} v C_p}, \Omega = \frac{\tilde{C} - \tilde{C}_{\infty}}{\tilde{C}_{w} - \tilde{C}_{\infty}}, \]

\[Gr = \frac{g \beta v \tilde{q}}{K v_0^4}, S = \frac{Q v^2}{K v_0^2}, G_m = \frac{g \tilde{b}_x (\tilde{C}_{w} - \tilde{C}_{\infty})}{v_0^3}, Sc = \frac{v}{D}, M = \frac{\sigma B_0^2 v}{\rho v_0^2}, Pr = \frac{\mu C_p}{k}, P_m = \frac{v}{\eta} \]

where \(Ec\) is the Eckert number, \(Gr\) is the Grashof number for heat transfer, \(S\) is the heat source parameter, \(G_m\) is the Grashof number for mass transfer, \(Sc\) is the Schmidt number, \(M\) is the Hartmann number, \(Pr\) is the Prandtl number and \(P_m\) is the magnetic Prandtl number.

The non-dimensional governing equations are

\[k_1 \frac{d^3 u}{dy^3} + \frac{d^2 u}{dy^2} + \frac{du}{dy} = -Gr \theta - Gm \Omega - \frac{M \frac{db_x}{P_m}}{dy} \]  

(5.2.7)
\[
\frac{d^2 \theta}{dy^2} + Pr \frac{d\theta}{dy} - S \theta = -EcPr \left( \frac{du}{dy} \right)^2 - k_1 EcPr \frac{du}{dy} \frac{d^2 u}{dy^2} - \frac{MEcPr}{Pm^2} \left( \frac{db_x}{dy} \right)^2 \tag{5.2.8}
\]
\[
\frac{d^2 b_x}{dy^2} + Pr \frac{db_x}{dy} = -Pr \frac{du}{dy} \tag{5.2.9}
\]
\[
\frac{1}{Sc} \frac{d^2 \phi}{dy^2} + \frac{d\phi}{dy} - k\phi = 0 \tag{5.2.10}
\]
subject to boundary conditions:
\[
y = 0: u = 0, \frac{d\theta}{dy} = -1, \phi = 1, b_x = 0
\]
\[
y \to \infty: u = U, \theta = 0, \phi = 0, b_x = H \tag{5.2.11}
\]

5.3 METHOD OF SOLUTION

The solution of the equation (5.2.10) subject to the boundary conditions (5.2.11) is
\[
\phi = e^{-A_1 y} \tag{5.3.1}
\]
Now, in order to solve the equations (5.2.7)-(5.2.9) under the boundary conditions given by (5.2.11), it is assumed that the solutions of the equations to be of the form
\[
u = u_0 + Ec\theta_1 + Ec^2b_x + \cdots \tag{5.3.2}
\]
\[
\theta = \theta_0 + Ec\theta_1 + Ec^2b_x + \cdots \tag{5.3.3}
\]
\[
b_x = \phi_0 + Ec\phi_1 + Ec^2b_x + \cdots \tag{5.3.4}
\]
where \( Ec \ll 1 \).

Substituting (5.3.2)-(5.3.4) in the equations (5.2.7)-(5.2.9) and equating the coefficient of the same degree terms and neglecting terms of \( O(Ec^2) \), the following differential equations are obtained:

Zeroth-order equations:
\[
k_1u_0 + \dot{u}_0 + u_0 = -Gr\theta_0 - Gme^{-A_1 y} -\frac{M}{Pm} b'_x \tag{5.3.5}
\]
\[
\theta_0 + Pr\theta_1 - S\theta_0 = 0 \tag{5.3.6}
\]
\[
b_x + Pm\phi_0 = -Pm\phi_0 \tag{5.3.7}
\]

First-order equations:
\[
k_1u_1 + \dot{u}_1 + u_1 = -Gr\theta_1 - \frac{M}{Pm} b'_x \tag{5.3.8}
\]
\[
\theta_1 + Pr\theta_1 - S\theta_1 = -Pr\phi_0^2 - k_1 Pr\theta_0 u_0 - \frac{MPr}{Pm^2} b'_x \tag{5.3.9}
\]
\[ b_{x_1}'' + Pmb'_{x_1} = -Pmu_1 \]  
(5.3.10)

corresponding to the boundary conditions:
\[ y = 0: u_0 = 0, u_1 = 0, \theta_0 = -1, \theta_1 = 0, b_{x_0} = 0, b_{x_1} = 0 \]
\[ y \to \infty: u_0 = U, u_1 = 0, \theta_0 = 0, \theta_1 = 0, \psi = 0, b_{x_0} = H, b_{x_1} = 0 \]  
(5.3.11)

Using multi-parameter perturbation technique and taking \( k_1 \ll 1 \) (as for small shear rate \( k_1 \) is very small), we assume
\[ u_0 = u_{00} + k_1 u_{01} \]  
(5.3.12)
\[ u_1 = u_{10} + k_1 u_{11} \]  
(5.3.13)
\[ b_{x_0} = b_{x_{00}} + k_1 b_{x_{01}} \]  
(5.3.14)
\[ b_{x_1} = b_{x_{10}} + k_1 b_{x_{11}} \]  
(5.3.15)
\[ \theta_1 = \theta_{10} + k_1 \theta_{11} \]  
(5.3.16)

Now, using equations (5.3.12)-(5.3.16) in equations (5.3.5), (5.3.7), (5.3.8), (5.3.9) and (5.3.10) and equating the coefficients of like powers of \( k_1 \) and neglecting the higher power of \( k_1 \), we get the following set of differential equations:

