Comparison of Paris potential results for the zero and continuous particle spectrum

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The single particle energy is calculated self-consistently in nuclear matter using the Paris potential. Binding energy and wave functions are compared for this choice and the zero particle spectrum. A gain in binding energy of 6 MeV is obtained at normal density of which 3 MeV is due to the $^3S_1$-$^3D_1$ channel alone. The wave function for the $^3S_1$-$^3D_1$ channel shows marked enhancement in the 0.9–3.0 fm region. The saturation point is shifted to 1.6 fm$^{-1}$ with a binding energy of about 21 MeV. Real parts of optical model parameters are also deduced and discussed.

I. INTRODUCTION

The Brueckner-Bethe-Goldstone (BBG) theory of nuclear matter has been extensively and critically studied recently. Comparisons of numerical calculations with different potentials using different choices of a single particle spectrum have been proposed. We have made one such comparison of the binding energy per nucleon with the Paris potential in the lowest order. For the single-particle spectrum we have used (i) the conventional gap choice with self-consistent hole energy and zero particle potential and (ii) the continuous choice treating the hole and particle potentials on an equal footing.

The more recent second prescription for the single-particle energy leads to an increment in the binding energy by a few MeV in the lowest order itself. Its effects in the higher order diagrams are not unambiguously known.

A discussion on the relationship between the potential of a particle in the intermediate state and the three body cluster is given in the work of Banerjee and Sprung. The justification for taking $U(a)=0$, $a>k_F$ was historically due to the belief that the three body cluster diagrams contributed negligibly. Recent calculations by Day show that this is not so; with $U(a)=0$, the three body cluster term yields substantial additional binding energy. By taking, on the other hand, the continuous choice for $U(a)$, it is possible that one is already taking a large part of the three-body cluster into the lowest order calculation.

It is conceivable that once the three-body cluster terms are calculated for the two choices, the difference in the two prescriptions would be much less, but this is a large task and clearly beyond the scope of the present paper.

The nuclear matter $G$ matrix satisfies the Bethe-Goldstone (BG) equation given by

$$G = V - VQG,$$  \hspace{1cm} (1.1)

where $V$ is the potential matrix; $Q$ is the Pauli operator which blocks the already occupied states below the Fermi momentum, $k_F$; $e$ is the energy denominator defined by

$$e = e(p) + e(q) - W,$$  \hspace{1cm} (1.2)

where $p$ and $q$ refer to momenta above $k_F$; and $W$ is the starting energy.

For the gap choice the single-particle potential is the generally accepted Hartree-Fock (HF) potential

$$U(m) = \sum_{\mathbf{K}F} \langle mn | G | mn \rangle, m < k_F$$

$$= 0 \text{ for } m > k_F.$$  \hspace{1cm} (1.3)

In this case $e(p)$ and $e(q)$ of Eq. (1.2) are the kinetic energies only. One advantage is the well-known cancellation of one class of cluster diagrams.

The continuous choice is given by the Hartree-Fock potential for the hole as well as for particle states.
TABLE I. Calculation of single particle energy at various momenta \( k \) for \( k_F = 1.10 \text{ fm}^{-1} \). All units are in MeV unless otherwise specified.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k ) (fm(^{-1}))</td>
<td>( U_0 = -60 \text{ MeV}, M^* = 625 \text{ MeV} )</td>
<td>( U_0 = -56 \text{ MeV}, M^* = 725 \text{ MeV} )</td>
<td>( U_0 = -56 \text{ MeV}, M^* = 723 \text{ MeV} )</td>
</tr>
<tr>
<td>0</td>
<td>-55.96</td>
<td>-59.94</td>
<td>-59.85</td>
</tr>
<tr>
<td>0.75</td>
<td>-51.95</td>
<td>-56.02</td>
<td>-55.94</td>
</tr>
<tr>
<td>1.50</td>
<td>-42.56</td>
<td>-46.85</td>
<td>-46.79</td>
</tr>
<tr>
<td>2.25</td>
<td>-32.38</td>
<td>-36.69</td>
<td>-36.60</td>
</tr>
<tr>
<td>3.00</td>
<td>-19.23</td>
<td>-23.02</td>
<td>-22.96</td>
</tr>
</tbody>
</table>

\[ U(m) = \sum_{n \leq k_F} \langle m \mid G \mid mn \rangle, \quad m < k_F. \]  

