Gravity-induced large grand-unification mass in SU(5) with higher-dimensional operators

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Following the recent method due to Hill, Shif, and Wetterich, we investigate the impact of higher-dimensional operators ($d \geq 9$) induced by gravity on the mass dimension of extra dimensions on the minimal SU(5) grand unified theory. Modifications caused by the $d < 5$ operator are found to be ruled out, even if the compactification scale $M_{C}$ is as low as $10^{3}$ GeV, as they require $\sin^2 \theta_{u} \lesssim 0.20$ in conflict with the present world average. The addition of a six-dimensional operator is found to allow only high unification masses $M_{u} \sim 0.1 - 1$ TeV, with $M_{C} \sim 10^{7} - 10^{9}$ GeV and $\sin^2 \theta_{u} \gtrsim 0.22 - 0.24$. The grand unification coupling constant is also found to be significantly smaller.

I INTRODUCTION

Gauge theories of the Kaluza-Klein type offer the exciting possibility of unification with gravity through the introduction of higher dimensions leading to the four-dimensional structure of the present Universe as a result of the compactification of extra dimensions. Attempts have been made to generate effective four-dimensional gauge theories, such as the standard model and grand-unified theories (GUTs), from the isometry group of the compactified manifold to identify the observed fermions as the chiral representations of the effective gauge theories, and to compute the gauge couplings in terms of the characteristic length scales of extra spatial dimensions. Although superstring theory is expected to provide a realistic gauge unification of all basic interactions, a lot of interest still remains in conventional GUTs, with and without gravity induced effects. Although most of the GUTs with intermediate symmetries have been in trouble to generate the observed fermions of the standard model, the introduction of higher-dimension operators induced by gravity with dimensions $d \geq 4$ and scaled by powers of $M_{C}$ is subject only to the symmetries of low energy theory and is known to occur, for example in the presence of gravitational instantons in quantization for $M_{C} \sim M_{I}$. The five-dimensional operators are seen to arise naturally as a result of compactification of extra dimensions in a Kaluza-Klein type theory. The impact of such an operator on the quark to lepton mass ratio $m_{D}/m_{L}$ predicted by the minimal SU(5) model was examined by Ellis and Gaillard.

In the case of a supersymmetric SU(5) GUT, significant modifications to $\tau_{R}$ and $\sin^2 \theta_{L}$ have been noted with $M_{C} \sim M_{I} \sim 10^{16}$ GeV. In the SO(10) GUT, with $SU(3)_{c} \times SU(2)_{L} \times SU(2)_{R} \times U(1)_{B}$ as intermediate symmetry, the masses of the $H_{R}$ gauge bosons have been brought down to the order of the Higgs scale leading to possible observation of low mass parity restoration in the future. If the minimal SU(5) GUT of the compactification scale is allowed to be about two orders lower than $M_{I}$, which is possible within certain Kaluza-Klein-type theories, Shif and Wetterich have observed a very significant increase of $\tau_{R}$, so as to be compatible with the experimental limit for the $p_{-e}^{-}\tau_{R}$ mode.

In this paper we use the method due to Hill, Shif, and Wetterich and Hill to compute modifications of the unification mass at high scale ($M_{u} > 10^{15}$ GeV), it is natural to suppose that there could be significant modification to the GUT predictions by gravity induced corrections. It is the purpose of this paper to compute such modifications to the minimal GUT predictions.
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Lagrangian, and the unification mass should be of the order (0.1 - 1) $M_{G}$, where $M_{G}$ could be anywhere between $10^{17}$ and $10^{19}$ GeV. As our main result, we then examine the modifications caused by adding a $d=6$ operator in the Lagrangian. We find that the only permissible values of the unification mass should be of the order (0.1 - 1) $M_{G}$, where $M_{G}$ could be anywhere between $10^{17}$ and $10^{19}$ GeV. Interestingly enough, the allowed values of the electroweak mixing angle can be made consistent with the currently available world average with $\sin^{2}\theta_{W}$, $\simeq 0.22 - 0.24$ for every value of the unification mass. Another interesting aspect of the present analysis is that the bare-grand-unification coupling $\alpha_{G}$ turns out to be nearly 2 orders of magnitude smaller than the earlier results. We also obtain perturbative and positivity bounds on certain parameters and mention a new relation among them.