**Zeroth-order equations:**
\[ u_{00}'' + u_{00}' = -GrB_1e^{-A_2\gamma} - Gme^{-A_1\gamma} - \frac{M}{Pm} b_{x_{00}} \]  
(5.3.17)
\[ b_{x_{00}}'' + Pmb_{x_{00}}' = -Pmu_{00}' \]  
(5.3.18)
\[ u_{10}'' + u_{10}' = -Gr\theta_{10} - \frac{M}{Pm} b_{x_{10}}' \]  
(5.3.19)
\[ \theta_{10}'' + Pr\theta_{10}' - S\theta_{10} = -Pr u_{00}' - \frac{MPr}{Pm^2} b_{x_{00}}' \]  
(5.3.20)
\[ b_{x_{10}}'' + Pmb_{x_{10}}' = -Pmu_{10}' \]  
(5.3.21)

**First-order equations:**
\[ u_{00}'' + u_{01}'' + u_{01}' = -\frac{M}{Pm} b_{x_{01}}' \]  
(5.3.22)
\[ b_{x_{01}}'' + Pmb_{x_{01}}' = -Pmu_{01}' \]  
(5.3.23)
\[ u_{10}'' + u_{11}'' + u_{11}' = -Gr\theta_{11} - \frac{M}{Pm} b_{x_{11}}' \]  
(5.3.24)
\[ \theta_{11}'' + Pr\theta_{11}' - S\theta_{11} = -2Pr u_{00}' u_{10} - Pr u_{00}' u_{00} - 2\frac{MPr}{Pm^2} b_{x_{00}}' b_{x_{01}}' \]  
(5.3.25)
\[ b_{x_{11}}'' + Pmb_{x_{11}}' = -Pmu_{11}' \]  
(5.3.26)

subject to boundary conditions:

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\[ y = 0: u_{00} = 0, u_{01} = 0, u_{10} = 0, u_{11} = 0, \theta_{00} = -1, \theta_{01} = 0, \theta_{10} = 0, \theta_{11} = 0, \]
\[ b_{x_{00}} = 0, b_{x_{01}} = 0, b_{x_{10}} = 0, b_{x_{11}} = 0 \]
\[ y \to \infty: u_{00} = U, u_{01} = 0, u_{10} = 0, u_{11} = 0, \theta_{00} = 0, \theta_{01} = 0, \theta_{10} = 0, \theta_{11} = 0, \]
\[ b_{x_{00}} = H, b_{x_{01}} = 0, b_{x_{10}} = 0, b_{x_{11}} = 0 \]

(5.3.27)

The solutions of the equations (5.3.6), (5.3.17)-(5.3.26) subject to the boundary conditions (5.3.11) and (5.3.27) are:

\[ \theta_0 = B_1 e^{-A_2 y} \]

(5.3.28)

\[ b_{x_{00}} = H + B_2 e^{-A_2 y} + B_3 e^{-A_1 y} + B_4 e^{-A_3 y} \]

(5.3.29)

\[ b_{x_{01}} = B_{45} e^{-y} + B_{46} e^{-A_2 y} + B_{47} e^{-A_1 y} + B_{48} e^{-A_3 y} \]

(5.3.30)

\[ b_{x_{10}} = B_{31} e^{-A_1 y} + B_{20} e^{-A_3 y} + B_{21} e^{-2 y} + B_{22} e^{-2 A_1 y} + B_{23} e^{-2 A_2 y} + B_{24} e^{-2 A_3 y} \]
\[ + B_{25} e^{-(1+1) A_1 y} + B_{26} e^{-(1+2) y} + B_{27} e^{-(1+A_3) y} + B_{28} e^{-(A_1+A_2) y} \]
\[ + B_{29} e^{-(A_1+A_3) y} + B_{30} e^{-(A_2+A_3) y} \]

(5.3.31)

\[ b_{x_{11}} = B_{64} e^{-A_3 y} \]
\[ + B_{65} e^{-2 y} + B_{66} e^{-2 A_1 y} + B_{67} e^{-2 A_2 y} + B_{68} e^{-2 A_3 y} + B_{69} e^{-(1+A_1) y} \]
\[ + B_{70} e^{-(1+A_2) y} + B_{71} e^{-(1+A_3) y} + B_{72} e^{-(A_1+A_2) y} + B_{73} e^{-(A_1+A_3) y} \]
\[ + B_{74} e^{-(A_2+A_3) y} + B_{75} e^{-y} + B_{76} e^{-A_3 y} \]

(5.3.32)

\[ u_{00} = U + B_8 e^{-y} + B_9 e^{-A_1 y} + B_{10} e^{-A_2 y} + B_{11} e^{-A_3 y} \]

(5.3.33)

\[ u_{01} = B_{49} e^{-A_1 y} + B_{50} e^{-A_2 y} + B_{51} e^{-A_3 y} + B_{52} e^{-y} \]

(5.3.34)

\[ u_{10} = B_{32} e^{-A_1 y} + B_{33} e^{-A_2 y} + B_{34} e^{-2 y} + B_{35} e^{-2 A_1 y} + B_{36} e^{-2 A_2 y} + B_{37} e^{-2 A_3 y} \]
\[ + B_{38} e^{-(1+A_1) y} + B_{39} e^{-(1+A_2) y} + B_{40} e^{-(1+A_3) y} + B_{41} e^{-(A_1+A_2) y} \]
\[ + B_{42} e^{-(A_1+A_3) y} + B_{43} e^{-(A_2+A_3) y} + B_{44} e^{-y} \]

