Chapter 3

Fingerprint Recognition Techniques

In this chapter various steps involved in fingerprint recognition are discussed the steps associated with fingerprint orientation estimation, fingerprint segmentation, ridge frequency estimation, enhancement techniques and feature extraction methods are discussed. Also, fingerprint matching methods, one of the major challenges in the fingerprint authentication, are also discussed. The techniques explained are compared in terms of the accuracy and computational complexity.
3.1 Introduction

A fingerprint is a well oriented pattern of interleaved ridges and valleys. A ridge is represented in a fingerprint image as a dark region and a valley is represented as a white region. Typically ridges have a thickness of 100\(\mu\)m to 300\(\mu\)m and the ridge-valley has a thickness of about 500\(\mu\)m. Ridges and valleys often run in parallel, sometimes they bifurcate and sometimes they terminate. The various discontinuities and their small details are referred to as ‘minutiae. Francis Galton observed that the minutiae remain unchanged over an individual’s lifetime and categorized minutiae as per Table 3.1 [Galton, 1892]. According to American National Standards Institute finger minutiae has only four classes: terminations, bifurcations, trifurcations (cross overs) and undetermined. The Federal Bureau of Investigation (FBI) taxonomy is based on two classes: termination and bifurcation [Wegstein, 1982]. Each minutia is denoted by its calls, the x- and y-coordinates and the angle between the tangent to the ridge line at the minutia position and the horizontal axis (Fig. 3.1) [Maltoni, 2005]. In the figure, \(\theta\) is the angle that the minutia tangent forms with the horizontal axis and for a bifurcation minutia, \(\theta\) is now defined by means of the ridge ending minutia corresponding to the original bifurcation that exists in the negative image.

3.2 Fingerprint Image Processing and Feature Extraction

A direct comparison of two fingerprint images in an automated environment by means of correlation techniques is computationally expensive and unstable. Majority of the fingerprint recognition and classification algorithms require a feature extraction stage for identifying the main features of the fingerprint. Fingerprint features include singularities or minutia. Features may be used for matching of fingerprints or may be used for the derivation of other features.
Table 3.1  Galton’s seven minutiae categories

<table>
<thead>
<tr>
<th>Minutiae</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Termination</td>
<td><img src="image1" alt="Termination" /></td>
</tr>
<tr>
<td>Bifurcation</td>
<td><img src="image2" alt="Bifurcation" /></td>
</tr>
<tr>
<td>Lake</td>
<td><img src="image3" alt="Lake" /></td>
</tr>
<tr>
<td>Independent ridge</td>
<td><img src="image4" alt="Independent ridge" /></td>
</tr>
<tr>
<td>Spur</td>
<td><img src="image5" alt="Spur" /></td>
</tr>
<tr>
<td>Point or island</td>
<td><img src="image6" alt="Point or island" /></td>
</tr>
<tr>
<td>Crossover</td>
<td><img src="image7" alt="Crossover" /></td>
</tr>
</tbody>
</table>

Fig.3.1 a) Ridge Ending Minutia  b) Bifurcation Minutia

Fingerprint identification system will have to carry out several steps for feature extraction which include:

1. Ridge Orientation Estimation
2. Fingerprint Image Segmentation
3. Singularity and Core Detection
4. Fingerprint Enhancement
5. Minutiae Detection

The various techniques/methods available in this field are reviewed/explained in the following sections.

### 3.2.1 Ridge Orientation Estimation

The fingerprint orientation image (also called directional image) is first introduced by Grasselli [Grasselli, 1969]. The orientation image is a matrix $O$ whose elements encode the local orientations of fingerprint ridges. Orientation image is one of the key features used to filter, to find singularities in a fingerprint image and to classify fingerprint images. Let $[x, y]$ be a pixel in a fingerprint image. The local ridge orientation at $[x, y]$ is the angle $\theta_{xy}$ that the fingerprint ridges crossing through an arbitrary small neighbourhood centered at $[x, y]$ form with the horizontal axis. The fingerprint ridges are not directed, $\theta_{xy}$ is an un-oriented direction lying in $[0…180^\circ]$. Each element $\theta_{ij}$, corresponding to the node $[i,j]$ of a square-meshed grid located over the pixel $[x_i, y_j]$ denotes the local orientation (Fig. 3.2). The magnitude of $r_{ij}$ is associated with $\theta_{ij}$ to denote the reliability or consistency of the orientation. The value of $r_{ij}$ is low for noisy and corrupted regions and high for good quality regions in the fingerprint image.

The most common and simplest method for extracting local ridge orientation is based on computation of gradients in the fingerprint image.

#### 3.2.1.1 Orientation Estimation by gradient based method:

M. Kass and Witkin, [Kass, 1987] introduced the gradient based method in which the image gradient $G_{xy}$ at point $[x, y]$ of image $I$, is a two-dimensional vector $\begin{bmatrix} G_x(x, y) & G_y(x, y) \end{bmatrix}^T$, where $G_x$ and $G_y$ components are the derivatives of $I$ at the point $[x, y]$ with respect to the $x$ and $y$ directions respectively and can be computed using the classical Prewitt or Sobel convolution masks [Gonzales, 1992].
The gradient vector \( \begin{bmatrix} G_x(x, y) \\ G_y(x, y) \end{bmatrix} \) is defined as:

\[
\begin{bmatrix} G_x(x, y) \\ G_y(x, y) \end{bmatrix} = \nabla I(x, y) = \begin{bmatrix} \frac{\partial I(x, y)}{\partial x} \\ \frac{\partial I(x, y)}{\partial y} \end{bmatrix}
\]

(3.1)

where \( I(x, y) \) represents the gray-scale image. The directional field is perpendicular to the gradients. Gradients are orientations at pixel-scale whereas directional field describes orientation of ridge-valley structure. An averaging operation is done on the gradients to obtain the directional field. Gradients cannot be averaged in the local neighbourhood as opposite gradients will cancel each other. To solve this problem Kass and Witkin doubled the angle of the gradient vectors before averaging. Doubling makes opposite vectors points in the same direction and will reinforce each other, while perpendicular gradients will cancel each other. After averaging, the gradient vectors have to be converted back to their single-angle representation.