In Sec II we obtain general formulas for the unification mass and the electroweak mixing angle including five-dimensional operators and particular forms of still higher-dimensional operators in the Lagrangian. In Sec III we discuss earlier results with five-dimensional operators. In Sec IV we report numerical analysis including five- and six-dimensional operators. Our conclusions are stated in Sec V.

II FORMULAS FOR GAUGE COUPLINGS, UNIFICATION MASS, AND $\sin^{2}\theta_{W}$

As has been emphasized earlier, gravitational effects could induce nonrenormalizable operators of dimension $d \geq 5$, scaled by powers of $M_{G}$, and the Lagrangian, but only the impact of $d=5$ operators have been examined so far on the unification mass $M_{G}$ and $\sin^{2}\theta_{W}$. Such operators are restricted only by the symmetries of the theory at lower energies. Denoting $\delta$ as the scalar in the adjoint representation 24 of $SU(5)$, the effect of the nonrenormalizable operators can be included in the following manner in the $SU(5)$-invariant Lagrangian:

$$L = L_{0} + L_{NR},$$

$$L_{NR} = \sum_{d=5}^{5} \frac{L_{NR}^{(d)}}{M_{0}^{d-4}},$$

$$L_{NR}^{(d)} = \frac{1}{2} \frac{Y_{i}^{d-2}}{M_{0}^{d-4}} \text{Tr}(\phi^{d} \phi'^{d}),$$

$$L_{0} = \frac{1}{2} \text{Tr}(\phi^{2}),$$

$$L_{\nu} = \nu \dot{\nu} - \frac{1}{2} \text{Tr}(\phi^{2} \nu^{2}),$$

$$I_{\mu} = \partial_{\mu} a_{\mu} - \partial_{\mu} a_{\mu} - \alpha_{V} \text{Tr}(\nu^{2} - 1).$$

$$\text{Tr}(\lambda_{\alpha} \lambda_{\beta}) = \frac{1}{2} \delta_{\alpha \beta}. $$

In Eq (21), $I_{\mu}$ is the $\mu$-th component of the gauge field $A_{\mu}$, $\lambda_{\alpha}$ is the corresponding generator, and $\alpha_{V} = 1/2$, $\alpha_{V} = 1$, $\alpha_{V} = 1$ for the unknown parameters. In Refs. 17 and 20 the case, with the five-dimensional operator corresponding to $\eta^{35} \phi$, is of $\alpha_{V} = 0$ for $n \geq 2$. It may be noted that the expressions (2a) for higher-dimensional operators given in Eq (2c) is not the most general one, especially when $n \geq 2$. For example, with $n = 2$ other gauge-invariant operators not covered by (2a) are $\text{Tr}(\phi^{2} \phi^{2} F_{\mu \nu} F^{\mu \nu})$ and $\text{Tr}(\phi^{2} \phi^{2} \phi^{2} F^{\mu \nu} \phi^{\mu \nu})$ the latter being more troublesome for computations of the physical quantities of interest in this paper. We confine to the choice (2a) for the sake of convenience and obtaining modifications to $M_{G}$ and $\sin^{2}\theta_{W}$ with a constraint on the parameters as shown in Eq (14a) in Sec IV. Using the vacuum expectation value of $\phi$ as

$$\langle \phi \rangle = (\frac{1}{\sqrt{2}})^{2} \text{diag}(1, 1, 1, -4, -\frac{1}{4}),$$

