CHAPTER 4

FIELDS AND TORQUES IN NANO-MAGNETIC DEVICES

Data stored in the free layer of an MTJ can be affected by other external fields. In this chapter, we discuss the various fields and torques influencing the magnetization of the FL, internally from its own MTJ, and also from external MTJs in an array. The effect of the field causes thermal excitations on lattice, which exerts a torque indirectly on the magnetization of the FL. But torques, like a spin torque, affect the magnetization directly [J. Stohr 2006]. The effective torque makes the magnetization move out of the easy axis causing half-write bits in the FL.

4.1 Fields in nano-magnetic devices

As the dimension of the magnetic material decreases, the effect of the classical magnetostatic dipolar fields reduces and the quantum mechanical exchange field becomes more dominant. Since the dimension of an MTJ is in the range of 10-100 nm, it is necessary to understand the various fields affecting the FL. In this section, we review the demagnetization field, dipolar fields from the own PL, fields from FL of the neighbouring MTJs in a square array, and the exchange field.

4.1.1 Demagnetization field

It is convenient to view a magnet as a north pole with positive monopoles and a south pole with negative monopoles on its surface. Due to these surface poles, there are external
field lines from the north pole to the south pole. There is also an internal field $\vec{H}_d$, which tends to demagnetize the specimen. The magnitude of the demagnetizing field depends on the shape of the material. The larger the value of $\vec{H}_d$ along a particular direction, the more difficult it is to magnetize the magnet in that direction. Hence, an easy axis of magnetization is produced along the longer axis. For example, a thin film of FM material can be magnetized easily along the plane of the film, as the demagnetizing factor is very low in this direction. The ellipsoid shown in Figure 4.1 gets magnetized along the $z$ axis, with the application of an external magnetic field. The surface poles formed at the ends, in turn creates a demagnetization field $\vec{H}_d$, opposite to $\vec{M}$ along $-z$ direction. $\vec{H}_d$ along the $z$ direction, is the lowest compared to $\vec{H}_d$ along $x$ and $y$ directions making the magnetization bi-stable along the $\pm z$ directions. The effective field reduced by $\vec{H}_d$ in the presence of an external magnetic field $\vec{H}_{app}$ is given as,

$$\vec{H}_{eff} = \vec{H}_{app} - \vec{H}_d.$$ (4.1)

As we know, in the free layer of an MTJ, logic 0 and 1 are represented by the direction of magnetizations. $\vec{H}_{app}$ has to overcome the demagnetization field to change the magnetization of the magnet.

Figure 4.1: Demagnetization field of an ellipsoid along its long axis

The demagnetizing field $\vec{H}_d$ is given in terms of $\vec{M}$ and a constant of proportionality, the
demagnetization tensor $\vec{N}$ as,

$$\vec{H}_d = -\vec{N} \cdot \vec{M}. \quad (4.2)$$

The demagnetization factor is shape dependent and satisfies the condition

$$N_x + N_y + N_z = 1, \quad (4.3)$$

where $N_x$, $N_y$, and $N_z$ are the demagnetization factors in the three respective co-ordinates.

For example, for a sphere, symmetry requires that the demagnetization factors be

$$N_x = N_y = N_z = \frac{1}{3}. \quad (4.4)$$

For a thin film on the $x - z$ plane as shown in Figure 4.2a,

$$N_x = N_z \approx 0; \ N_y \approx 1, \quad (4.5)$$

and for a long cylinder as in Figure 4.2b,

$$N_x = N_y \approx \frac{1}{2}; \ N_z \approx 0. \quad (4.6)$$

Figure 4.2: (a) Thin sheet of film with hard axis along $\hat{y}$ (b) A cylindrical rod with easy axis along $\hat{z}$
In our work, we have assumed the free layer of the MTJ as a very flat oblate spheroid i.e., a circular disc whose thickness is much smaller than the radius as shown in Figure 4.3. The demagnetization factors for such a circular disc are [J. A. Osborn 1945],

\[ N_y = N_z = \frac{\pi}{4n} \left( 1 - \frac{4}{\pi n} \right), \tag{4.7a} \]
\[ \text{and} \quad N_x = 1 - \frac{\pi}{2n} + \frac{2}{n^2}, \tag{4.7b} \]

where \( n = c/a \), \( a \), \( b \) and \( c \) are the dimensions of the element. For the disc shaped free layer, \( a = b \) and \( a >> c \). From Equations 4.7a - 4.7b, it is obvious that, the value of \( N_x \) is much higher than \( N_y \) and \( N_z \). Hence, \( \vec{H}_d \) in \( z \) and \( y \) directions are much smaller to that in \( x \) direction, making it difficult to magnetize along the \( \hat{x} \) axis, the hard axis. \( \vec{H}_d \) becomes a crucial factor in designing the MTJ as a bi-stable device for storing data.