One feels that the small attractive potential felt by the nucleon above the Fermi sea may have a non-trivial influence on the \( G \) matrix. The following points then arise:

(i) What is the change in the binding energy?
(ii) What is its effect on the correlated wave function? We expect that the effect should be transparent in the intermediate and long range.
(iii) The continuous choice particle spectrum can be quite easily related to the optical potential \( U \) for a nucleon in nuclear matter. It can be expressed in terms of the energy \( E \) of the nucleon in a simple form \( U = A + BE \). The coefficients \( A \) and \( B \) can be compared with fits given by experimentalists.

In this paper we try to consider each of the above-mentioned points. The nucleon-nucleon potential chosen is the momentum-dependent Paris potential. The momentum dependence is quite strong in this potential. Without it, the phase shifts become 50% lower at low energy. Obviously, it is more convenient to work in the momentum space and to use the well-known matrix inversion technique to solve for the \( G \) matrix and for the correlated wave function.

II. FORMALISM

The correlated wave function \( \psi \) and the unper-terbed wave function \( \phi \) are related through the \( G \) matrix as

\[ \mathbf{\psi} = \mathbf{G} \mathbf{\phi} \]  

We proceed to solve Eq. (2.1) along with Eq. (1.1), which could be recast in the form

\[ \mathbf{G} = \begin{bmatrix} 1 + \mathbf{Q} & \mathbf{Q} \\ \mathbf{Q} & \mathbf{Q} \end{bmatrix}^{-1} \mathbf{V}, \]

\[ = \begin{bmatrix} 1 - \mathbf{Q} & \mathbf{Q} \\ \mathbf{Q} & \mathbf{Q} \end{bmatrix} \mathbf{V}, \]

where use has been made of Eq. (1.1). Defining

\[ \mathbf{F}^{-1} = 1 - \mathbf{Q} \mathbf{G}, \]

we have

\[ \mathbf{G} = \mathbf{F}^{-1} \mathbf{V}. \]

Now,

\[ \mathbf{\psi} = \left[ 1 - \mathbf{Q} \mathbf{G} \right] \mathbf{\phi}, \]

\[ = \mathbf{F}^{-1} \mathbf{\phi}, \]

where \( \mathbf{F}^{-1} \) is the adjoint matrix of \( \mathbf{F}^{-1} \). Partial wave decomposition of Eq. (1.1) leads to