(5.3.35)

\[ u_{11} = B_{77} e^{-A_1 y} + B_{78} e^{-A_2 y} + B_{79} e^{-2 y} + B_{80} e^{-2 A_1 y} + B_{81} e^{-2 A_2 y} + B_{82} e^{-2 A_3 y} \]
\[ + B_{83} e^{-(1+A_1) y} + B_{84} e^{-(1+A_2) y} + B_{85} e^{-(1+A_3) y} + B_{86} e^{-(A_1+A_2) y} \]
\[ + B_{87} e^{-(A_1+A_3) y} + B_{88} e^{-(A_2+A_3) y} + B_{89} e^{-y} \]

(5.3.36)

\[ \theta_{10} = B_{19} e^{-A_1 y} + B_{20} e^{-2 y} + B_{21} e^{-2 A_1 y} + B_{11} e^{-2 A_2 y} + B_{12} e^{-2 A_3 y} + B_{13} e^{-(1+A_1) y} \]
\[ + B_{14} e^{-(1+A_2) y} + B_{15} e^{-(1+A_3) y} + B_{16} e^{-(A_1+A_2) y} + B_{17} e^{-(A_1+A_3) y} \]
\[ + B_{18} e^{-(A_2+A_3) y} \]

(5.3.37)
\[
\theta_{11} = B_{53}e^{-(1+A_1)y} + B_{54}e^{-(1+A_2)y} + B_{55}e^{-(1+A_3)y} + B_{56}e^{-2y} + B_{57}e^{-2A_1y} \\
+ B_{68}e^{-(A_1+A_2)y} + B_{69}e^{-(A_1+A_3)y} + B_{60}e^{-2A_2y} + B_{61}e^{-(A_2+A_3)y} + B_{62}e^{-2A_3y} \\
+ B_{63}e^{-A_3y}
\]

where the values of the constants are given by

\[
\begin{align*}
A_1 &= \frac{S + \sqrt{Sc(Sc + 4k)}}{2}, & A_2 &= \frac{Pr + \sqrt{Pr^2 + 4S}}{2}, & B_1 &= \frac{1}{A_2}, \\
A_3 &= \frac{(Pm + 1) + \sqrt{(Pm + 1)^2 - 4(Pm - M)}}{2}, \\
B_2 &= \frac{PmGrB_1}{2}, \\
B_3 &= \frac{PmGm}{2}, \\
B_4 &= -H - B_2 - B_3, & B_5 &= \frac{-(Gm + A_1B_3)}{A_1(A_1 - 1)} ,
\end{align*}
\]

\[
\begin{align*}
B_6 &= \frac{-(GrB_1 + A_2B_2)}{A_2(A_2 - 1)}, & B_7 &= \frac{-B_6}{A_3 - 1}, & B_8 &= -U - B_5 - B_6 - B_7, \\
A_4 &= \frac{Pr + \sqrt{Pr^2 + 4S}}{2}, & B_9 &= \frac{-PrB_6^2}{4 - 2Pr - S}, & B_{10} &= \frac{-Pr\left(A_1^2B_6^2 + \frac{M}{Pm^2}A_1^2B_3^2\right)}{4A_1^2 - 2PrA_1 - S}, \\
B_{11} &= \frac{-Pr\left(A_1^2B_6^2 + \frac{M}{Pm^2}A_1^2B_3^2\right)}{4A_1^2 - 2PrA_1 - S}, \\
B_{12} &= \frac{-Pr\left(A_2^2B_7^2 + \frac{M}{Pm^2}A_2^2B_3^2\right)}{4A_2^2 - 2PrA_2 - S}, \\
B_{13} &= \frac{-2PrA_1B_7B_8}{(1 + A_1)^2 - Pr(1 + A_1) - S}, & B_{14} &= \frac{-2PrA_2B_6B_8}{(1 + A_2)^2 - Pr(1 + A_2) - S}, \\
B_{15} &= \frac{-2PrA_3B_7B_8}{(1 + A_3)^2 - Pr(1 + A_3) - S}, \\
B_{16} &= \frac{-Pr\left(2A_1A_2B_5B_6 + 2\frac{M}{Pm^2}A_1A_2B_2B_3\right)}{(A_1 + A_2)^2 - Pr(A_1 + A_2) - S}, \\
B_{17} &= \frac{-Pr\left(2A_1A_3B_5B_7 + 2\frac{M}{Pm^2}A_1A_3B_3B_4\right)}{(A_1 + A_3)^2 - Pr(A_1 + A_3) - S}, \\
B_{18} &= \frac{-Pr\left(2A_2A_3B_6B_7 + 2\frac{M}{Pm^2}A_2A_3B_2B_4\right)}{(A_2 + A_3)^2 - Pr(A_2 + A_3) - S},
\end{align*}
\]