Figure 4.3: Circular disc placed in \( y - z \) plane with hard axis along \( x \) axis.

\[ ^1 \text{For a flat oblate spheroid, we have a mathematical expression for demagnetization factors.} \]
4.1.2 Neighbours’ coupling field

In the simplest case, the field from a magnetic dipole $\vec{m}$ of length $2d$, at a point $P$ and distance $r$, as shown in Figure 4.4 is given in cylindrical coordinates as,

$$\vec{H}_r = \frac{1}{4\pi} \frac{2\vec{m} \cos \theta}{r^3},$$

$$\vec{H}_\theta = \frac{1}{4\pi} \frac{\vec{m} \sin \theta}{r^3}.$$ (4.8a, 4.8b)

Similarly, the field from a neighbouring MTJ ($\vec{H}_{MN}$) is a long range dipolar field. We study the effective dipolar fields due to all the four nearest neighbours in a square array. Since the effect of dipolar field is inversely proportional to the square of the distance, we ignore the effect of the next nearest neighbours.

Consider an array of disc shaped MTJ cells lying on $y - z$ plane as shown in Figure 4.5, with 'X’ being the reference and A, B, C and D, the nearest neighbours. Here A and B lie along the $\pm \hat{y}$ axis and C and D lie along the $\pm \hat{z}$ axis with respect to the reference. All the MTJs have their easy axis of magnetization along $\pm \hat{z}$ and hard axis along $\pm \hat{x}$. Depending on the orientation of magnetizations of these neighbouring cells, the magnetic field experienced
Figure 4.5: Top view of FLs in an array; X - Reference; A, B, C, D - Nearest neighbours.

The coupling between FLs of two MTJs, $i$ and $j$ as shown in Figure 4.6 is given by the coupling matrix [G. Csaba 2002],

$$C^{(ij)} = \frac{V^{(j)}}{4\pi r_{ij}^3} \begin{bmatrix} 3\hat{r}_x^2 - 1 & 3\hat{r}_x \hat{r}_y & 3\hat{r}_x \hat{r}_z \\ 3\hat{r}_y \hat{r}_x & 3\hat{r}_y^2 - 1 & 3\hat{r}_y \hat{r}_z \\ 3\hat{r}_z \hat{r}_x & 3\hat{r}_z \hat{r}_y & 3\hat{r}_z^2 - 1 \end{bmatrix}, \quad (4.9)$$

where, $r_{ij}$ is the distance between MTJs $i$ and $j$, $\hat{r}_i$ is the unit vector pointing from $i$ to $j$ and $V^{(j)}$ is the volume of the FL. Since, the field between any two FLs is dipolar in nature, we obtain each entry in the coupling matrix using the expression,

$$\vec{B} = \frac{\mu_0}{4\pi r_{ij}^3} [3(\vec{m} \cdot \hat{r}_{ij}) \hat{r}_{ij} - \vec{m}]. \quad (4.10)$$

**Origin of the coupling matrix**

Let us use an example and demonstrate how the coupling matrix given in Equation 4.9 is obtained from Equation 4.10. Assume that the moment is pointing towards $x$ direction i.e., $\vec{m}_x = m\hat{r}_x$. The field induced at a distance $r$ along the $x$ direction from the moment is
obtained as

\[
\vec{B} = \frac{\mu_0}{4\pi r_{ij}^3} [3(\vec{m}_x \cdot \hat{r}) \hat{r} - \vec{m}_x]
\]

\[
= \frac{\mu_0 m}{4\pi r_{ij}^3} [3(\hat{r}_x(\hat{r}_x + \hat{r}_y + \hat{r}_z))(\hat{r}_x + \hat{r}_y + \hat{r}_z) - \hat{r}_x]
\]

\[
= \frac{\mu_0 m}{4\pi r_{ij}^3} [3(\hat{r}_x \cdot \hat{r}_x)(\hat{r}_x + \hat{r}_y + \hat{r}_z) - \hat{r}_x]
\]

\[
= \frac{\mu_0 m_x}{4\pi r_{ij}^3} [(3\hat{r}_x^2 - 1) + (3\hat{r}_x\hat{r}_y + 3\hat{r}_x\hat{r}_z)]
\]

\[
= \frac{\mu_0 m_x}{4\pi r_{ij}^3} [(3\hat{r}_x^2 - 1) + 3\hat{r}_x\hat{r}_y + 3\hat{r}_x\hat{r}_z].
\] (4.11)