TABLE II. Calculation of single particle energy at various momenta \( k \) for \( k_F = 1.36 \text{ fm}^{-1} \). All units are in MeV unless otherwise specified.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>4</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k ) (fm(^{-1}))</td>
<td>( U_0 = -89.50 \text{ MeV}, M^* = 560 \text{ MeV} )</td>
<td>( U_0 = -88.65 \text{ MeV}, M^* = 670 \text{ MeV} )</td>
<td>( U_0 = -95.40 \text{ MeV}, M^* = 662 \text{ MeV} )</td>
</tr>
<tr>
<td>0</td>
<td>-88.66</td>
<td>-95.39</td>
<td>-94.89</td>
</tr>
<tr>
<td>0.75</td>
<td>-82.80</td>
<td>-89.63</td>
<td>-89.16</td>
</tr>
<tr>
<td>1.50</td>
<td>-69.47</td>
<td>-77.09</td>
<td>-76.53</td>
</tr>
<tr>
<td>2.25</td>
<td>-57.86</td>
<td>-67.28</td>
<td>-66.55</td>
</tr>
<tr>
<td>3.00</td>
<td>-32.53</td>
<td>-40.47</td>
<td>-40.10</td>
</tr>
</tbody>
</table>
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TABLE III. Calculation of single particle energy at various momenta \((k)\) for \(k_F = 1.60\) \(\text{fm}^{-1}\). All units are in MeV unless otherwise specified.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>(U_0 = -125) MeV</th>
<th>(U_0 = -130) MeV</th>
<th>(U_0 = -127) MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k) (fm(^{-1}))</td>
<td>(M^* = 600) MeV</td>
<td>(M^* = 591) MeV</td>
<td>(M^* = 596) MeV</td>
</tr>
<tr>
<td>0</td>
<td>-130.44</td>
<td>-127.14</td>
<td>-126.73</td>
</tr>
<tr>
<td>0.75</td>
<td>-123.53</td>
<td>-119.19</td>
<td>-118.78</td>
</tr>
<tr>
<td>1.50</td>
<td>-102.03</td>
<td>-98.53</td>
<td>-98.10</td>
</tr>
<tr>
<td>2.25</td>
<td>-76.22</td>
<td>-72.50</td>
<td>-72.01</td>
</tr>
<tr>
<td>3.00</td>
<td>-44.71</td>
<td>-41.08</td>
<td>-40.62</td>
</tr>
</tbody>
</table>

\[
\sum_{F} \left[ \delta_{11} + \frac{2}{\pi} \int d\kappa \kappa^2 V_{F}(\kappa, k') \frac{\bar{Q}(\bar{F}, k')}{M^*} \right] G_{ll_0}(k', k_0) = V_{l_0}(k, k_0),
\]

(2.10)

where \(\bar{F}\) is the average center of mass momentum, \(\bar{Q}\) is the angle averaged Pauli operator, and \(\epsilon\) is the energy denominator defined in Eq. (1.2).

The matrix inversion technique is invoked and with the help of the grid points \(i,j\), the \(F\) matrix in partial wave decomposition is now written as

\[
F_{ij}(k_0, k_0) = \delta_{ij} + \frac{2}{\pi} \int d\kappa \kappa^2 V_{ij}(\kappa, k_0) G_{ll_0}(k', k_0) = V_{l_0}(k, k_0),
\]

(2.11)

so that

\[
\sum_{F} \sum_{i,j} F_{ij}(k_0, k_0) G_{ll_0}(k, k_0) = V_{l_0}(k, k_0).
\]

(2.12)

Inversion of the \(F\) matrix not only yields \(G\) but also the correlated wave \(\psi\) from Eq. (2.8). The correlated wave in partial waves now becomes

\[
\psi_{ll}(k_0, r) = \sum_{k'} f_{ll}(k', r) F_{ll_0}(k', k_0).
\]

(2.13)

In Eq. (2.12), \(l\) is the entrance channel with momentum \(k_0\), whereas the exit channel \(l'\) corresponds to momentum \(k'\). Once we know \(\psi\), it is trivial to compute the correlation function

\[
f_{ll_0}(k_0, r) = \psi_{ll_0}(k_0, r) / j_{l_0}(k_0, r)
\]

(2.14)

The single-particle HF potential for the gap choice, Eq. (1.3), is given by

\[
U(k_m) = \frac{8}{\pi} \sum_{n, \ell} (2J + 1)(2T + 1) \int_0^{(k_F - k_m)/2} dk \kappa^2 G_{LL}(\bar{F}; k, k) \left( \frac{k_F^2 - k_m^2}{4} - k(k - k_m) \right) G_{LL}(\bar{F}; k, k)
\]

(2.15)