denoting $g_{C}, \epsilon_{C}$ and $g_{V}$ as the $SU(3)_{C}, SU(2)_{W}$, and $U(1)_{Y}$ coupling constants, respectively, and the gravity induced changes as

$$g_{C}^{2} = \sum_{n=1}^{2} \epsilon_{C}^{n},$$

$$\epsilon_{1} = -\frac{3}{2} \epsilon_{1}^{11} + \frac{1}{2} \epsilon_{1}^{12} - \frac{1}{2} \epsilon_{1}^{13} + \frac{1}{2} \epsilon_{1}^{13},$$

$$\epsilon_{2} = -\frac{3}{2} \epsilon_{2}^{11} + \frac{1}{2} \epsilon_{2}^{12} - \frac{1}{2} \epsilon_{2}^{13} + \frac{1}{2} \epsilon_{2}^{13},$$

where the ellipsis in (5) includes the effect of operators $d > 7$ and

$$\epsilon_{n}^{d} = \frac{1}{\sqrt{15}} \frac{\epsilon_{n}}{M_{G}},$$

Using $\alpha_{V} = \epsilon_{V} / 4\pi$, where $\epsilon_{V}$ is the bare GUT coupling and the relation

$$\alpha_{V} = 6 \pi / 5 \alpha_{G} + \frac{1}{12} \frac{\epsilon_{V}}{M_{G}},$$

Eq (6) can be rewritten as

$$\eta^{d} = 2 \frac{\pi \alpha_{V}}{2} \frac{M_{G}}{M_{0}} \frac{\epsilon_{V}}{M_{G}}.$$
\[ \ln \frac{M_t}{M_W} = \frac{1}{D} \left( 1 + \epsilon_t - \frac{5 \epsilon_3}{3} \right) \left( 1 - \frac{8 \alpha}{3 \alpha_t} \right) \left( 1 - \frac{8 \alpha}{3 \alpha_t} \right) \]

\[ \sin^2 \theta_W = \sin^2 \theta_W^{(1)} + \frac{21 \alpha}{4 \alpha_t} \epsilon_t \]

where \( M^{(1)}_t \) and \( \sin^2 \theta_W^{(1)} \) denote the one-loop predictions of the minimal model, without gravity induced effects, including only one set of 24 + 5 of Higgs fields and three generations of fermions.

\[ \ln \frac{M_t}{M_W} = \frac{6}{67 \alpha} \left( 1 - \frac{8 \alpha}{3 \alpha_t} \right) \]

\[ \sin^2 \theta_W = \frac{23}{134} + \frac{109 \alpha}{201 \alpha_t} \]

In Eqs (11) and (12), \( \alpha^{-1}(M_W) = 127.54 \) and \( \alpha_t = \left( \frac{4 \pi}{3} \right) \alpha \) corresponding to \( \Lambda_{\text{MS}} = 160 \) MeV, where MS denotes the modified minimal subtraction scheme. Solutions in the similar forms including only the five-dimensional operators have been obtained in Refs 17 and 20. Here we note that the effects of all higher-dimensional operators are contained in the parameters \( \epsilon_t, \epsilon_3, \) and \( \alpha_t \) as illustrated in Eq (5). Thus, Eq (11) through Eq (5), can, in principle, account for all gravity induced corrections due to higher-dimensional operators.

### III. Solutions with Five-Dimensional Operators

In this section we briefly review the earlier solutions obtained with \( d = 5 \) operator noting that they are ruled out because of experimental constraints on \( r_p \) and \( \sin^2 \theta_W \). Such a conclusion was already reached by Hill\(^{17}\) with \( M_C = 10^{19} \) GeV. We, therefore, discuss Shafi-Wetterich\(^{20}\) solutions with \( M_C = 10^{17} \) GeV which SU(5) has been stated to survive the then existing experimental data. Using \( \epsilon^{(1)} = \epsilon^{(1)} = \epsilon = \eta \) and \( \epsilon = \eta = \eta = \epsilon \) in Eqs (5) and (11) yields \( \epsilon_t = \epsilon, \epsilon_3 = -3 \epsilon / 2, \) and \( \alpha_t = -\epsilon / 2 \) and \( D = 1 - 3 \epsilon / 76 \), as in Ref 20, leading to