We observe that Equation 4.11 represents the first row of the coupling matrix. Similarly for the moments \(m_y\) and \(m_z\), we obtain the second and third rows in Equation 4.9. Substituting the known facts that,

\[
\vec{H} = \frac{\vec{B}}{\mu_0},
\]

\[
\text{and } \vec{M} = \frac{\vec{m}}{V},
\]

in Equation 4.11, we obtain

\[
\begin{bmatrix}
\vec{H}_x \\
\vec{H}_y \\
\vec{H}_z
\end{bmatrix}
= \frac{\nu^j}{4\pi r_{ij}^3}
\begin{bmatrix}
3\hat{r}_x^2 - 1 & 3\hat{r}_x\hat{r}_y & 3\hat{r}_x\hat{r}_z \\
3\hat{r}_y\hat{r}_x & 3\hat{r}_y^2 - 1 & 3\hat{r}_y\hat{r}_z \\
3\hat{r}_z\hat{r}_x & 3\hat{r}_z\hat{r}_y & 3\hat{r}_z^2 - 1
\end{bmatrix}
\begin{bmatrix}
\vec{M}_x \\
\vec{M}_y \\
\vec{M}_z
\end{bmatrix}.
\] (4.13)

For example, in Figure 4.5, consider the nearest neighbour, ‘A’ located along +y direction.
to 'X’. The dipolar field induced on the FL of reference can be obtained as

$$\vec{B}_y = \frac{\mu_0}{4\pi r_{ij}^3} \left( 3 (\vec{m}_z \cdot \hat{r}_y) \hat{r}_y - \vec{m}_z \right),$$

$$= \frac{\mu_0}{4\pi r^3} (-\hat{m}_z),$$

$$= \frac{\mu_0|m|}{4\pi r^3} (-\hat{r}_z).$$  \hspace{1cm} (4.14)

With the direction of the magnetization of MTJ B in $-y$ direction, the dipolar field induced on 'X’ is,

$$\vec{B}_y = \frac{\mu_0|m|}{4\pi r^3} (\hat{r}_z).$$  \hspace{1cm} (4.15)

Similarly, the nearest neighbour C and D along $\pm z$ direction can induce a dipolar field given by

$$\vec{B}_z = \frac{\mu_0|m|}{4\pi r^3} (\pm 2 \hat{r}_z).$$  \hspace{1cm} (4.16)

We consider only the static fields due to neighbour’s magnetizations. The magnetization of the reference changes due to these induced magnetic fields. As a result, the polarity, and not the magnitude of the field induced in X changes, provided, the distance between the neighbour and the reference does not change.

Incorporating the effect of the neighbours’ field into the coupling matrix given in Equa-
tion 4.9, we obtain the coupling field due to B along +y

\[
\begin{bmatrix}
H_x \\
H_y \\
H_z
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 \\
\frac{V(j)}{4\pi r^3_{ij}} & 0 & 0 & -1 \\
0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix}.
\] (4.17)

The coupling field due to MTJs, C, or D, along ±\(\hat{z}\) direction is

\[
\begin{bmatrix}
H_x \\
H_y \\
H_z
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 \\
\frac{V(j)}{4\pi r^3_{ij}} & 0 & 0 & 0 \\
0 & 0 & -2
\end{bmatrix} \begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix}.
\] (4.18)

Hence from Equations 4.17 and 4.18, we understand that, the effect of the neighbouring field depends on the direction of the magnetization as well as the location of the neighbour.

The effective dipolar field of all the 4 neighbours \(\vec{H}_{MN}\), in a particular orientation, can be expressed as an algebraic sum of the individual fields as,

\[
\vec{H}_{MN} = \sum_{j(\text{neighbours})} C^{(ij)} \vec{M}^{(j)}.
\] (4.19)

We have extensively studied the effect of all the possible combinations of orientations of magnetizations of all the 4 neighbours. As expected, we found that the orientation of neighbours shown in Figures 4.7a and 4.7b respectively, require the maximum and minimum switching currents to switch X.
4.1.3 Field due to pinned layer

The pinned layer’s magnetic field ($H_{PL}$) also has a long range magnetostatic influence on the free layer. So, similar to $H_{MN}$, we obtain the coupling field induced by PL on FL. From Figure 4.8, we observe that the PL lies along $-x$ axis with respect to the FL. The magnetostatic dipolar field induced on the FL can be obtained as

$$\vec{B}_x = \frac{\mu_0}{4 \pi r^3_{ij}} \left(3 (\vec{m}_z \cdot \hat{r}_x) \hat{r}_x - \vec{m}_z\right),$$

$$= \frac{\mu_0 M V}{4 \pi r^3} \left[-\hat{r}_z\right], \quad (4.20)$$

where $M$ and $V$ are the magnetization and volume of FL. We have included the effect of this dipolar field from pinned layer while studying the switching characteristics of the FL.