The gradient vectors are estimated first in Cartesian co-ordinate system and is given by \([G_x, G_y]\). For the purpose of doubling the angle and squaring the length, the gradient vector is converted to polar system, which is given by
\[
[\rho \ \phi]^T \text{ where } -\frac{1}{2} \pi < \phi \leq \frac{1}{2} \pi
\]

\[
\begin{bmatrix}
\rho \\
\phi
\end{bmatrix} = \begin{bmatrix}
\sqrt{G_x^2 + G_y^2} \\
\tan^{-1} \frac{G_y}{G_x}
\end{bmatrix}
\]

(3.2)

The gradient vector is converted back to its Cartesian as:

\[
\begin{bmatrix}
G_x \\
G_y
\end{bmatrix} = \begin{bmatrix}
\rho \cos \phi \\
\rho \sin \phi
\end{bmatrix}
\]

(3.3)

The average squared gradient \[\begin{bmatrix}
\overline{G_{sx}} \\
\overline{G_{sy}}
\end{bmatrix}\] is given by

\[
\begin{bmatrix}
\overline{G_{sx}} \\
\overline{G_{sy}}
\end{bmatrix} = \begin{bmatrix}
\sum_W G_x^2 - G_y^2 \\
\sum_W 2G_xG_y
\end{bmatrix} = \begin{bmatrix}
G_{xx} - G_{yy} \\
2G_{xy}
\end{bmatrix}
\]

(3.4)

where

\[
G_{xx} = \sum_W G_x^2
\]

\[
G_{yy} = \sum_W G_y^2
\]

\[
G_{xy} = \sum_W G_xG_y
\]

(3.5)

are estimates for the variances and crosscovariances of \(G_s\) and \(G_o\), averaged over the window \(W\). The average gradient direction \(\phi\) is given by:

\[
\phi = \frac{1}{2} \angle (G_{xx} - G_{yy}, 2G_{xy})
\]

(3.6)

where \(\angle (x, y)\) is defined as:
Fingerprint and orientation images are shown in fig. 3.3

![Fingerprint Image](image1.png) ![Orientation Field](image2.png)

Fig.3.3 a) Fingerprint image b) Orientation field

Orientation field can be used to enhance the fingerprint by means of directional filters like Gabor filter. The major drawback of gradient-based orientation estimators is their failure in the near-zero gradient regions, i.e. in the ridge tops and valley bottoms.

### 3.2.2 Estimation of Local Ridge Frequency

Ridge frequency is one of the feature parameter used for enhancement using Gabor filter. The local ridge frequency (or density) $f(x,y)$ at [x, y] is the inverse of the number of ridges per unit length along a hypothetical segment centered at [x, y] and orthogonal to the local ridge orientation $\theta_{xy}$. The local ridge frequency has variations across different fingers; also vary across different regions in the same fingerprint.
Hong et al. [Hong, 1998] estimated local ridge frequency by counting the average number of pixels between two consecutive peaks of gray-levels along the direction normal to the local ridge orientation (Fig. 3.4). For this purpose a section of fingerprint orthogonal to local orientation is taken and the frequency $f_{ij}$ is computed as follows:

1. A 32 X 16 oriented window centered at $[x, y]$ is defined in the ridge coordinate system.
2. The $x$-signature of the gray-levels is obtained by accumulating, for each column $x$, the gray-level values of the corresponding pixels in the oriented window.
3. $f_{ij}$ is found out as the inverse of the average distance between two consecutive peaks of the $x$-signature.

Even though the method is simple and fast, it is difficult to reliably detect consecutive peaks of gray-levels in the spatial domain in noisy fingerprint images. Yang et al. [Yang, 2003] suggested making use of a fitting method based on first and second order derivatives as an alternative to extract ridge distances from the $x$-signature.

![Fig.3.4 Scheme for ridge frequency estimation.](image)
In the method proposed by Maio and Maltoni [Maio, 1998] the ridge pattern is modeled as a sinusoidal-shaped surface, and by using variation theorem, the frequency is estimated (Fig. 3.5). If the variation \( V \) of a function \( h \) in the interval \( [x_1, x_2] \) are small, the variation may be expressed as a function of the average amplitude \( \alpha_m \) and the average frequency \( f \).

\[
V(h) = \frac{1}{2} \alpha_m \cdot f
\]  

(3.7)

**Fig.3.5** Variation of the function \( h \) in the interval \( [x_1, x_2] \) is the sum of amplitudes \( \alpha_1, \alpha_2, \ldots, \alpha_8 \). If the function is periodic or the function amplitude does not change significantly within the interval of interest, the average amplitude \( \alpha_m \) can be used to approximate the individual \( \alpha \) values.

\[
f = \frac{V(h)}{2 \cdot (x_2 - x_1) \cdot \alpha_m}
\]  

(3.8)

Maio and Maltoni (1998a) proposed a practical method based on the above analysis. The variation and the average amplitude of a two-dimensional ridge pattern are estimated from the first and second-order partial derivatives and the local ridge frequency is computed from Equation (3.8).