\[ \ln \frac{M_t}{M_W} = \frac{6}{67 - 38 \epsilon} \alpha^{-1} \left( 1 - \frac{8 \alpha}{3 \alpha_t} \right) \]

\[ \sin^2 \theta_W = \frac{23}{2} + \frac{109 \alpha}{3 \alpha_t} - \frac{41 + 116 \alpha}{\alpha_t} \]

For different assumed values of the parameter \( \epsilon \) the gauge coupling constant \( \alpha_t, \) unification mass \( M_{\text{unif}} \), and \( \sin^2 \theta_W \) are computed as has been done before. The basic parameter \( \eta \) occurring in the Lagrangian is calculated using the relation

\[ \eta = \frac{25 \pi}{2} \left( 67 - 38 \epsilon \right) \left( \frac{M_t}{M_U} \right) \]

It is worth mentioning the new additional fact that the \( \epsilon \) parameter can be bounded from above and below, in this case, using the positivity and perturbative constraints on \( \alpha_t \). From Eq (13a), the positivity of \( \alpha_t \) suggests that \( \epsilon < 0.25 \) and 1.76, whereas the perturbative constraint \( \alpha_t < 1 \) yields with \( \alpha_t = 0.1088 \) and \( \alpha_t = 1.2754 \), \( \epsilon > -70 \). The lower bound is dominated by \( \alpha_t^{-1} \) and does not vary significantly in the allowed range of \( \alpha_t, (M_H) \) corresponding to \( \Lambda_{\text{MS}} = 160 \pm 100 \) GeV.

Numerical solutions for the unification mass, \( r_p, \) \( \sin^2 \theta_W, \) \( \alpha_t^{-1}, \) and \( \eta \) for different values of the \( \epsilon \) parameter are presented in Table 1. For calculating \( \eta \), the value of the compactification scale has been taken to be \( M_C = 10^{17} \) GeV as before. The uncertainty in \( \Lambda_{\text{MS}} \) and the matrix elements for \( p = e^+ e^- \) in order that \( \epsilon_t \) agrees with the experimental limit, \( r_p \geq 3 \times 10^{17} \) yr, it is clear that \( \epsilon = 0.015 \) which needs \( \sin^2 \theta_W \geq 0.203 \), even though \( M_C \) is allowed to be as low as \( 10^{16} \) GeV. Such lower values of \( \sin^2 \theta_W \) in the modified solutions, needed for the stability of the proton, are clearly in contradiction with the recent world average \( \sin^2 \theta_W = 0.23 \pm 0.005 \)
IV. NEW SOLUTIONS WITH 11 V,- AND SIX-DIMENSIONAL OPERATORS

As we mentioned in Sec III, modifications with $d=5$ in the minimal GUT seem to be ruled out as they require $\sin^2\theta_W < 0.203$, in order to yield $\tau > 3 \times 10^{12}$ yr. But, following the similar philosophy as in Refs 17 and 20, we investigate whether inclusion of still higher-dimensional operators could predict $\tau$ and $\sin^2\theta_W$ consistent with the available experimental data. As the next dimensional operators could predict $\tau$ and $\sin^2\theta_W$, we investigate whether inclusion of still higher-dimensional operators could predict $\tau$ and $\sin^2\theta_W$, we investigate whether inclusion of still higher-dimensional operators could predict $\tau$ and $\sin^2\theta_W$, we investigate whether inclusion of still higher-dimensional operators could predict $\tau$ and $\sin^2\theta_W$, we investigate whether inclusion of still higher-dimensional operators could predict $\tau$ and $\sin^2\theta_W$, we investigate whether inclusion of still higher-dimensional operators could predict $\tau$ and $\sin^2\theta_W$, we investigate whether inclusion of still higher-dimensional operators could predict $\tau$ and $\sin^2\theta_W$, we investigate whether inclusion of still higher-dimensional operators could predict $\tau$ and $\sin^2\theta_W$, we investigate whether inclusion of still higher-dimensional operators could predict $\tau$ and $\sin^2\theta_W$, we investigate whether inclusion of still higher-dimensional operators could predict $\tau$ and $\sin^2\theta_W$, we investigate whether inclusion of still higher-dimensional operators could predict $\tau$ and $\sin^2\theta_W$, we investigate whether inclusion of still higher-dimensional operators could predict $\tau$ and $\sin^2\theta_W$, we investigate whether inclusion of still higher-dimensional operators could predict $\tau$ and $\sin^2\theta_W$.