Figure 4.8: Magnetostatic field from PL to FL
4.1.4 Exchange field

Exchange energy is the main cause for ordering of the molecular magnetic moments in a ferromagnetic material. The effect of exchange field \( H_{\text{ex}} \) is very much greater than that of the dipolar field in nano magnets. Weiss proposed that a molecular field existing between the neighbouring moments makes them get aligned parallel to one another [N. A. Spaldin 2003]. This field is caused by exchange interactions, which is a consequence of Pauli’s exclusion principle, Hund’s rule and Coulomb interaction between electrons.

Consider two electrons in a system. If they are of different spins, they can be in the same atomic and molecular orbital. Due to this, their wave functions overlap and hence the electrons have a higher Coulomb repulsion. When the spins are alike, they must occupy different orbitals and hence, have less Coulomb repulsion. This is the reason the electrons occupy orbitals with one electron per orbital, before electrons of opposite spins are paired with them, as given by Hund’s first rule. We see that the electrostatic energy of the atomic system depends on the relative orientation of spins and exchange energy is the difference in energy. The exchange field is very large and is of the order of \( 10^3 \) T. Heisenberg’s representation of the exchange coupling between two spins \( S_i \) and \( S_j \) is given by

\[
E_{\text{ex}} = -2J\vec{S}_i \cdot \vec{S}_j = -2JS_i S_j \cos \theta, \tag{4.21}
\]

where, \( J \) is the exchange constant and \( \theta \) is the angle between the 2 spins. For \( J > 0 \), a ferromagnetic coupling exists between the spins and for \( J < 0 \), an antiferromagnetic coupling exists.

In a multi layered heterostructure as in an MTJ, the concept of exchange coupling is better explained by spin current due to the itinerant \( 3d \) electrons [P. Bruno 1993, P. Bruno 1994, J. C. Slonczewski 1993]. Electrons in the PL get spin polarized and their polarizations are assumed to be conserved in the barrier. These tunneling spin polarized electrons interact with the FL electrons, causing an exchange interaction as shown in Figure 4.9. As the thickness of
the barrier decreases, the exchange energy increases, aiding switching of the FL in parallel with the PL with less external energy. The exchange energy is zero when all the moments in a material are parallel to each other.

![Figure 4.9: Exchange field between PL and FL](image)

**Exchange field due to a mono atomic layer of PL on FL**

Using the two current model, the exchange energy, due to the tunneling of a single electron per unit area, is given in terms of reflection coefficients of Bloch waves at the interface [P. Bruno 1993, J. C. Slonczewski 1995] as

\[
\mathcal{E}_{ex} = \frac{\hbar^2 \kappa^5 (\kappa^2 - k_+ k_-) (k_+ - k_-)^2 (k_+ + k_-) e^{-2\kappa t}}{2 \pi^2 m_e t^2 (\kappa^2 + k_+^2)(\kappa^2 + k_-^2)^2},
\]  

(4.22)

where, \(k_+\) and \(k_-\) are the wave vectors of spin-up and spin-down electrons, \(\kappa\) is the decay constant within the barrier and \(t\) is the thickness of the barrier.

**Barrier height on exchange energy**

As the tunneling current depends on the barrier height, we expect that the exchange energy also gets influenced by the variation in barrier height. Consider an Fe/I/Fe multilayer for the MTJ, with Fe as the FM material and I as the insulator. We use \(k_+ = 0.973/\text{Å}\) and \(k_- = 0.37/\text{Å}\) for BCC iron [M. B. Stearns 1977] with \(E_F = 2.1\text{ eV}\). We study the effects of
the varying barrier heights ($U_0$) on the exchange energy, for the MTJ structure with insulator thickness of 0.8 nm. From Figure 4.10, we observe that, for low values of barrier height, $E_{\text{ex}}$ is negative, indicating that an antiferromagnetic coupling exists between the FL and the PL and for $U_0 > 3.5$ eV, $E_{\text{ex}}$ is positive, indicating a ferromagnetic coupling between the FL and the PL. We enlarge the region marked 'X', and in the inset, we observe that the exchange energy peaks at a barrier height of about 4 eV and later decreases with increase in barrier height. For still higher values of $U_0$ for example above 5 eV, the exchange coupling becomes negligible. We understand that, the exchange coupling is dependent on barrier height of the insulating material, but, unfortunately, the control over barrier height is difficult, as it is process and material dependent [B. D. Schrag 2000].