### 3.2.3 Fingerprint Image Segmentation

Segmentation is the process of separating the foreground regions in the image from the background regions [Gonzalez, 2007]. The foreground region contains the ridges and valleys which is the area of interest in a fingerprint image. The background corresponds to the regions outside the borders of the fingerprint.
area, which do not contain any valid fingerprint information. When minutiae extraction algorithms are applied to the background regions of an image, it results in the extraction of noisy and false minutiae. Thus, segmentation is employed to discard these background regions, which facilitates the reliable extraction of minutiae.

In a fingerprint image, the background regions generally exhibit a very low variance in gray value and the foreground regions have a very high variance. As the fingerprint images are striated patterns, using a global or local thresholding technique does not allow the fingerprint area to be effectively isolated. Variance thresholding method proposed by Mehtre [Mehtre, 1993] is one of the simplest methods available in the literature. In this method, fingerprint image is divided into blocks and the gray scale variance is calculated for each block in the image. If the variance is less than the global threshold, then the block is assigned to be a background region; otherwise it is assigned to be part of the foreground. Considering a block of size $w \times w$, the variance is given by:

$$V(k) = \frac{1}{W^2} \sum_{i=0}^{W-1} \sum_{j=0}^{W-1} [I(i,j) - M(k)]^2$$  \hspace{1cm} (3.9)

where $V(k)$ is the variance for the block $k$, $I(i,j)$ is the gray level value at pixel $(i,j)$, and $M(k)$ is the mean gray level value for the block $k$. Fig. 3.6 shows the variance thresholding based segmented image with $w = 16$ and $w = 8$.

![Fig.3.6](image)

**Fig.3.6** a) Fingerprint image b) Segmented image with $w = 16$ c) Segmented image with $w = 8
Fig 3.7 Segmentation of a fingerprint image proposed by Ratha et al. a) original image b) variance field c) quality image derived from the variance field d) Segmented Image

Ratha et al. [Ratha, 1995] assigned each $16 \times 16$ block to the foreground or the background, according to the variance of gray-levels in the orthogonal direction to the ridge orientation. Also a quality index from the block variance is derived; underlying assumption is that the noisy regions have no directional dependence, whereas regions of interest exhibit a very high variance in a direction orthogonal to the orientation of ridges and very low variance along ridges. Fig.3.7 shows some of the results obtain by the algorithm.
Maio and Maltoni [Maio, 1997] segmented foreground and background by using the average magnitude of the gradient in each image block because of the reason that the fingerprint area is rich in edges due to the ridge/valley alternation, the gradient response is high in the fingerprint area and small in the background. Shi et al. [Shi, 2004] proposed a method based on gray-scale statistics in which contrast-based segmentation is preceded by a non-linear histogram manipulation aimed at decreasing the contrast in ambiguous regions.

3.2.4 Singularity and Core Detection

Singularities are another set of important fingerprint structures that have both global and local properties [Neil, 2004]. Globally, a singularity is a region of a fingerprint where the ridge pattern makes it visually prominent. There are two types of fingerprint singularities: cores and deltas. Locally, a core is the turning point of an inner-most ridge and a delta is a place where two ridges running side-by-side diverge. Core and delta points are best illustrated by examples (Fig. 3.8). Singularities play important role in determining a fingerprint’s class. For example, left loops (as in Fig. 3.8) have one core point near the centre of the print and one delta point to the lower right. Singularities also have other uses, such as fingerprint

![Fig. 3.8 Fingerprint Singularities: Core above and delta below](image)

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alignment and as a coarse discriminating feature. In this work, singularities are the key features used to identify fingerprints and hence a discussion of some of the techniques for singularity detection is followed. There are several approaches proposed in the literature, for singularity detection, which operate on the fingerprint orientation image. The most commonly used method was proposed by Kaswogo and Tojo (Kawogoe, 1984) based on Poincaré index.

3.2.4.1 Singularity detection using Poincaré index:

Let \( \mathbf{G} \) be a vector field and \( C \) be a curve immersed in \( \mathbf{G} \); then the Poincaré index \( P_{G,C} \) is defined as the total rotation of the vectors of \( \mathbf{G} \) along \( C \) as shown in Fig. 3.

\[ P_{G,C} = \sum \theta_i \]

**Fig. 3.9** The Poincaré index computed over a curve \( C \) immersed in a vector field \( \mathbf{G} \) [Maltoni, 2005]

Let \( \mathbf{G} \) be the discrete vector field associated with a fingerprint orientation image \( \mathbf{D} \) and let \( [i,j] \) be the position of the element \( \theta_j \) in the orientation image; then the Poincaré index \( P_{G,C} \) at \( [i,j] \) is computed as follows

- The curve \( C \) is a closed path defined as an ordered sequence of some elements of \( \mathbf{D} \), such that \( [i,j] \) is an internal point;
- \( P_{G,C} \) is calculated by algebraically summing the orientation differences between adjacent elements of \( C \). On close curves, Poincaré index assumes only one of the discrete values: 0°, ±180° and ±360°. In the case of fingerprint singularities:
Fig. 3.11. Singularity detection by using the Poincaré index method. [Maltoni, 2005]

Fig. 3.10 Computation of the Poincaré index in the 8-neighborhood of points belonging to a whorl, loop, and delta singularity respectively [Maltoni, 2005]