TABLE IV. Calculation of single particle energy at various momenta \((k)\) for \(k_F = 1.75\) \(\text{fm}^{-1}\). All units are in MeV unless otherwise specified.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>(U_0 = -148) MeV</th>
<th>(U_0 = -141) MeV</th>
<th>(U_0 = -145) MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k) (fm(^{-1}))</td>
<td>(M^* = 445) MeV</td>
<td>(M^* = 495) MeV</td>
<td>(M^* = 495) MeV</td>
</tr>
<tr>
<td>0</td>
<td>140.03</td>
<td>144.90</td>
<td>144.90</td>
</tr>
<tr>
<td>0.75</td>
<td>130.32</td>
<td>133.83</td>
<td>135.22</td>
</tr>
<tr>
<td>1.50</td>
<td>103.29</td>
<td>108.31</td>
<td>108.29</td>
</tr>
<tr>
<td>2.25</td>
<td>-68.80</td>
<td>-74.03</td>
<td>-74.03</td>
</tr>
<tr>
<td>3.00</td>
<td>-33.86</td>
<td>-39.42</td>
<td>-39.42</td>
</tr>
</tbody>
</table>
If $k_m > k_F$, obviously the first integral should be omitted.

As pointed out by Sprung, the $G$ matrix is a slowly varying function of $P$, and to simplify the numerical computation one can use the same $P$ as in nuclear-matter binding:

$$4P_{eF}^2 = 2.4k_F^2 \left[ 1 - \frac{k}{k_F} \right] \left[ 1 + \frac{k^2/k_F^2}{3(2 + k/k_F)} \right].$$

(2.16)

This choice is expected to be valid at high values of $k$ but is not satisfactory for $k$ just above $k_F$. This implies that the single particle spectrum at $k \approx k_F$ is not calculated very accurately in our formalism. However, in any case, at $k = k_F$ the effective mass

\begin{table}
\centering
\caption{Single particle energies at $k_F = 1.36$ fm$^{-1}$ ($U_p = -95.40$ MeV; $M^* = 662$ MeV). All units are in MeV unless otherwise specified.}
\begin{tabular}{cccccccc}
\hline
$k$ (fm$^{-1}$) & 0 & 0.75 & 1.50 & 2.25 & 3.00 & 3.50 & 4.25 & 5.00 & 5.75 & 6.50 \\
\hline
$S_0$ & -42.67 & -37.76 & -25.79 & -13.08 & -3.43 & 1.11 & 5.87 & 9.88 & 15.18 & 32.11 \\
$P_1$ & 7.96 & 8.26 & 8.81 & 8.34 & 8.25 & 8.40 & 8.78 & 9.54 & 10.63 & 12.44 \\
$P_0$ & 17.66 & 19.05 & 21.87 & 22.62 & 24.92 & 27.32 & 32.31 & 40.28 & 53.69 & 92.65 \\
$D_1$ & 1.98 & 2.56 & 3.63 & 4.22 & 4.87 & 5.14 & 5.29 & 5.36 & 5.52 & 6.46 \\
$P_0$ & -10.00 & -13.00 & -20.42 & -28.72 & -29.60 & -27.02 & -21.01 & -13.30 & -5.97 & 0.62 \\
$F_0$ & -0.55 & -0.92 & -1.76 & -2.47 & -2.18 & -1.30 & 0.99 & 4.17 & 7.95 & 12.15 \\
$D_2$ & 0.15 & 0.11 & -0.30 & -1.30 & -2.07 & -2.24 & -2.02 & -0.98 & 0.64 & 2.63 \\
$G_1$ & 0.14 & 0.29 & 0.70 & 1.24 & 1.97 & 2.43 & 3.01 & 3.35 & 3.35 & 3.14 \\
\hline
Total & -94.89 & -89.16 & -76.53 & -66.55 & -40.10 & -17.59 & 15.55 & 54.70 & 103.13 & 196.21 \\
\hline
\end{tabular}
\end{table}
suffers an enhancement\textsuperscript{1} which we have neglected.
The binding energy should not depend particularly on this as it is insensitive to small local variations in the single-particle potential $U$.