Figure 4.10: Variation of exchange energy with barrier height for an insulator of thickness 0.8 nm. Inset: Enlarged portion of the plot marked 'X'

**Effect of barrier thickness on exchange energy**

Apart from the exchange field being dependent on the barrier height, the barrier thickness also influences the magnetization of an MTJ’s FL. From Equation 4.22, we observe that
\( \mathcal{E}_{\text{ex}} \) is proportional to the thickness of the barrier. So, we plot the exchange energy per unit area to the variations in barrier thickness in Figure 4.11 for a barrier height of 3.5 eV, assuming a ferromagnetic coupling. With an increase in barrier thickness, the exchange energy decreases as expected.

![Graph showing variation of exchange constant with barrier thickness for \( U_0 = 3.5 \text{eV} \).](image)

Figure 4.11: Variation of exchange constant with barrier thickness for \( U_0 = 3.5 \text{eV} \).

Equation 4.22 gives the exchange energy per unit area caused by a single electron tunneling across the barrier. We extend these results to obtain the exchange field exerted by the PL over the FL in an MTJ structure. Since the exchange field is short ranged, we consider that the exchange field exists only between monolayers of the PL and the FL and is obtained from Equation 4.22 as,

\[
H_{\text{ex}} = -\frac{\mathcal{E}_{\text{ex}}}{\mu_0 M_s d} \times \text{(moment of interfacial atoms)} \text{ A/m},
\]

(4.23)

where, \( d \) is the thickness of the FL and \( M_s \) is its saturation magnetization. \( M_s \) is the saturation magnetization of the material of FL. The exchange field is mutual and can exist from FL to PL. But, as the PL is usually pinned by means of an antiferromagnetic pinning layer, we ignore \( H_{\text{ex}} \) from FL over PL. Later, in our analysis of an MTJ, we make use of Equation 4.23
to study the effect of the exchange field.

4.2 Torques

Torques affect magnetic moments directly, unlike fields which exert a torque through lattice vibrations [J. Stohr 2006]. In a CPP structure, the current flowing perpendicularly through the device causes the switching of the free layer. The spin polarized electrons from the pinned layer carry the angular momentum of the PL, which exerts a torque on free layer electrons. This spin transfer torque causes a change in the angular momentum of spins of electrons in the FL and hence the switching of FL. The magnetization of FL either keeps precessing and/or switching, depending on the direction and magnitude of the tunnel current. Apart from a spin transfer torque, we also introduce another important torque called the Ampere torque, which is caused by the spin polarized tunnel current.

4.2.1 Spin torque

Spin transfer torque in CPP

The spin transfer torque is due to externally driven spin polarized tunneling electrons flowing through the MTJ. Spin transfer torque has commercial applications in STTRAMs and high frequency oscillators [J. A. Katine 2008]. The principle of switching by means of spin transfer torque [S. M. Rezende 2005, M. D. Stiles 2006] can be understood from Figure 4.12.

In Figure 4.12a, the magnetization in PL and FL are initially antiparallel, and the external bias allows electrons to tunnel from PL to FL. The unpolarized electrons flowing from an external source into the PL gets polarized in the direction of the PL’s magnetization. These polarized parallel electrons from the PL tunnel through the barrier, while the antiparallel
electrons get reflected. It is assumed that spin is conserved during the tunneling process, as the dimension of the barrier is much smaller than the spin relaxation length. These spin polarized electrons carrying the spin angular momentum of the PL, exert a torque ($\tau_s$) on the FL. This switches the FL moment when the current exceeds a threshold value making FL and PL parallel. If the initial magnetizations of FL and PL were parallel, the torque exerted on FL stabilizes the magnetization of FL. Hence, depending on the relative magnetization of the FL with the reference PL, the torque either stabilizes or switches the FL’s magnetization.

We assume initial magnetizations of FL and PL to be parallel, and the electron flow due to bias is from FL to PL as shown in Figure 4.12b. The electrons get spin polarized in the FL, and tunnel through the barrier to reach the PL. But, since the PL is pinned by the AFM layer and only parallel spins are transmitted, the electrons with antiparallel spins are reflected back from the PL. This spin current in turn, exerts a torque on the FL moment which switches the magnetization of the FL, making PL and FL antiparallel. Similarly, when the initial magnetizations of FL and PL are antiparallel, the electrons from FL to PL stabilizes the magnetization of the FL.