\text{(5.3.38)}
\[ B_{19} = -\frac{1}{A_4} \left( 2B_9 + 2A_1B_{10} + 2A_2B_{11} + 2A_3B_{12} + (1 + A_1)B_{13} + (1 + A_2)B_{14} \right. \]
\[ + (1 + A_3)B_{15} + (A_1 + A_2)B_{16} + (A_1 + A_3)B_{17} + (A_2 + A_3)B_{18} \left. \right) \]
\[ B_{20} = -\frac{PmGrA_4B_{19}}{-A_4^2 + (Pm + 1)A_4^2 - (Pm - M)A_4}, \]
\[ B_{21} = -\frac{-2PmGrB_9}{-8 + 4(Pm + 1) - 2(Pm - M)}, \]
\[ B_{22} = -\frac{-2PmGrA_1B_{10}}{-8A_1^3 + 4A_1^2(Pm + 1) - 2A_1(Pm - M)}, \]
\[ B_{23} = -\frac{-2PmGrA_2B_{11}}{-8A_2^3 + 4A_2^2(Pm + 1) - 2A_2(Pm - M)}, \]
\[ B_{24} = -\frac{-2PmGrA_3B_{12}}{-8A_3^3 + 4A_3^2(Pm + 1) - 2A_3(Pm - M)}, \]
\[ B_{25} = -\frac{-(1 + A_1)^2 + (Pm + 1)(1 + A_1) - (Pm - M)}{-PmGrB_{13}}, \]
\[ B_{26} = -\frac{-(1 + A_2)^2 + (Pm + 1)(1 + A_2) - (Pm - M)}{-PmGrB_{14}}, \]
\[ B_{27} = -\frac{-(1 + A_3)^2 + (Pm + 1)(1 + A_3) - (Pm - M)}{-PmGrB_{15}}, \]
\[ B_{28} = -\frac{-(A_1 + A_2)^2 + (Pm + 1)(A_1 + A_2) - (Pm - M)}{-PmGrB_{16}}, \]
\[ B_{29} = -\frac{-(A_1 + A_3)^2 + (Pm + 1)(A_1 + A_3) - (Pm - M)}{-PmGrB_{17}}, \]
\[ B_{30} = -\frac{-(A_2 + A_3)^2 + (Pm + 1)(A_2 + A_3) - (Pm - M)}{-PmGrB_{18}}, \]
\[ B_{31} = -(B_{20} + B_{21} + B_{22} + B_{23} + B_{24} + B_{25} + B_{26} + B_{27} + B_{28} + B_{29} + B_{30}), \]
\[ B_{32} = \frac{MA_3B_{31}}{PmA_3(A_3 - 1)}, B_{33} = \frac{M}{PmA_4B_{20} - GrB_{19}} A_4(A_4 - 1), B_{34} = \frac{2M}{PmA_4B_{21} - GrB_{20}} \]
\[ B_{35} = \frac{2M}{PmA_4B_{22} - GrB_{10}} A_1(2A_1 - 1), B_{36} = \frac{2M}{PmA_2B_{23} - GrB_{11}} A_2(2A_2 - 1), \]
\[ B_{37} = \frac{2M}{PmA_3B_{24} - GrB_{12}} \frac{A_1}{A_2(2A_3 - 1)}, B_{38} = \frac{M}{PmA_4B_{25} - GrB_{13}} \frac{1 + A_1}{A_1(1 + A_1)}, \]
\[ B_{39} = \frac{M}{PmA_4B_{26} - GrB_{14}} \frac{(1 + A_2)}{A_2(1 + A_2)}, B_{40} = \frac{M}{PmA_3B_{27} - GrB_{15}} \frac{(1 + A_3)}{A_3(1 + A_3)}, \]
\[ B_{41} = \frac{M (A_1 + A_2) B_{28} - GrB_{16}}{(A_1 + A_2)(A_1 + A_2 - 1)} , \]

\[ B_{42} = \frac{M (A_1 + A_3) B_{29} - GrB_{17}}{(A_1 + A_3)(A_1 + A_3 - 1)} , B_{43} = \frac{M (A_2 + A_3) B_{30} - GrB_{18}}{(A_2 + A_3)(A_2 + A_3 - 1)} , \]

\[ B_{44} = -(B_{32} + B_{33} + B_{34} + B_{35} + B_{36} + B_{37} + B_{38} + B_{39} + B_{40} + B_{41} + B_{42} + B_{43}) , \]

\[ B_{45} = \frac{-B_{8} P_m}{M} , \]

\[ B_{46} = -\frac{A_1^2 B_6 P_m}{A_1^2 + A_2^2 (P_m + 1) - A_2 (P_m - M)} , \]

\[ B_{47} = -\frac{A_2^2 B_6 P_m}{A_2^2 + A_2^2 (P_m + 1) - A_2 (P_m - M)} , \]

\[ B_{48} = -(B_{45} + B_{46} + B_{47}) , B_{49} = \frac{M}{P_m} A_1 A_1 B_{46} + A_1^2 B_5}{A_1 (A_1 - 1)} , B_{50} = \frac{M}{P_m} A_2 A_2 B_{47} + A_2^2 B_6}{A_2 (A_2 - 1)} , \]

\[ B_{51} = \frac{M}{P_m} A_3 B_{48} + A_3^2 B_7}{A_3 (A_3 - 1)} , B_{52} = -(B_{49} + B_{50} + B_{51}) , \]

\[ B_{53} = -\frac{Pr \left( 2 A_1 B_6 B_{49} + 2 A_1 B_6 B_{52} - A_1^2 B_6 B_8 - A_1 B_1 B_8 - 2 \frac{M}{P_m} A_1 A_1 B_{45} \right)}{(1 + A_1)^2 - Pr(1 + A_1) - S} , \]

\[ B_{54} = -\frac{Pr \left( 2 A_2 B_6 B_{50} + 2 A_2 B_6 B_{52} - A_2^2 B_6 B_8 - A_2 B_1 B_8 - 2 \frac{M}{P_m} A_2 A_2 B_{45} \right)}{(1 + A_2)^2 - Pr(1 + A_2) - S} , \]

\[ B_{55} = -\frac{Pr \left( 2 A_3 B_6 B_{51} + 2 A_3 B_7 B_{52} - A_3^2 B_7 B_8 - A_3 B_1 B_8 - 2 \frac{M}{P_m} A_3 A_3 B_{45} \right)}{(1 + A_3)^2 - Pr(1 + A_3) - S} , \]