Note that the relation (14a) is also valid in the $d=5$ case.

The basic parameters of the Lagrangian are the $\eta$ parameters, rather than the $\epsilon$ parameters. Except the positivity and perturbativity constraints on $\epsilon$, as already discussed in this paper, there seems to be no theoretical constraint on the $\eta$ parameters. But, in order that the modified Lagrangian makes some sense, the following general criteria on the parametrical data. As the next, and we treat the solutions as acceptable when either criteria (i) and (iii) are satisfied and (i) and (iv) are satisfied. The magnitude of $\eta^{(n)}$, $n=1,2$, is not very large, but although the individual values of the $\eta$ parameters may differ, one parameter is that they are of the same order. But if the gravity-induced corrections might be acting as the terms in a perturbation series, for reasons later to be explained, we consider the possibility that $\eta^{(1)}$ may be the only one with $\eta^{(2)}$.

Now we discuss briefly how the criterion (i) has been satisfied by earlier solutions with $d=5$ operators. Since $\eta^{(1)}$ and $\eta^{(2)}$ are of the same order, it is clear from Eq (18) that if $M_\eta \ll M_G$, then $|\eta^{(1)}| \gg |\eta^{(2)}|$. For example, with $M_\eta = 10^{15}$ GeV, we obtain $|\eta^{(1)}| = 4 \times 10^6$, $|\eta^{(2)}| = 10^{10}$ (or $4 \times 10^6$, $-10^{10}$), and $|\eta^{(2)}| = 10^{10}$ (or $4 \times 10^6$, $-10^{10}$), for $M_\eta = 10^{15}$, $10^{16}$, and $10^{17}$ GeV, respectively. Further, the combinations $(M_\eta, \eta^{(1)}) = (10^{15}, 10^{16})$ GeV, $(10^{14}, 10^{15})$ GeV, and $(10^{16}, 10^{17})$ GeV, correspond to the parameter values $(\alpha, \eta_3) = (3.07, 5.56), (1.29, -3.8)$, and $-10.7$

<table>
<thead>
<tr>
<th>$\eta^{(1)}$</th>
<th>$\eta^{(2)}$</th>
<th>$M_\eta$ (GeV)</th>
<th>$\sin \theta_W$</th>
<th>$\tau_\eta$</th>
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<td>-0.995</td>
<td>-0.9956</td>
<td>-0.3131</td>
<td>-0.6035</td>
</tr>
</tbody>
</table>

TABLE 11 Parameters $\epsilon_t$, $\epsilon_s$, $\epsilon_t$, and $\epsilon_s$, computed using the loop normalization group equations and corrections due to $d=5$ and 6 operators. Relations among $\epsilon$ parameters are given in Eq (14).
My~0 IMc