![Figure 4.12: Spin torque experienced by FL electrons due to (a) electron flow from PL to FL (b) electron flow from FL to PL. [S. Maekawa 2006]](image)

The spin transfer torque ($\tau_s$) exerted on the FL [J. C. Slonczewski 1996, J. C. Slonczewski 1999]
is,

\[
\frac{d\vec{M}}{dt} = \frac{\gamma \hbar J \eta}{2eM_s^2d}[\vec{M} \times (\vec{M} \times \vec{M}_{PL})], \tag{4.24}
\]

where, \(\eta\) is the spin transfer efficiency or the spin polarization factor, \(d\), the thickness of the free layer, \(\vec{M}_{PL}\), the direction of magnetization of PL, \(\gamma\) is the gyromagnetic ratio. The spin polarization factor at the ferromagnet PL depends on the currents due to spin-up and spin-down electrons and is given in terms of their wave vectors as [J. C. Slonczewski 1989],

\[
\eta = \frac{(k_+ - k_-)(\kappa^2 - k_+k_-)}{(k_+ + k_-)(\kappa^2 + k_+k_-)}, \tag{4.25}
\]

These spin polarized electrons tunnel through the barrier and exert a spin transfer torque on FL electrons given by,

\[
\vec{\tau}_s = \frac{\gamma \hbar J}{2eM_s^2d}(k_+ - k_-)(\kappa^2 - k_+k_-)[\vec{M} \times (\vec{M} \times \vec{M}_{PL})], \tag{4.26}
\]

where, the tunnel current density

\[
J = \frac{V_{app}}{R_T A}, \tag{4.27}
\]

and \(A\) is the cross-sectional area of the structure. The tunnel current required being sourced by the external bias, we study the effect of switching of FL with respect to the \(V_{app}\). As we have observed in Equation 3.35, the tunneling resistance is dependent on the transition probability \(\tilde{\tau}\) of electrons [J. G. Lu 2004] given by

\[
R_T = \frac{\hbar}{\pi \tilde{T}^2 e^2 D_1 D_2}, \tag{4.28}
\]

where, \(D_1\) and \(D_2\) are the density of states (DOS) of the electrons in FL and PL. Since, both spin torque \(\left(\tau_s\right)\) and exchange field \(H_{ex}\) have a great influence on the magnetization of FL and also depend on the current density (Equations 4.22 and 4.26), we proceed to study the impact of varying barrier thickness on \(\tau_s\) and \(H_{ex}\).
4.2.2 Effect of barrier thickness on $E_{\text{ex}}$ and $\tau_s$

Theoretical estimates of the exchange field and the spin torque [J. C. Slonczewski 2002, J. C. Slonczewski 2005] were developed under the two current approximation, where the spin-up and spin-down currents are treated as independently ballistic. We present an analysis of the exchange energy between the FL and PL as well as the spin torque, both of which are determined by the spin polarized tunneling of electrons. The exchange energy, $E_{\text{ex}}$, and the spin torque, $\tau_s$, which decide the switching current requirement for flipping the memory element, depend on the barrier height [J. C. Slonczewski 1989]. Unfortunately, barrier heights are material and process dependent and are not easily controlled.

We make the following observations:

- In Equation 4.28, $R_T$ is inversely proportional to the transition probability and the DOS in the two FM layers.

- $\tilde{T}$ for a 1D system depends on the tunneling probability ($T$), which we derive later, in Section 7.2.

- The DOS depends on the wave vectors of the spin-up and spin-down electrons ($k_+, k_-$).

- With a decrease in the thickness of the barrier, $T$ increases and hence, $R_T$ decreases for a constant applied voltage.

- With the decrease in resistance, the tunnel current through the device increases without an increase in $V_{\text{app}}$.

The increase in the current density $J$ causes an increase in the spin torque. $\tau_s$, $H_{\text{ex}}$ and $J$, all increase with a reduction in barrier thickness but at different rates. Since both $E_{\text{ex}}$ and $\tau_s$ depend on barrier thickness, it is possible to optimize the barrier thickness to reduce the switching voltage.
Consider a ferromagnetic coupling between FL and PL. We assume the barrier height to be 3.5 eV and plot the exchange field and the spin torque for varying barrier thicknesses with a constant applied potential, in Figure 4.13. We see that both $\tau_s$ and $H_{ex}$ increase with a decrease in barrier thickness, without an increase in external bias. Note that a higher value of $\tau_s$ will allow faster switching with less precession. Hence, by reducing the barrier thickness, we reduce the external bias required. However, the barrier thickness cannot be reduced beyond a limit as, at very low barrier thickness, the exchange energy becomes comparable to the thermal energy, and switching of the FL occurs even without an external applied voltage. Furthermore, pin holes in the insulating oxide pose a technological challenge in making reliable devices.