\[ B_{56} = -\frac{Pr(2 B_5 B_{52} - B_8)}{4 - 2Pr - S} , B_{57} = -\frac{Pr \left( 2 A_1^2 B_5 B_{49} - A_1^3 B_5^2 - 2 \frac{M}{P_m} A_1 A_1 B_5 B_{46} \right)}{4A_1^2 - 2A_1 Pr - S} , \]

\[ B_{58} = -\frac{Pr \left( 2 A_1 A_2 B_5 B_{50} + 2 A_1 A_2 B_6 B_{49} - A_1 A_2^2 B_5 B_6 - A_2 A_1^2 B_2 B_8 - \frac{M}{P_m} A_1 A_2 B_{46} - \frac{M}{P_m^2} A_1 A_2 A_2 B_{47} \right)}{(A_1 + A_2)^2 - Pr(A_1 + A_2) - S} , \]

\[ B_{59} = -\frac{Pr \left( 2 A_1 A_3 B_5 B_{51} + 2 A_1 A_3 B_7 B_{49} - A_1 A_3^2 B_5 B_7 \right)}{(A_1 + A_3)^2 - Pr(A_1 + A_3) - S} , \]

\[ B_{60} = -\frac{Pr \left( 2 A_2^2 B_4 B_{50} - A_2^3 B_6^2 - 2 \frac{M}{P_m} A_2^2 B_2 B_{47} \right)}{4A_2^2 - 2A_2 Pr - S} , \]
\[ B_{61} = \frac{\left( 2A_2A_3B_6B_{51} + 2A_2A_3B_7B_{50} - A_2A_3^2B_6B_7 - A_3A_2^2B_6B_7 \right) - \frac{M}{Pm^2} A_2A_3B_2B_{48} - \frac{M}{Pm^2} A_2A_3B_4B_{47}}{(A_2 + A_3)^2 - Pr(A_2 + A_3) - S}, \]

\[ B_{62} = \frac{-Pr \left( 2A_2^2B_7B_{51} - A_3^2B_7^2 - \frac{M}{Pm^2} A_3^2B_4B_{48} \right)}{4A_3^2 - 2A_3Pr - S}, \]

\[ B_{63} = \frac{-1}{A_4} \left( (1 + A_1)B_{53} + (1 + A_2)B_{54} + (1 + A_3)B_{55} + 2B_{56} + 2A_1B_{57} + (A_1 + A_2)B_{58} + (A_1 + A_3)B_{59} + 2A_2B_{60} + (A_2 + A_3)B_{61} + 2A_3B_{62} \right), \]

\[ B_{64} = \frac{Pm(GrB_{63} - A_3^2B_{53})}{-A_3^2 + (Pm + 1)A_3^2 - (Pm - M)A_4}, \]

\[ B_{65} = \frac{Pm(GrB_{56} - 8B_{34})}{-8 + 4(Pm + 1) - 2(Pm - M)}, \]

\[ B_{66} = \frac{Pm(GrB_{57} - 8A_3^2B_{55})}{-8A_3^2 + 4A_3^2(Pm + 1) - 2A_3(Pm - M)}, \]

\[ B_{67} = \frac{Pm(GrB_{60} - 8A_3^2B_{36})}{-8A_3^2 + 4A_3^2(Pm + 1) - 2A_3(Pm - M)}, \]

\[ B_{68} = \frac{Pm(GrB_{62} - 8A_3^2B_{37})}{-8A_3^2 + 4A_3^2(Pm + 1) - 2A_3(Pm - M)}, \]

\[ B_{69} = \frac{Pm(GrB_{53} - (1 + A_1)^3B_{38})}{-(1 + A_1)^3 + (Pm + 1)(1 + A_2)^2 - (Pm - M)(1 + A_1)}, \]

\[ B_{70} = \frac{Pm(GrB_{54} - (1 + A_2)^3B_{39})}{-(1 + A_2)^3 + (Pm + 1)(1 + A_2)^2 - (Pm - M)(1 + A_2)}, \]

\[ B_{71} = \frac{Pm(GrB_{55} - (1 + A_3)^3B_{40})}{-(1 + A_3)^3 + (Pm + 1)(1 + A_3)^2 - (Pm - M)(1 + A_3)}, \]

\[ B_{72} = \frac{Pm(GrB_{58} - (A_1 + A_2)^3B_{41})}{-(A_1 + A_2)^3 + (Pm + 1)(A_1 + A_2)^2 - (Pm - M)(A_1 + A_2)}, \]

\[ B_{73} = \frac{Pm(GrB_{59} - (A_1 + A_3)^3B_{42})}{-(A_1 + A_3)^3 + (Pm + 1)(A_1 + A_3)^2 - (Pm - M)(A_1 + A_3)}, \]

\[ B_{74} = \frac{Pm(GrB_{61} - (A_2 + A_3)^3B_{43})}{-(A_2 + A_3)^3 + (Pm + 1)(A_2 + A_3)^2 - (Pm - M)(A_2 + A_3)}, \]

\[ B_{75} = \frac{-PmB_{66}}{-1 + (Pm + 1) - (Pm - M)}, \]

\[ B_{76} = -(B_{64} + B_{65} + B_{66} + B_{67} + B_{68} + B_{69} + B_{70} + B_{71} + B_{72} + B_{73} + B_{74} + B_{75}), \]
\[ B_{77} = \frac{A_3^2 B_{32} + \frac{M}{p_m} B_{75}}{A_3 (A_3 - 1)}, \quad B_{78} = \frac{A_4^3 B_{33} - Gr B_{63} + \frac{M}{p_m} A_4 B_{64}}{A_4 (A_4 - 1)}, \]