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\[
P_{G,C} = \begin{cases} 
0, & \text{if } [i,j] \text{ does not belong to any singular region} \\
360^\circ, & \text{if } [i,j] \text{ does not belong to a whorl type singular region} \\
180^\circ, & \text{if } [i,j] \text{ does not belong to a loop type singular region} \\
-180^\circ, & \text{if } [i,j] \text{ does not belong to a delta type singular region} 
\end{cases}
\]

Fig. 3.10 shows three portions of orientation images. The path defining C is the ordered sequence of the eight elements \( d_k \) \((k=0...7)\) surrounding \([i, j]\). The direction of the elements \( d_k \) is chosen as follows: \( d_0 \) is directed upward; \( d_k \)(\(k=1...7)\) is directed so that the absolute value of the angle between \( d_k \) and \( d_{(k+1)\mod 8} \) is less than or equal to \( 90^\circ \). \( P_{G,C} \) is then computed as

\[
P_{G,C}(i,j) = \sum_{k=0...7} \text{angle}(d_k, d_{(k+1)\mod 8})
\]  
(3.10)

An example of singularities detected by the above method is shown in fig. 3.11.
An alternative method was proposed by Bazen and Gerez [Bazen, 2002] for locating singular points; according to Green’s theorem, a closed line integral over a vector field can be calculated as a surface integral over the rotation of this vector field. Instead of summing angle differences along a closed path, the authors compute the “rotation” of the orientation image and then perform a local integration (sum) in a small neighborhood of each element.

Singularity detection in noisy or low-quality fingerprints is difficult and the Poincaré method may lead to the detection of false singularities [Maltoni, 2005] as given in fig. 3.12. By regularizing the orientation image through a local averaging is often quite effective in preventing the detection of false singularities. Karu and Jain [Karu, 1996] proposed smoothing of orientation image iteratively through averaging until a valid number of singularities is detected by the Poincaré index (fig. 3.12c).

![Fig 3.12.](image)

**Fig 3.12.** a) A poor quality fingerprint; b) the singularities of fingerprint in a) are extracted through Poincaré method (circles highlight the false singularities); c) the regularized orientation.

3.2.4.2 Singularity detection based on local characteristics of orientation image

The reliability \( r \) of the estimate can be derived by the concordance (or coherence) of the orientation vectors \( \mathbf{d} \) in the local window \( W \) (Kass and Witkin
(1987); Bazen and Gerez (2002)). In fact, due to the continuity and smoothness of fingerprint ridges, sharp orientation changes often denote unreliable estimation. Kass and Witkin (1987) defined the coherence as the norm of the sum of orientation vectors divided by the sum of their individual norms. This scalar always lies in \([0,1]\). Its value is 1 when all the orientations are parallel to each other (maximum coherence) and 0 if they point in opposite directions (minimum coherence):

\[
r = coherence(\theta) = \frac{\sum w d}{\sum w |d|}
\]

The above equation simplifies to:

\[
r_{ij} = coherence(\theta_{ij}) = \sqrt{(G_{xx} - G_{yy})^2 + 4G_{xy}^2}
\]

The coherence defined was used by Cappelli et al. [Cappelli, 1999] to coarsely locate singular regions. Fig. 3.13 shows the coherence map computed over 3 × 3 neighbourhoods.

### 3.2.4.3 Core detection and fingerprint registration

The core position may be defined as the location of the north most loop. When the core point is detected with the aim of registering fingerprint images, its location may be quite critical and an error at this stage often leads to a failure of subsequent processing like matching. On the other hand, if the core has to be used

![Orientation image and the corresponding coherence map. Dark regions identifies the singularities](image)
only for fingerprint registration, it is not important to find the north most loop exactly and any stable point in the fingerprint pattern is suitable.

Wegstein [Wegstein, 1982] proposed an automatic method, known as R92, searches a core point independently of the other singularities. The core is searched by scanning the orientation image to find well-formed arches; a well-formed arch is denoted by a sextet (set of six) of adjacent elements whose orientations comply with several rules controlled by many parameters. One sextet is chosen among the valid sextets by evaluating the orientation of the elements in adjacent rows. The exact core position is then located through interpolation (Fig. 3.1).

**Fig.3.14.** The core point “+” located on the chosen sextet

This algorithm was a fundamental component of the fingerprint identification systems used by the FBI and is still extensively used by other researchers like Candela et al. [Candela, 1995]. Other registration techniques include methods proposed by Novikov and Kot [Novikov, 1998] in which the core is defined as the crossing point of the line normal to the ridges as shown in fig.3.15 and used Hough transform to determine its coordinates. In the figure, the straight lines normal to the ridges identify a valid registration point that corresponds to the center of curvature.

**Fig.3.15.** Fingerprint registration using Hough Transform method [Maltoni, 2005]
3.2.5 Fingerprint Enhancement

The feature extraction and minutia detection relies heavily on the quality of the input fingerprint image. A good quality fingerprint image is characterized by the ridges and valleys alternate and flow in a locally constant direction. The ridges can be easily detected in that case and minutiae can be precisely located in the image. But due to skin conditions, sensor noise, incorrect finger pressure, and inherently low-quality fingers, approximately 10% of the fingerprints are of poor quality and the ridge pattern is very noisy and corrupted. Fingerprint images have the following degradation due to several reasons:

- The ridges have small breaks or gaps
- Parallel ridges are not well separated due to the presence of noise which links parallel ridges, resulting in their poor separation.
- Cuts, ceases and bruises on the finger.

The results of the above degradation in fingerprint image are:

- A significant number of spurious minutiae are extracted
- A large number of genuine minutiae are missed and
- Large errors in the location of minutiae are introduced.

In order to overcome the degradation in quality of fingerprint image an enhancement step is carried out in all the fingerprint authentication systems.