![Figure 4.13: Variation of $\tau_s$ and $H_{ex}$ for varying barrier thicknesses assuming an insulator with $U_0 = 3.5$ eV.](image)

**4.2.3 Ampere torque**

As with any current carrying conductor, the tunnel current causes an Oersted field, which in turn induces a torque, called the Ampere torque ($\vec{\tau}_A$). $\vec{\tau}_A$ can influence the magnetization of FL. The effect of this Ampere torque has to be incorporated into the LLGE which governs the dynamics of a magnetic moment. To obtain the effect of the Ampere torque
under the single domain approximation, we consider the magnetized disc to have infinitesimal areas of magnetization. Each of these infinitesimal areas can be assumed to be a dipole. Assuming that the FL rotates coherently, we calculate the torque experienced by these individual dipoles and then find the effective torque on the entire disc integrating over the surface of the disc. First, we start with finding the Oersted field inside the circular disc.

**The effect of Oersted field in a circular conductor**

Consider a circular conductor of radius $R$ as shown in Figure 4.14. From Equation 2.6, at a distance $r$ from the centre of the conductor, the Oersted field can be obtained as,

$$\oint \vec{H} \cdot d\vec{l} = \frac{I\pi r^2}{\pi R^2},$$

$$H \ 2\pi r = \frac{I r^2}{R^2},$$

$$H = \frac{I r}{2\pi R^2}.$$  \hspace{1cm} (4.29)

Assuming a uniform current density, $J = I/(2\pi R^2)$, flowing through the conductor, the magnetic field at a distance $r$ from the centre of the circular conductor is,

$$\vec{H} = \frac{J r \hat{a}_\phi}{2}.$$  \hspace{1cm} (4.30)
Torque experienced by a magnetized circular disc

The FL is assumed to be a circular disc placed on the $x - y$ plane with the easy bistable orientation along $\pm x$ direction and the hard axis along $z$ axis, as shown in Figure 4.15. The infinitesimal magnetizations ($d\vec{M}$) in the FL are the same at any instant of time due to the single domain approximation. But, we know from Equation 4.30 that the Oersted field is minimum at the centre of the conductor and maximum on the circumference. Hence, the torque experienced by these individual dipoles are not the same. We calculate the torque experienced by $d\vec{M}$ due to $\vec{H}_\phi$. The magnetization $d\vec{M}$, being along $\pm x$ direction, is

$$d\vec{M} = dM\hat{a}_x.$$  \hspace{1cm} (4.31)

The torque acting on $d\vec{M}$ due to the Oersted field is

$$d\vec{\tau} = d\vec{M} \times \vec{H}.$$ \hspace{1cm} (4.32)

As the direction of the Oersted field is along $\hat{a}_\phi$ as shown in Figure 4.14, to find the torque, we transform the magnetization vector from the cartesian coordinates to the cylindrical co-
ordinates and obtain,
\[ d\vec{M} = dM [\hat{a}_r \cos \phi - \hat{a}_\phi \sin \phi]. \]  \hspace{1cm} (4.33)

We obtain the infinitesimal torque \( d\vec{\tau} \)
\[ d\vec{\tau} = dM [\hat{a}_r \cos \phi - \hat{a}_\phi \sin \phi] \times [H\hat{a}_\phi] \]
\[ = (dM\hat{a}_r \cos \phi) \times \left[ \frac{J r}{2} \hat{a}_\phi \right], \]
\[ d\vec{\tau} = \frac{J r dM}{2} \cos \phi \hat{a}_z. \]  \hspace{1cm} (4.34)

From the above equation, we observe that the torque experienced by \( d\vec{M} \) is dependent on the distance of the magnetization vector from the centre of the disc in space coordinate, and the angle of the magnetization vector.

Before calculating the effective Ampere torque on the FL, we discuss the effect of assuming a single domain approximation. In Figure 4.16a, the direction of all the \( d\vec{M} \)s being along \( +x \), we can assume the magnetization of FL \( (\vec{M}) \), as a single dipole along the \( +x \) direction. At any instant of time, \( \vec{M} \) also takes the same direction as that of \( d\vec{M} \), as shown in Figure 4.16b.