$Q$ used in the $G$ matrix is given by

$$Q(P, k') = 0 \text{ for } k'^2 + \frac{P^2}{2} < k_F^2$$

$$= \min \left[ 1, \frac{k'^2 + \frac{P^2}{4} - k_F^2}{P k'} \right] \text{ otherwise.}$$

(2.18)

This formula is usually derived for $P < 2k_F$, but it holds for any value of $P$\textsuperscript{1}.

The point $k = k_F$ is avoided in a principal value integration in Eq. (1.4). This is achieved in matrix inversion through a subtraction

$$-Q(P, k_0) \sum_{l=1}^{N} \frac{2 \theta^2 w_l k^2}{M} \varepsilon.$$
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\[ e = 2 \left( \frac{\hbar^2 c^2 k^2 + M^2 c^4}{2} \right)^{1/2} + U \left[ k^2 + \frac{p^2}{4} \right]^{1/2} \]

\[ U = \left( k^2 + \frac{p^2}{4} \right)^{1/2} k_F \]

(2.21)

III. NUMERICAL RESULTS AND OUTLOOK

The single particle energies for the continuous choice approximation are found self-consistently up to 3.0 fm\(^{-1}\) for four densities (Tables I–IV). A detailed breakdown for each channel is given in Table V up to \( k = 6.5 \) fm\(^{-1}\) at \( k_F = 1.36 \) fm\(^{-1}\). The potential energy becomes negligible in comparison with the kinetic energy around a high \( k \) value ~ 6 fm\(^{-1}\) and the contribution to the binding energy also becomes very small. The main motivation was to see if at such a high value of momentum, the single particle energy turns out to be constant or if it starts to decrease as it is supposed to do for a hard-core potential.\(^1\) But we found that for the Paris potential, the single particle energy is still an increasing function even at this momentum (Table V). Table VI shows the individual channel contributions to the nuclear potential energy with the continuous single particle spectrum. The gap choice results are also displayed in this table for comparison. The difference in the \( ^1S_0 \) binding is small. We find a larger increase in the \( ^3S_1 - ^3D_1 \) binding compared to other authors.\(^3\) At \( k_F = 1.36 \) fm\(^{-1}\), the Paris potential yields a total gain of more than 6 MeV with the continuous choice, and of this about 5 MeV is due only to the \( ^3S_1 - ^3D_1 \) channel. We have plotted the \( ^3S_1 - ^3D_1 \) defect wave function at the same Fermi momentum in Fig. 1 with the gap choice as well as the continuous choice to understand this difference.

From 0.9 fm on, the continuous choice \( ^3S_1 - ^3D_1 \) wave function is significantly larger than the gap choice result. The effect is less pronounced for the \( ^1S_0 \) channel (Fig. 2). The saturation curve for the continuous choice is given in Fig. 3. The binding energy is about 21 MeV at \( k_F = 1.55 \) fm\(^{-1}\) at saturation. Contributions from higher partial waves \( l \geq 5 \) are not included.

One can find the potential energy \( U \) as a function of the total energy \( E \). In this form it can be interpreted as the potential felt by an incident particle in nuclear matter—this being the real part of the optical potential felt by it. The Liege group\(^6\) analyzed \( U \) as a function of the density \( \rho \) and energy \( E \) in the form

\[ U = \sum_{l=0} a_{lj} \rho^{1/2} E^{l-1} \quad (3.1) \]

Neglecting the \( j \neq 3 \) contribution, a more approximate relation is

\[ U = A + B E \quad (3.2) \]

In Tables VII and VIII we compare the \( a_{lj} \) and the \( A \) and \( B \) parameters of the Liege group obtained for the Reid potential with those obtained by us for the Paris potential. The Paris potential depth \( A \) is larger and the energy dependence is smaller. The larger value of \( A \) can be understood by the fact that the saturation point for the Paris potential lies below the continuous Coester band, though, like the other modern potentials (Bonn, Beth-Johnson), it also misses the empirical binding energy, being in a parallel "second" Coester band.\(^9\)