Generally for a given fingerprint image, the fingerprint areas resulting from the segmentation step may be divided into three categories [Maltoni, 2003] (Fig. 3.16).

- **Well-defined region:** Well-defined region: ridges can be clearly differentiated from each another.
- **Recoverable region:** ridges are corrupted by a small amount of gaps, creases, smudges, links, and the like, but they are still visible and the neighboring regions provide sufficient information about their true structure.
- **Unrecoverable region**: ridges are corrupted by such a severe amount of noise and distortion that no ridges are visible and the neighboring regions do not allow them to be reconstructed.

![Fig. 3.16. Fingerprint image containing regions of different quality: a) well-defined region b) a recoverable region; c) an unrecoverable region. [Maltoni, 2005](#)]](image)

The aim of enhancement algorithm is to improve the clarity of the ridge structures in the recoverable regions and mark the unrecoverable regions as too noisy for further processing. Fingerprint images are well-defined pattern and general purpose image enhancement techniques do not produce satisfying and definitive results. In the pre-processing stage, histogram manipulation, contrast stretching, normalisation and Weiner filtering have been shown to be effective. There are two methodologies for fingerprint image enhancements: Pixel-wise enhancement and contextual-wise enhancement.

### 3.2.5.1 **Pixel-wise enhancement**:

In a pixel-wise image processing operation the new value of each pixel depends only on its previous value and some global parameters (but not on the value of the neighboring pixels). Normalization is one of the pixel-wise enhancement method carried out as pre-processing. The method proposed by Hong et al. [Hong, 1998] determines the new intensity value of each pixel in an image as

\[
I'[x, y] = \begin{cases} 
    m_0 + \sqrt{(I[x, y] - m)^2 \cdot v_0/v} & \text{if } I[x, y] > m \\
    m_0 - \sqrt{(I[x, y] - m)^2 \cdot v_0/v} & \text{otherwise}
\end{cases}
\]

(3.13)
where $m$ and $\nu$ are the image mean and variance and $m_0$ and $\nu_0$ are the desired mean and variance after the normalization. The effect of normalization is shown in Fig.3.17.

![Fig.3.17. Normalization method using Hon et al. [Maltoni, 2005] ($m_0 = 100$ and $\nu_0 = 100$)](image)

Kim and Park [Kim, 2002] introduced a block-wise implementation of Equation (9) where $m$ and $\nu$ are the block mean and variance, respectively, and $m_0$ and $\nu_0$ are adjusted for each block according to the block features. A similar adaptive normalization was proposed by Zhixin and Govindaraju [Zhixin, 2006]. However, this kind of normalization involves pixel-wise operations and does not change the ridge and valley structures. In particular, it is not able to fill small ridge breaks, fill intra-ridge holes, or separate parallel touching ridges.

### 3.2.5.2 Contextual filtering:

Unlike in a conventional image filtering, where only a single filter is used for convolution throughout the image, in contextual filtering the filter characteristics change according to the local context. Usually, a set of filters is pre-computed and one of them is selected for each image region. In fingerprint enhancement, the context is often defined by the local ridge orientation and local ridge frequency. In fact, the sinusoidal-shaped wave of ridges and valleys is mainly defined by a local orientation and frequency that varies slowly across the fingerprint area. An appropriate filter that is tuned to the local ridge frequency and orientation can efficiently removes the undesired noise and preserve the true ridge and valley structure.
The first contextual filter was proposed by O’Gorman and Nickerson [Gorman, 1988, 1989]. A mother filter based on four main parameters of fingerprint images at a given resolution: minimum and maximum ridge width, and minimum and maximum valley width. The filter is a bell-shaped, elongated along the ridge direction, and cosine tapered in the direction normal to the ridges (Fig. 3.18). The local ridge frequency is assumed constant and therefore, the selective parameter is only the local ridge orientation. Once the mother filter has been generated, a set of 16 rotated versions (in steps of 22.5°) is derived. The image enhancement is performed by convolving each point of the image with the filter in the set whose orientation best matches the local ridge orientation. Depending on some input parameters, the output image may be gray-scale or binary.

![Fig.3.18. The mother filter response proposed by O’Gorman and Nickerson [Gorman, 1989]](image)

Sherlock et al. [Sherlock, 1992, 1994] performed contextual filtering in the Fourier domain. The filter is defined in the frequency domain by the function:

\[ H(\rho, \theta) = H_{radial}(\rho) \cdot H_{angle}(\theta) \]  

(3.14)

where \( H_{radial} \) depends only on the local ridge spacing \( \rho = \frac{1}{f} \) and \( H_{angle} \) depends only on the local ridge orientation \( \theta \). Both \( H_{radial} \) and \( H_{angle} \) are defined as bandpass filters and are characterized by a mean value and a bandwidth. A set of \( n \) discrete filters is derived by their analytical definition. The Fourier Transform \( P_i, i=1\ldots n \) of the filters is pre-computed and stored. Filtering of an input fingerprint image \( I \) is performed as follows:

- The FFT \( F \) of \( I \) is computed.
- Each filter \( P_i \) is point-by-point multiplied by \( F \), thus obtaining in filtered image transforms \( PF_i, i=1\ldots n \) (in the frequency domain).
Inverse FFT is computed for each PF\(_i\) resulting in \(n\) filtered images \(\text{PI}_i\), \(i = 1 \ldots n\) (in the spatial domain).

The enhanced image \(\text{I}_{\text{enh}}\) is obtained by setting, for each pixel \([x, y]\), \(\text{I}_{\text{enh}}[x, y] = \text{PI}_k[x, y]\), where \(k\) is the index of the filter whose orientation is the closest to \(\theta_{xy}\). The entire scheme is shown in fig. 3.19.