\[ B_{79} = \frac{8 B_{34} - Gr B_{56} + 2 \frac{M}{p_m} B_{65}}{2}, \quad B_{80} = \frac{8 A_1^3 B_{35} - Gr B_{57} + 2 \frac{M}{p_m} A_1 B_{66}}{2A_1 (2A_1 - 1)}, \]

\[ B_{81} = \frac{8 A_2^3 B_{36} - Gr B_{60} + 2 \frac{M}{p_m} A_2 B_{67}}{2A_2 (2A_2 - 1)}, \quad B_{82} = \frac{8 A_3^3 B_{37} - Gr B_{62} + 2 \frac{M}{p_m} A_3 B_{68}}{2A_3 (2A_3 - 1)}, \]

\[ B_{83} = \frac{(1 + A_1)^3 B_{38} - Gr B_{53} + \frac{M}{p_m} (1 + A_1) B_{69}}{A_2 (1 + A_2)}, \]

\[ B_{84} = \frac{(1 + A_2)^3 B_{39} - Gr B_{54} + \frac{M}{p_m} (1 + A_2) B_{70}}{A_2 (1 + A_2)}, \]

\[ B_{85} = \frac{(1 + A_3)^3 B_{40} - Gr B_{55} + \frac{M}{p_m} (1 + A_3) B_{71}}{A_3 (1 + A_3)}, \]

\[ B_{86} = \frac{(A_1 + A_2)^3 B_{41} - Gr B_{58} + \frac{M}{p_m} (A_1 + A_2) B_{72}}{(A_1 + A_2) (A_1 + A_2 - 1)}, \]

\[ B_{87} = \frac{(A_1 + A_3)^3 B_{42} - Gr B_{59} + \frac{M}{p_m} (A_1 + A_3) B_{73}}{(A_1 + A_3) (A_1 + A_3 - 1)}, \]

\[ B_{88} = \frac{(A_2 + A_3)^3 B_{43} - Gr B_{61} + \frac{M}{p_m} (A_2 + A_3) B_{74}}{(A_2 + A_3) (A_2 + A_3 - 1)}, \]

\[ B_{89} = -(B_{77} + B_{78} + B_{79} + B_{80} + B_{81} + B_{82} + B_{83} + B_{84} + B_{85} + B_{86} + B_{87} + B_{88}) \]

### 5.4 Results and Discussion

The fluid velocity is given by

\[ u = (u_{00} + k_1 u_{01}) + Ec (u_{10} + k_1 u_{11}) \quad (5.4.1) \]

The non-dimensional shearing stress \( \sigma \) at the plate \( y=0 \) is given by

\[ \sigma = \frac{du}{dy} + k_1 \frac{d^2 u}{dy^2} |_{y=0} = M_1 + k_1 M_2 + Ec (M_3 + k_1 M_4) + k_1 ((M_5 + k_1 M_6) + Ec (M_7 + k_1 M_8)) \quad (5.4.2) \]

The non-dimensional heat flux \( Nu \) at the plate \( y=0 \) in terms of Nusselt number is given by
\[ Nu = \frac{d\theta}{dy} \bigg|_{y=0} = N_1 + Ec (N_2 + k_1 N_3) \] (5.4.3)

The non-dimensional mass flux \( Sh \) at the plate \( y=0 \) in terms of Sherwood number is given by

\[ Sh = \frac{d\phi}{dy} \bigg|_{y=0} = -A_1 \] (5.4.4)

where the values of the constants are

\[ M_1 = -(B_8 + A_1 B_5 + A_2 B_6 + A_3 B_7) \]
\[ M_2 = -(A_1 B_{50} + A_2 B_{51} + A_3 B_{52} + B_{53}) \]
\[ M_3 = -(A_3 B_{32} + A_4 B_{33} + 2B_{34} + 2A_1 B_{35} + 2A_2 B_{36} + 2A_3 B_{37} + (1 + A_1) B_{38} + (1 + A_2) B_{39} + (1 + A_3) B_{40} + (A_1 + A_2) B_{41} + (A_1 + A_3) B_{42} + (A_2 + A_3) B_{43} + B_{44}) \]
\[ M_4 = -(A_3 B_{77} + A_4 B_{78} + 2B_{79} + 2A_1 B_{80} + 2A_2 B_{81} + 2A_3 B_{82} + (1 + A_1) B_{83} + (1 + A_2) B_{84} + (1 + A_3) B_{85} + (A_1 + A_2) B_{86} + (A_1 + A_3) B_{87} + (A_2 + A_3) B_{88} + B_{89}) \]
\[ M_5 = B_8 + A_1^2 B_5 + A_2^2 B_6 + A_3^2 B_7 \]
\[ M_6 = A_1^2 B_{50} + A_2^2 B_{51} + A_3^2 B_{52} + B_{53} \]
\[ M_7 = A_3^2 B_{32} + A_4^2 B_{33} + 4B_{34} + 4A_1^2 B_{35} + 4A_2^2 B_{36} + 4A_3^2 B_{37} + (1 + A_1)^2 B_{38} + (1 + A_2)^2 B_{39} + (1 + A_3)^2 B_{40} + (A_1 + A_2)^2 B_{41} + (A_1 + A_3)^2 B_{42} + (A_2 + A_3)^2 B_{43} + B_{44} \]
\[ M_8 = A_3^2 B_{77} + A_4^2 B_{78} + 4B_{79} + 4A_1^2 B_{80} + 4A_2^2 B_{81} + 4A_3^2 B_{82} + (1 + A_1)^2 B_{83} + (1 + A_2)^2 B_{84} + (1 + A_3)^2 B_{85} + (A_1 + A_2)^2 B_{86} + (A_1 + A_3)^2 B_{87} + (A_2 + A_3)^2 B_{88} + B_{89} \]