Inclusion of isobar degrees of freedom with these modern potentials may yield a better binding energy and at the same time may bring the potential depth parameter \( A \) closer to the conventional value of 55 MeV or so as suggested by experiment (Becchetti and Greenlees\(^6\)). Such calculations would also yield the isobar optical potential which can be derived in a way analogous to what we have outlined in the present paper. The isobar optical potential, which is our next target, is an object of interest in pion physics.
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Half shell reaction matrices and high energy phase shifts for the Paris potential

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We have calculated high energy phase shifts for the Paris potential for $E_{\text{lab}} \geq 330$ MeV and compared these with experimental results for all channels with $L \leq 3$. The effect of the momentum dependence on these phase shifts is studied. The half shell reaction matrices for these channels are also presented.

Nous avons calculé les déphasages à haute énergie, pour le potentiel Paris, avec $E_{\text{lab}} \geq 330$ MeV. Les résultats du calcul ont été comparés avec les données expérimentales pour tous les canaux avec $L \leq 3$. L'effet de ces déphasages de la dépendance de l'impulsion est étudié. Les matrices de réaction demi-couche pour ces canaux sont présentées.

Introduction

A phenomenological potential is generally obtained by fitting low energy two-body scattering data. The recent Paris potential (1) is no exception, although part of it is derived from theory. The Paris potential is fitted to Livermore phase shifts (2) up to 330 MeV and in addition it fits the raw scattering data with $\chi^2 = 1.99$ for proton–proton ($p-p$) scattering and 2.17 for neutron–proton ($n-p$) scattering (3). The Paris group has replaced the energy dependence in their potential by a momentum dependence of the form $p^4/\ln p$. The potential is indeed more general because of its nonlocality. The interplay between the momentum dependent and momentum independent parts can significantly affect the calculated half shell reaction matrix and the high energy phase shifts in a way that may not be the same as in low energy two-body scattering data. This phenomenological $p^4$ behaviour is determined by two-body scattering data only up to 330 MeV. Recently, the Paris group carried out an analysis of higher energy data up to 800 MeV (5) by using either a relativistic wave equation with an optical potential for inelasticities or a phase shift analysis with inelasticity parameters. For carrying out this programme, one has to use the unpublished data on either the long-range imaginary part of the optical potential or the peripheral inelasticity parameters. Both are extracted from the two-proton exchange potential by the Paris group. It is natural to wish to see the extent to which the published potential itself fits the experimental phase shifts beyond 330 MeV. It should be pointed out that the Paris potential (1) does not contain any inelasticities and the inelastic parts of the phase shifts are subtracted in the phase shift analysis. Therefore, the phase shifts we calculate and the experimental phase shifts are roughly on the same footing. The other possible test of the high energy behaviour can come from the application of half shell reaction matrices to study $NN\gamma$ and $d(p, 2p)n$ reactions (4). The importance of such checks for the high energy behaviour of the Paris potential has been stressed recently by Maxwell and Smith (5) as being highly relevant for neutron matter studies where the density is about ten times that of normal nuclear matter.

In this paper we study the behaviour of reaction matrices in all channels up to $L = 3$. The calculated high energy phase shifts have been compared with the experimental values. A discussion of the results follows in the next section.

Results and discussion

The $R$ matrix which we calculate is given by

$$R(p, q; S) = V_i(p, q) + \frac{2}{\pi} \int \frac{V_i(p, k)R_i(k, q; S)}{S - k^2} k^2 dk$$

It is related to the $T$ matrix used, for example, in the $d(p, 2p)n$ reaction by

$$T_i(p, q; k^2) = R_i(p, q; k^2) = \frac{i k(\delta(k^2)R_i(p, k^2)R_i(q, k^2) + 1 + i kR_i(k, k^2))}{1 + i kR_i(k, k^2)}$$

where $\delta(k^2)$ is a unit step function, zero for negative $k^2$. Equation [1] is solved by the matrix inversion method, the details of which are already published elsewhere (6–8). The phase shifts are calculated from the on-shell $R$ matrix elements.
TABLE 1. Phase shifts ($\delta$) in degrees with $V_0$ (momentum independent part of the potential) and $V$ (total potential):