![Fig. 3.19. Fingerprint enhancement scheme proposed by Sherlock et al. [Sherlock, 1994].](image)

### 3.2.5.2.1 Gabor filter based enhancement

Gabor filter based enhancement is the most effective method in this scenario and was proposed by Hong et al. [Hong, 1998]. A Gabor filter is defined by a sinusoidal plain wave modulated by a Gaussian. The even symmetric two-dimensional Gabor filter has the equation given by:

\[
g(x, y; \theta, f) = \exp \left\{ - \frac{1}{2} \left[ \frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} \right] \right\} \cos(2\pi f \cdot x\theta) \quad (3.15)
\]

where \(\theta\) is the orientation of the filter, and \([x_\theta, y_\theta]\) are the coordinates of \([x, y]\) after a clockwise rotation of the Cartesian axes by an angle of \((90^\circ - \theta)\), i.e.,

\[
\begin{bmatrix} x_\theta \\ y_\theta \end{bmatrix} = \begin{bmatrix} \cos(90^\circ - \theta) & \sin(90^\circ - \theta) \\ -\sin(90^\circ - \theta) & \cos(90^\circ - \theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} 
\]  

(3.16)
$f$ is the frequency of the sinusoidal plane wave and $\sigma_x$ and $\sigma_y$ are the standard deviations of the Gaussian envelope along the $x$- and $y$- axes respectively. Fig.3.20 shows the graphical representation of the Gabor filter.

Fig. 3.20 Graphical representation of the Gabor filter defined by the parameters $\theta = 135^\circ$, $F=1/5$, and $\sigma_x=\sigma_y=3$ [Maltoni, 2005].

Gabor filters have the four parameters $(\theta, f, \sigma_x, \sigma_y)$ to be specified. The frequency of the filter is governed by the ridge frequency and the orientation is determined by the local ridge orientation. The values of $\sigma_x$ and $\sigma_y$ are to be selected in an empirical way. Large values make the filter robust, but are also more likely to create spurious ridges and valleys. Small values, the filters are less likely to introduce spurious ridges but less effective in removing the noise. In practice, $\sigma_x = \sigma_y = 4$. Also, instead of computing the filter function for the given $\theta$ and $f$ for each pixel, a set \{g_{ij}(x,y): i=1...n_0, j=1...n_f\} of filters are a-priori created and stored, where $n_0$ is the number of discrete orientations \{\theta_i: i=1...n_0\} and $n_f$ the number of discrete frequencies \{f_j: j=1...n_f\}. Then each pixel \[x,y\] of the image is convolved, in the spatial domain, with the filter $g_{ij}(x,y)$ such that $\theta_i$ is the discretized orientation closest to $\theta$, and $f_j$ is the discretized frequency closest to $f_{xy}$. Fig. 3.21 shows an example of the filter set for $n_0 = 8$ and $n_f = 3$.

There are several modifications for Gabor filters that can be found in the literature.
3.2.5.2.2 FFT based enhancement

Another contextual filtering without requiring explicitly computing local ridge orientation and frequency was proposed by Watson et al. [Watson, 1994] and Wills and Myers [Wills, 2001].

![A bank of 24 Gabor filters](image)

Fig. 3.21 A bank of 24 Gabor filters

The fingerprint is divided into $32 \times 32$ image blocks and each block is enhanced separately; the Fourier transform of the block is multiplied by its power spectrum raised to a power $k$:

$$I_{\text{enh}}[x,y] = F^{-1}\{F[I[x,y]] \times |F[I[x,y]]|^k\}$$  \hspace{1cm} (3.16)

The power spectrum contains information about the underlying dominant ridge orientation and frequency and the multiplication has the effect of enhancing the block accordingly. Watson et al. set $k=0.6$ and Willis and Myers proposed a value of $k=1.4$.

3.2.6 Minutiae Detection

Majority of fingerprint identification system rely on minutiae matching and detection of a reliable method for minutiae detection is the final and most important task in fingerprint feature extraction. The first step utilised in minutiae detection phase is to convert the gray-scale image into a binary image. Binarisation processes greatly benefit from a priori enhancement. The binary images are
subjected to a thinning stage which reduces the thickness of ridges to one pixel wide, resulting in a skeleton image. The process is shown in fig. 3.22.

![Fingerprint Image](image1.png) ![Binarised Image](image2.png) ![Thinned Image](image3.png)

Fig. 3.22. a) Fingerprint Image b) Binarised Image c) Thinned Image

3.2.6.1 Binarisation methods

One of the easiest approaches in binarisation is to use a global threshold \( t \) and by setting the pixels whose gray-level is lower than \( t \) to 0 and the remaining pixels to 1. Contrary to that, a local threshold technique in which the \( t \) is changed locally by adapting its value to the average local intensity is also used for binarisation. For fingerprints or very poor quality, a local threshold method cannot always guarantee acceptable results and more effective fingerprint-specific solutions are necessary.

An iterative application of a Laplacian operator and a pair of dynamic thresholds was proposed by Moayer and Fu [Moayer, 1986]. The image is convolved through a Laplacian operator and the pixels whose intensity lies outside the range bounded by two thresholds are set to 0 and 1 respectively.