\[ N_1 = -A_2 B_{11}, \]
\[ N_2 = -(A_4 B_{19} + 2B_9 + 2A_1 B_{10} + 2A_2 B_{11} + 2A_3 B_{12} + (1 + A_1) B_{13} + (1 + A_2) B_{14} + (1 + A_3) B_{15} + (A_1 + A_2) B_{16} + (A_1 + A_3) B_{17} + (A_2 + A_3) B_{18}) \]
\[ N_3 = -(1 + A_1) B_{53} + (1 + A_2) B_{54} + (1 + A_3) B_{55} + 2B_{56} + 2A_1 B_{57} + (A_1 + A_2) B_{58} + (A_1 + A_3) B_{59} + 2A_2 B_{60} + (A_2 + A_3) B_{61} + 2A_3 B_{62} + A_4 B_{63} \]

The purpose of the present study is to bring out the effects of elastico-viscous parameter on mixed convective MHD flow of a visco-elastic fluid past an infinite vertical porous surface by imposing the effect of induced magnetic field in the
governing fluid flow system. The elastico-viscous effect is exhibited through the non-dimensional parameter $k_1$. The non-zero values of the parameter $k_1$ characterize the visco-elastic fluid and $k_1=0$ represents the Newtonian fluid flow phenomenon.

In order to get physical insight into the problem, the fluid velocity $u$ is depicted against $y$ in the figures 5.2-5.8. The variation of the shearing stress $\sigma$ against various flow parameters viz. $P_m$, $Pr$, $G_m$, $S$, $Sc$, $M$ is illustrated in the figures 5.9-5.14. Figures 5.15-5.17 reveal the variation of Nusselt number against the flow parameters $Gr$, $Pr$ and $Sc$. The numerical calculations are to be carried out for $U=1$, $Ec=0.001$, $k=0.5$ throughout the discussion. The various combinations of flow parameters are given in the table 5.1.

Figures 5.2 to 5.8 represent the pattern of fluid velocity $u$ against the distance $y$ for various values of other flow parameters. The graphs show that the fluid velocity boosts up considerably in the neighbourhood of the plate and then it starts to converge to free stream velocity for both Newtonian and visco-elastic fluids. The elasticity factor of Walters liquid (Model $B'$) diminishes the speed of the fluid in comparison with Newtonian fluid.

Figures 5.2 and 5.3 represent the variation of fluid velocity against Grashof number for heat transfer ($Gr$) and Grashof number for mass transfer ($G_m$). Grashof number studies the behaviour of free convection and it is defined as the ratio of buoyancy force to viscous force. It plays an important role in both heat and mass transfer mechanisms. $Gr$ characterizes the free convection parameter for heat transfer and $G_m$ characterizes the free convection parameter for mass transfer. In both the cases, it is observed that fluid velocity boost up to a considerable amount and then follows a steady path.

The effect of $Pr$ on the fluid flow is illustrated in Figure 5.4. With the rising value of Prandtl number the fluid velocity experiences a decelerating trend. This phenomenon is observed in both Newtonian and elastico-viscous fluid.

The magnetic Prandtl number ($P_m$) signifies the relative importance of momentum diffusion and magnetic diffusion as it is defined as the ratio of momentum diffusivity to magnetic diffusivity. The effects of $P_m$ on Newtonian and non-Newtonian fluids have been observed in figure 5.5. It shows that the rising value of $P_m$ accelerates
the fluid flow but due to the presence of elasticity, the visco-elastic fluid flow experiences a declined trend during the enhancement of magnetic Prandtl number.

The effects of heat source parameter and Hartmann number on fluid velocity have been cited in figures 5.6 and 5.7. An inclined trend is observed for both kinds of fluid velocity with the growing nature of magnetic parameter and heat source parameter. The maximum effect of both the parameters on visco-elastic fluid and Newtonian fluid is seen in the neighbourhood of the plate.

Schmidt number signifies the ratio of momentum diffusivity to concentration diffusivity. The role of Schmidt number on the fluid velocity is illustrated in figure 5.8. Increasing value of Schmidt number increases the velocity of the Newtonian fluid as well as visco-elastic fluid. Also, the velocity of the visco-elastic fluid subdues with the enhancement of Schmidt number in comparison with simple Newtonian fluid.

Figures 5.9 to 5.14 exhibit the variation of shearing stress $\sigma$ against various flow parameters. In addition to this, the visco-elastic effects on shearing stress are also measured in these graphs. The elasticity factor present in the visco-elastic fluid subdues the shearing stress at the plate in comparison with Newtonian fluid.

Figures 5.9 and 5.10, characterize the variations of shearing stress against Prm and Pr. The positive values of Prandtl number signify the dominant effect of viscosity. It is noticed that the shearing stress formed by the visco-elastic fluid flow is negative, which interprets that the viscous drag experiences a reverse direction.

Figure 5.11 and 5.12 show that the magnitude of shearing stress is reduced along with the amplified values of Grashof number for mass transfer Gm and heat source parameter S for Newtonian as well as non-Newtonian cases.