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\delta$ at $E_{lab} = 25$ MeV</th>
<th>$\delta$ at $E_{lab} = 330$ MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>$23.764$</td>
<td>$48.774$</td>
</tr>
<tr>
<td>$D_1$</td>
<td>$62.300$</td>
<td>$80.965$</td>
</tr>
<tr>
<td>$P_0$</td>
<td>$-3.215$</td>
<td>$-2.840$</td>
</tr>
<tr>
<td>$P_1$</td>
<td>$-4.566$</td>
<td>$11.670$</td>
</tr>
<tr>
<td>$S_3$</td>
<td>$-7.255$</td>
<td>$7.097$</td>
</tr>
<tr>
<td>$D_3$</td>
<td>$0.790$</td>
<td>$0.780$</td>
</tr>
<tr>
<td>$D_3$</td>
<td>$3.982$</td>
<td>$3.984$</td>
</tr>
</tbody>
</table>

The Paris potential is now regarded as the best available nucleon-nucleon $(N-N)$ potential. It has a very strong momentum dependent part. One simple way to see this is to calculate the phase shifts for the momentum independent part $V_0$ alone and compare this with the phase shifts obtained from the total potential $V$. This is shown in Table 1 for a relatively low energy of 25 and 330 MeV. It is obvious that at high energies the momentum dependent part will be even more important.

The $S_0$ and $S_1$ channels are the two most important ones. In the $S_0$ and $S_1$ channels, the calculated high energy phase shifts are in good agreement with the experimental data of Arndt et al. (9) and Bugg and co-workers (10). The $S_1$ high energy phase shifts, however, do not agree well with those of MacGregor et al. (11) (Figs. 1, 2). The mixing parameter $\varepsilon$, and the phase shift for $D_1$ lie midway between the data of refs. 9 and 11 and are rather close to the data of ref. 10 (Figs. 3, 4). The $D_2$ and $D_3$ channels (Figs. 5, 6) show comparatively better fits. It was found that in the $D_3$ channel, neglect of the coupling to the $D_1$ channel produces repulsive phase shifts (Table 2), showing the importance of the tensor coupling in this channel (Fig. 7). The fit to $\varepsilon$ is reasonable (Fig. 8). The phase shifts for the $P_1$ channel are presented in Fig. 9. The fit

![Fig. 1. Phase shifts for the $S_0$ channel. Curve I: Calculated $(n-p)$ phase shifts for the Paris potential. Curve II: Calculated $(p-p)$ phase shifts for the Paris potential. Curve III: Experimental phase shifts of ref. 9. Curve IV: Experimental phase shifts of ref. 11. Circles (with error bars) refer to the data of Bugg and co-workers (10).]

TABLE 2. Phase shifts ($\delta$) in degrees with and without the tensor force in the $D_1$ channel

<table>
<thead>
<tr>
<th>$E_{lab}$ (MeV)</th>
<th>$\delta$ with tensor force</th>
</tr>
</thead>
<tbody>
<tr>
<td>$25$</td>
<td>$-0.14$</td>
</tr>
<tr>
<td>$50$</td>
<td>$-0.43$</td>
</tr>
<tr>
<td>$95$</td>
<td>$-0.88$</td>
</tr>
<tr>
<td>$142$</td>
<td>$-1.25$</td>
</tr>
<tr>
<td>$210$</td>
<td>$-1.76$</td>
</tr>
<tr>
<td>$330$</td>
<td>$-5.06$</td>
</tr>
</tbody>
</table>

Present Absent

The fits are all reasonable.
Figs. 17-20. Phase shifts for the channels mentioned. Curve I: Calculated phase shifts for the Paris potential. Curve II: Experimental phase shifts of ref. 9. Curve III: Experimental phase shifts of ref. 11. Abscissa: Lab. energy in MeV. Ordinate: 