Ratha, Jain and Chen method: They used peak detection in the gray-level profiles along sections orthogonal to ridge orientation. A 16 × 16 oriented window centered around each pixel \([x,y]\) is considered. The gray-level profile is obtained by projection of the pixel intensities onto the central section. The profile is smoothed through a local averaging the peaks and the two neighboring pixels on either side of each peak constitute the foreground of the resulting binary images as in fig. 3.23.
Even though many approaches are proposed by researchers, contextual filtering based method proposed by Ó Gorman and Nickerson [Gorman, 1998] and Sherlock et al. [Sherlock , 1994] generates regular binary pattern. A good contextual based filter image can be locally thresholded to produce a regular binary ridge pattern.

Fig.3.23. Gray-level profile obtained through projection of pixel intensities on the segment centered at \([x, y]\) and normal to the ridge orientation \(\theta_{xy}\).

### 3.2.6.2 Thinning of Binarised image

Minutiae detection is usually carried out by thinning the binarised image into a one pixel wide.

Morphology based operation: Method proposed by using successive deletion of pixels from all sides of the image. Each of the four sides are eroded away according to some set template (Fig.3.24) [Kasaei, 1997]. If the image matches with template, the middle pixel is removed. Initially the two north images are processed, and then the other compass points are overlaid. One all eight matrices have been sampled on the entire image; the process is repeated again on the newly formed image. Processing stops when no more pixels can be deleted.
Table 3.1. Thinning matrix

<table>
<thead>
<tr>
<th>North</th>
<th>South</th>
<th>East</th>
<th>West</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0</td>
<td>1 1 X</td>
<td>0 X 1</td>
<td>X X 0</td>
</tr>
<tr>
<td>X 1 X</td>
<td>X 1 X</td>
<td>0 1 1</td>
<td>1 1 0</td>
</tr>
<tr>
<td>X 1 1</td>
<td>0 0 0</td>
<td>0 X X</td>
<td>1 X 0</td>
</tr>
<tr>
<td>X 0 0</td>
<td>X 1 X</td>
<td>0 0 X</td>
<td>X 1 X</td>
</tr>
<tr>
<td>1 1 0</td>
<td>0 1 1</td>
<td>0 1 1</td>
<td>1 1 0</td>
</tr>
<tr>
<td>X 1 X</td>
<td>0 0 X</td>
<td>X 1 X</td>
<td>X 0 0</td>
</tr>
</tbody>
</table>

After achieving a binary skeleton, minutiae can be detected by computing a crossing number [Arcelli, 1984]. The crossing number $cn(p)$ of a pixel in a binary image is designed as the half of the sum of differences between pairs of adjacent pixels with 8-neighborhood of $p$.

$$cn(p) = \frac{1}{2} \sum_{i=1,8} |val(p_{i \mod 8}) - val(p_{i-1})|$$  \hspace{1cm} (3.17)

where $p_0$, $p_1$, ..., $p_7$ are the pixels belonging to an ordered sequence of pixels defining the 8-neighborhood of $p$ and $val(p) \in \{0,1\}$ is the pixel value. For an intermediate ridge point, $cn(p)=2$. $cn(p)=1$ for a termination minutia and $cn(p)=3$ for a bifurcation, crossover etc.

Other approaches in thinning are: Hung [Hung, 1993] used the algorithm by Arcelli and Baja [Arcelli, 1984]; Ratha et al. [Ratha, 1995] adopted a technique included in the HIPS library [Landy, 1984], Mehtre [Mehrete, 1993] employed the parallel algorithm described in Tamura [Tamura, 1978] and Coetzee and Botha [Coetzee, 1993] used the method by Baruch [Baruch, 1988]. In Ji et al. (2007) the skeleton is computed through a constrained PCNN (Pulse Coupled Neural Network) where the orientation image is used to constrain the thinning direction of PCNN thus allowing to reduce bothersome artifacts such as the short spikes that conventional thinning algorithms often produce.
3.3 Fingerprint Matching

Fingerprint matching involves the measurement of degree of similarity between two given fingerprints and to give a score between 0 and 1. Most of the fingerprint matching algorithms use feature matching rather than direct grayscale fingerprint image comparison. The fingerprint image acquired during enrollment is denoted as the template (T) and the fingerprint to be matched as the input (I). The fingerprint feature extraction and matching algorithms are usually quite similar for both fingerprint verification and identification problems. This is because the fingerprint identification problem (i.e., searching for an input fingerprint in a database of N fingerprints) can be implemented as a sequential execution of N one-to-one comparisons (verifications) between pairs of fingerprints. Matching can be simplified by indexing the fingerprint into many classes.

3.3.1 Correlation-Based Techniques

Let T and I be the two fingerprint images corresponding to the template and the input fingerprint, respectively. Then an intuitive measure of their diversity is the sum of squared differences (SSD) between the intensities of the corresponding pixels:

\[
SSD(T, I) = \|T - I\|^2 = (T - I)^T(T - I) = \|T\|^2 + \|I\|^2 - 2T^TI
\]  
(3.18)

where the superscript \(T\) denotes the transpose of a vector. If the terms \(\|T\|^2\) and \(\|I\|^2\) are constant, the diversity between the two images is minimized when the cross-correlation (CC) between T and I is maximized:

\[
CC(T, I) = T^TI
\]  
(3.19)

The cross-correlation is a measure of image similarity. The rotation and displacement of the image has to be considered and the similarity has to be calculated accordingly. Let \(I^{(\Delta x, \Delta y, \theta)}\) represent a rotation of the input image I by an angle \(\theta\) around the origin and shifted by \(\Delta x\) and \(\Delta y\) pixels in directions x and y respectively; then the similarity between the two fingerprint images T and I can be measured as:
Minutiae-Based Matching Method

3.3.2.1 Problem formulation

Let $T$ and $I$ be the representation of the template and input fingerprints, respectively. The feature vector is of variable length and the elements are fingerprint minutiae. Each minutia is represented by a number of attributes, including its location in the fingerprint image, orientation, type etc... Each minutia is represented as a triplet $m = \{x, y, \theta\}$ that indicates the x,y minutia location coordinates and the minutia angle $\theta$

\[ T = \{T_1, T_2, \ldots, T_m\}, \quad m_i = \{x_i, y_i, \theta_i\}, i = 1 \ldots m \]

\[ I = \{I_1', I_2', \ldots, I_n'\}, \quad m'_j = \{x'_j, y'_j, \theta'_j\}, j = 1 \ldots n, \]

where $m$ and $n$ denote the number of minutiae in $T$ and $I$, respectively.