Figure 5.13 shows the effects of Schmidt number on shearing stress. It is noticed that the magnitude of shearing stress increases with the increasing value of Sc for visco-elastic fluid but an opposite behaviour is observed in case of Newtonian fluid. The intensity of transverse magnetic field is shown in figure 5.14. Hartmann number subdues the shearing stress of visco-elastic fluid in comparison with the Newtonian fluid.

Nusselt number studies the rate of heat transfer through the fluid system. Here we have investigated the nature of Nusselt number on the flat plate. The graphical presentations of rate of heat transfer are given in figures 5.15 to 5.17. Visco-elasticity
factor present in the complex fluid flow system subdues the rate of heat transfer in comparison with the Newtonian fluid flow system.

Figure 5.15 characterizes the pattern of rate of heat transfer against Gr. It shows that increasing values of Gr modify the rate of heat transfer of Newtonian fluid in comparison with visco-elastic fluids. Figures 5.16 and 5.17 analyze the effects of heat source parameter S and Schmidt number Sc on variation of Nusselt number. In both the cases, it is observed that Nusselt number for non-Newtonian fluid increases with the rising values of other flow parameters.

Table 5.2 demonstrates the variation of induced magnetic field $b_x$ for various combinations of flow parameters (Table 5.1). From the table 5.2, it is observed that $b_x$ enhances with the increasing values of the visco-elastic parameter $k_1$ in comparison with the Newtonian fluid for different cases. Also, the induced magnetic field $b_x$ increases with the increase of Grashof number Gr (cases I & II) for visco-elastic fluid, but remains same for viscous fluid. Due to the increase of all other flow parameters (cases I & III, IV, V, VI, VII, VIII) the values of induced magnetic field slightly diminishes for non-Newtonian cases but modify in case of Newtonian cases.

5.5 CONCLUSION:

The study concludes the following results:

- The fluid velocity shows an enhancement trend in the neighbourhood of the plate and then follows a steady path.
- The visco-elasticity factor decelerates the speed of fluid flow in comparison with the Newtonian fluid.
- The shearing stress formed at the plate is subdued with the growing trend of visco-elastic parameter.
- The increasing values of Gm, S and M lessen the shearing stress formed by visco-elastic fluid.
- The shearing stress modifies with the increasing values of Schmidt number Sc.
- The rate of heat transfer enhances with the enhancement of S and Sc.
- The rate of mass transfer is not significantly affected by visco-elastic parameter.
Table 5.1: Various combinations of flow parameters

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<th>Gr</th>
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Table 5.2: The values of induced magnetic field $b_z$. 

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Figure 5.1: The physical model of the problem

Figure 5.2: Variation of transient velocity $u$ versus $y$ for $Pr=5$, $Pm=4$, $Gm=7$, $S=2$, $Sc=5$, $M=5$.

Figure 5.3: Variation of transient velocity $u$ versus $y$ for $Pr=5$, $Pm=4$, $Gr=8$, $S=2Sc=5$, $M=5$. 81
Figure 5.4: Variation of transient velocity $u$ versus $y$ for $Gr=8$, $Pm=4$, $Gm=7$, $S=2$, $Sc=5$, $M=5$.

Figure 5.5: Variation of transient velocity $u$ versus $y$ for $Pr=5$, $Gr=8$, $Gm=7$, $S=2$, $Sc=5$, $M=5$.

Figure 5.6: Variation of transient velocity $u$ versus $y$ for $Pr=5$, $Pm=4$, $Gm=7$, $Gr=8$, $Sc=5$, $M=5$. 

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Figure 5.7: Variation of transient velocity $u$ versus $y$ for $Pr=5$, $Pm=4$, $Gm=7$, $S=2$, $Sc=5$, $Gr=8$.

Figure 5.8: Variation of transient velocity $u$ versus $y$ for $Pr=5$, $Pm=4$, $Gm=7$, $S=2$, $Gr=8$, $M=5$.

Figure 5.9: Variation of shearing stress $\sigma$ at the plate $y=0$ versus $Pm$ for $Pr=5$, $Gr=8$, $Gm=7$, $S=2$, $Sc=5$, $M=5$. 
Figure 5.10: Variation of shearing stress $\sigma$ at the plate $y=0$ versus $Pr$ for $Pr=4$, $Gr=8$, $Gm=7$, $S=2$, $Sc=5$, $M=5$.

Figure 5.11: Variation of shearing stress $\sigma$ at the plate $y=0$ versus $Gm$ for $Pr=5$, $Gr=8$, $Pm=4$, $S=2$, $Sc=5$, $M=5$.

Figure 5.12: Variation of shearing stress $\sigma$ at the plate $y=0$ versus $S$ for $Pr=5$, $Gr=8$, $Gm=7$, $Pm=4$, $Sc=5$, $M=5$. 
Figure 5.13: Variation of shearing stress $\sigma$ at the plate $y=0$ versus $Sc$ for $Pr=5$, $Gr=8$, $Gm=7$, $S=2$, $Pm=4$, $M=5$.

Figure 5.14: Variation of shearing stress $\sigma$ at the plate $y=0$ versus $M$ for $Pr=5$, $Gr=8$, $Gm=7$, $S=2$, $Sc=5$, $Pm=4$.

Figure 5.15: Nusselt number versus $Gr$ for $Pr=5$, $Sc=5$, $Gm=7$, $S=2$, $M=5$, $Pm=4$. 

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Figure 5.16: Nusselt number versus $S$ for $Pr=5$, $Gr=8$, $Gm=7$, $Sc=5$, $M=5$, $Pm=4$.

Figure 5.17: Nusselt number versus $Sc$ for $Pr=5$, $Gr=8$, $Gm=7$, $Sc=2$, $M=5$, $Pm=4$. 