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(Continued on next page)
| \( k (\text{fm}^{-1}) \) | \( ^1S_0 \) | \( ^3S_1 \) | \( ^1D_1 \) | \( ^3D_1 \) | \( ^1D_2 \) | \( ^1G_1 \) | \( ^1F_1 \) | \( ^1F_3 \) | \( ^1P_1 \) | \( ^1P_3 \) | \( ^1P_0 \) | \( ^3P_1 \) | \( ^3P_3 \) | \( ^3P_0 \) | \( ^3P_2 \) | \( ^3P_2 \) | \( ^3F_2 \) | \( ^3F_4 \) | \( ^3G_4 \) | \( ^3G_3 \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.17            | -0.099          | -0.369          | 0.001           | -0.880          | 0.000           | 0.037           | 0.009           | 0.036           | -0.024          | 0.000           | 0.023           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           |
| 0.83            | -0.062          | -0.301          | 0.016           | -0.653          | 0.078           | 0.165           | 0.036           | 0.165           | -0.107          | -0.001          | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           |
| 1.68            | 0.051           | -0.097          | 0.018           | -0.138          | 0.072           | 0.165           | 0.036           | 0.215           | -0.157          | 0.034           | 0.049           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           |
| 2.00            | 0.100           | -0.051          | -0.051          | 0.230           | 0.247           | 0.098           | 0.272           | -0.151          | 0.018           | 0.018           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           |
| 2.33            | 0.173           | 0.103           | -0.132          | -0.200          | 0.309           | 0.143           | 0.256           | -0.131          | 0.041           | -0.001          | 0.009           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           |
| 2.71            | 0.232           | 0.196           | -0.209          | -0.036          | 0.169           | 0.183           | 0.186           | 0.230           | -0.097          | 0.061           | -0.007          | 0.023           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           |
| 3.49            | 0.272           | 0.278           | -0.235          | -0.076          | 0.103           | 0.139           | 0.196           | 0.178           | -0.020          | 0.066           | -0.003          | 0.039           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           |
| 4.51            | 0.192           | 0.217           | -0.133          | -0.088          | 0.055           | 0.104           | 0.118           | 0.196           | 0.045           | 0.044           | 0.002           | 0.036           | 0.000           | 0.000           | 0.000           | 0.000           |
| 5.29            | 0.094           | 0.122           | -0.049          | -0.039          | 0.031           | 0.078           | 0.052           | 0.052           | 0.029           | 0.003           | 0.026           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           |
| 6.02            | -0.017          | 0.042           | 0.034           | -0.014          | 0.015           | 0.051           | 0.007           | 0.014           | 0.040           | 0.021           | 0.002           | 0.016           | 0.000           | 0.000           | 0.000           |
| 7.98            | -0.043          | -0.036          | 0.033           | 0.011           | 0.005           | -0.004          | -0.026          | -0.191          | 0.013           | -0.001          | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           |
| 10.53           | 0.098           | 0.004           | 0.001           | 0.001           | -0.002          | 0.000           | -0.003          | -0.001          | 0.000           | -0.006          | 0.010           | 0.000           | 0.000           | 0.000           | 0.000           |
| 12.48           | 0.022           | 0.011           | 0.001           | 0.000           | 0.000           | 0.000           | 0.001           | 0.003           | 0.004           | 0.018           | 0.007           | 0.011           | 0.000           | 0.000           | 0.000           |
| 13.83           | 0.023           | 0.008           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           |
| 16.96           | 0.018           | 0.003           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           |
| 21.04           | 0.011           | 0.003           | 0.001           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           |
| 24.17           | 0.007           | 0.006           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           | 0.000           |

Table 3: Half-shell reaction matrices \( (R) \) in \( \text{fm}^{-1} \) at \( E_{reac} = 330 \text{ MeV} \) \( (a_0 = 1.995 \text{ fm}^{-1}) \)