In the presence of an Oersted field, the torque experienced by \( d\vec{M} \) depends on its distance from the centre of the disc as seen in Equation 4.34. With reference to the Figure 4.17, we observe that due to single domain approximation, the torque on elements named A in all the four quadrants is the same as they are equidistant from the centre. Hence, to find the effective torque on FL, we integrate \( d\vec{\tau} \) as
\[ \int d\vec{\tau} = 4 \int_0^R \int_0^{\pi/2} \frac{J r M}{2} \frac{r dr d\phi}{\pi R^2} \cos \phi \hat{a}_z, \]
\[ \vec{\tau} = \frac{4 MJ}{2 \pi R^2} \int_0^R \int_0^{\pi/2} r^2 \hat{a}_z \cos \phi dr d\phi, \]
\[ = \frac{2MJ}{3\pi} R \hat{a}_z. \]  \hspace{1cm} (4.35)
Figure 4.16: Single domain approximation

Figure 4.17: Pairs of micromagnetic elements approximated as a single dipole
Table 4.1: Calculated values of various fields on MTJ

<table>
<thead>
<tr>
<th>Excitation</th>
<th>Direction</th>
<th>Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_d$</td>
<td>$\parallel$ to $x$ axis</td>
<td>$-1.7 \times 10^6$ A/m</td>
</tr>
<tr>
<td></td>
<td>$\parallel$ to $\pm y$ or $z$ axis</td>
<td>$-1.5 \times 10^4$ A/m</td>
</tr>
<tr>
<td>$H_{ex}$</td>
<td>$\vec{M}_{PL}$</td>
<td>$8 \times 10^3$ A/m</td>
</tr>
<tr>
<td>$\tau_s$</td>
<td>$\vec{M} \times (\vec{M} \times \vec{M}_{PL})$</td>
<td>$4.6 \times 10^3$ A/m-s</td>
</tr>
<tr>
<td></td>
<td>$J = 10^{11}$ A/m$^2$</td>
<td></td>
</tr>
<tr>
<td>$H_{PL}$</td>
<td>$\perp$ to $\vec{M}$</td>
<td>$\pm 286$ A/m</td>
</tr>
<tr>
<td>$\tau_A$</td>
<td>$\vec{M} \times (\vec{J} \times \vec{M})$</td>
<td>$0.5 \times 10^3$ A/m-s</td>
</tr>
<tr>
<td>$H_{MN}$</td>
<td>$\parallel$ to $\vec{M}$</td>
<td>$\pm 840$ A/m</td>
</tr>
<tr>
<td></td>
<td>$\perp$ to $\vec{M}$</td>
<td>$\pm 420$ A/m</td>
</tr>
</tbody>
</table>

The effective field, which we now refer to as the Oersted field on the FL is obtained from the torque as

$$\vec{H}_{Amp} = \frac{2 J R}{3 \pi} \hat{a}_\phi,$$  \hspace{1cm} (4.36)

or equivalently,

$$\vec{H}_{Amp} = \frac{2}{3 \pi} R (\vec{J}_z \times \hat{a}_r),$$ \hspace{1cm} (4.37)

which is convenient for inclusion in the LLGE in Section 5.1.1.

Now let us consider a disc shaped MTJ lying on $y - z$ plane with their easy axes of magnetization along $\pm \hat{z}$ and hard axes along $\hat{x}$ in a square array of MTJs, with only nearest neighbours’ interactions, as shown in Figure 4.5. We present a comparative study of calculated values of all the fields and torques acting on an MTJ in Table 4.1. We consider a disc shaped MTJ structure of Fe ($8$ nm) / Al$_2$O$_3$ / Fe ($4$ nm), whose radius is assumed to be $150$ nm.

In Table 4.1, $\vec{M}$ indicates the magnetization of the free layer. We observe that the value
of demagnetization field along the $x$ axis is very much higher than in the $y$ and $z$ directions, due to the shape anisotropy. The exchange field $H_{ex}$ is always along the direction of $\vec{M}_{PL}$, the magnetization of pinned layer. $H_{PL}$, the magnetostatic field from PL, is perpendicular to the direction of $\vec{M}$. Since, we have assumed the easy axes to be along $\pm z$ axis, the field from the nearest neighbours can be either parallel or perpendicular to $\vec{M}$. $\vec{H}_{MN}$ from the neighbours lying along the $\pm z$ i.e, in parallel with the reference, induces the maximum field on the reference. We assumed a current density of $J = 10^{11}$ A/m$^2$ flowing through the device to study the effect of the spin and the Ampere torques. The effect of the Ampere torque is comparable to that of the spin torque. Hence, the effect of the Ampere torque has to be considered, while studying the magnetization dynamics.

Summary

The various fields influencing the MTJ in an STTRAM were discussed in detail in this chapter along with Slonzewski’s spin torque. We also understood that the tunneling current gives rise to an Ampere torque and hence under single domain approximation derived an expression for the same to be included in the LLGE. In the next section, we shall develop a compact model of the MTJ and using SPICE simulation, study the effect of these fields and torques in influencing the switching threshold voltages.