The matching between minutia $m'_j$ in $I$ and a minutia $m_i$ in $T$ is maximum if the spatial distance ($sd$) between them is smaller than a given tolerance $r_0$ and the direction difference ($dd$) between them is smaller than an angular tolerance, $\theta_0$:

\[ sd(m'_j, m_i) = \sqrt{(x'_j - x_i)^2 + (y'_j - y_i)^2} \leq r_0 \quad \text{and} \]

\[ dd(m'_j, m_i) = \min(|\theta'_j - \theta_i|, 360^o - |\theta'_j - \theta_i|) \leq \theta_0 \]

Aligning of the two fingerprints is a mandatory step to maximize the number of matching minutiae. Aligning two fingerprints require displacement, rotation and other geometrical transformations:

\[
S(T, I) = \max_{\Delta x, \Delta y, \theta} CC(T, I^{(\Delta x, \Delta y, \theta)}) \tag{3.20}
\]

$CC(T, I^{(\Delta x, \Delta y, \theta)})$
- Scale has to be considered when the resolution of the two fingerprints varies (e.g., the two fingerprint images have been taken by scanners operating at different resolutions).

- Other distortion-tolerant geometrical transformations could be useful to match minutiae in case one or both of the fingerprints is affected by severe distortions.

Let $\text{map}(\ )$ be the function that maps a minutia $\mathbf{m}'_j$ (from $\mathbf{I}$) into $\mathbf{m}''_j$, according to a given geometrical transformation; for example, by considering a displacement of $[\Delta x, \Delta y]$ and a counterclockwise rotation $\theta$ around the origin:

$$\text{map}_{\Delta x, \Delta y, \theta}(\mathbf{m}'_j = \{x'_j, y'_j, \theta'_j\}) = \mathbf{m}''_j = \{x''_j, y''_j, \theta''_j + \theta\}$$

where

$$\begin{bmatrix} x''_j \\ y''_j \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x'_j \\ y'_j \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

Let $\text{mm}(\ )$ be an indicator function that returns 1 in the case where the minutiae $\mathbf{m}''_j$ and $\mathbf{m}_i$ match according to Equations (3.22) and (3.23):

$$\text{mm}(\mathbf{m}'_j, \mathbf{m}_i) = \begin{cases} 1, & \text{sd}(\mathbf{m}'_j, \mathbf{m}_i) \leq r_0 \text{ and } dd(\mathbf{m}'_j, \mathbf{m}_i) \leq \theta_0 \\ 0, & \text{otherwise} \end{cases} \quad (3.24)$$

Then the matching problem can be formulated as

$$\maximize_{\Delta x, \Delta y, \theta, P} \sum_{i=1}^{m} \text{mm}(\text{map}_{\Delta x, \Delta y, \theta}(\mathbf{m}'_{P(i)}), \mathbf{m}_i), \quad (3.25)$$

where $P(i)$ is an unknown function that determines the pairing between $\mathbf{I}$ and $\mathbf{T}$ minutiae; in particular, each minutia has either exactly one mate in the other fingerprint or has no mate at all:

1. $P(i) = j$ indicates that the mate of the $\mathbf{m}_i$ in $\mathbf{T}$ is the minutia $\mathbf{m}'_j$ in $\mathbf{I}$.

2. $P(i) = \text{null}$ indicates that minutia $\mathbf{m}_i$ in $\mathbf{T}$ has no mate in $\mathbf{I}$. 

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3. A minutia $m'_j$ in $I$, has no mate in $T$ if $P(i) \neq j \ \forall \ i = 1...m$.

4. $\forall i=1...m, k=1...m, i \neq k \Rightarrow P(i) \neq P(k)$ or $P(i) = P(k) = \text{null}$

### 3.3.2.2 Similarity score

In the manual matching performed by forensic experts, the number of matching minutiae is the main output of the comparison. Automatic matching system converts this number into a similarity score by normalizing the number of matching minutiae by the average number of minutiae in $T$ and $I$:

$$score = \frac{k}{(m + n)/2}$$

(3.26)

$k$ is the number of matching minutiae.

The definition of optimal rules for combining various similarity contributions into a single score can be complex. Learning-based techniques where the rule and its parameters are optimized to best separated genuine and imposter scores have been proposed by Jea and Govindaraju [Jea, 2005], Srinivasan et al. [Srinivasan, 2006], Feng [Feng, 2008], Lumini and Nanni [Lumini, 2008].

### 3.4 Conclusion

In this chapter, various steps in fingerprint recognition techniques were discussed. The major ideas behind fingerprint image normalization, segmentation, contextual based fingerprint enhancement, FFT based fingerprint etc. were discussed. The idea behind fingerprint matching and matching score computation were explained. In the next chapter development of a novel fingerprint recognition system based on global singularity features is discussed. The state-of-the art minutiae based technique is used to compare the performance of the system developed.