But, as mentioned earlier, criterion (in), and 1 36, respectively Thus, combining criteria (1) and (ii), and the ratio 1 772/77, 1 51, 87, I 51, solutions which satisfy either |η^2/η^1| or slightly ruled out because of these highly undesirable values of the parameters and their ratio, we conclude that if the addition of a six-dimensional operator is going to make any sense, in the presence of a five-dimensional operator, the solutions with M( much less than M^c^c^c. As a result, a small numerical value of a^c is understood by noting that

\[ \alpha_c = \frac{67 + 25 \varepsilon_c - 42 \varepsilon_c \alpha_c}{3 \alpha_c} \]

where the numerator tends to be small as ε_c ~ ε_L ~ - 1. The small value of α_c decreases the proton decay rate resulting in a very significant increase in τ_p. Thus, according to the present observations, the enhancement in τ_p occurs due to two sources - largeness of M(U) and smallness of a_c. Introducing a factor of 10^18 enhancement due to smallness of α_c has been included if, on the other hand, we confine to the most general expectation, M(U) ~ M(P) ~ 10^15 GeV, the GUT does not seem to have unification significantly below M(U) ~ 10^18 GeV.

Before closing this section it might be necessary to

### Table III:

<table>
<thead>
<tr>
<th>M_i (GeV)</th>
<th>η^1</th>
<th>η^2</th>
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<th>(η^1)/η^2</th>
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</thead>
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<td>-0.0356</td>
</tr>
<tr>
<td>10^18</td>
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<td>1.461×10^-4</td>
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<td>-0.0003</td>
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<tr>
<td>10^19</td>
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<td>1.188×10^-4</td>
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<td>10^18</td>
<td>0.239</td>
<td>1.039×10^-4</td>
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### Table IV:

<table>
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<tr>
<th>M_c (GeV)</th>
<th>M(U) (GeV)</th>
<th>sin^2θ_u</th>
<th>a_c</th>
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<th>η^2</th>
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</table>
clarify certain points regarding the self-consistency of the treatment of compactification effects through an expansion in higher-dimensional operators. As evident from Table II, $e^{11}$ and $e^{12}$ are related to $e^{10}$ by Eq. (8). This might give the impression that the expansions for $e_{c}$, $e_{f}$, and $e_{f}$ expressed in Eq. (5) are not converging. But we have taken only the first two out of a large number of terms in the series in Eq. (5) to show that they fully account for the available data on $\sin^{2}\theta_{W}$, and large values of $M_{U}$. This implies that, so far as the available values of $\sin^{2}\theta_{W}$ and allowed values of $e^{10}$ are concerned, $e^{11}=0$ for $n>2$, thus guaranteeing convergence of the series and the self-consistency of the method for $M_{U} \approx (10^{-1} - 1)M_{p}$. Another way of looking into the convergence of expansions is the following: Since the first two terms are capable of explaining the available experimental values of $\sin^{2}\theta_{W}$, for $M_{U}$ in the range $10^{16} - 10^{19}$ GeV, it is certainly true that at least the same values of $\sin^{2}\theta_{W}$ and $M_{U}$ are possible by taking larger number of terms such that $|e^{11}| < |e^{10}|$, for $n>2$, thus guaranteeing convergence of the series and self-consistency of the method adopted.

V. SUMMARY, CONCLUSION, AND DISCUSSION

The minimal SU(5) model predicts a proton lifetime about 2-3 orders less than the observed lower limit and $\sin^{2}\theta_{W} \approx 0.21$. Including gravity-induced effects through $d=5$ operators, scaled by the compactification mass, although Hill\(^{11}\) obtained quite lower values of $\sin^{2}\theta_{W}$ and, hence, ruled out any modification with $M_{C} \sim M_{p}$, Shafi and Wetterich\(^{10}\) found that the minimal GUT could be saved by such compactification effects if $M_{C} = 10^{17}$ GeV. But, as we have noted here, these solutions can be consistent with experiments provided $\sin^{2}\theta_{W} < 0.203$ which seem to disagree with the present world average $\sin^{2}\theta_{W} = 0.21\pm 0.005$.

To investigate further, whether the SU(5) predictions can be improved by spontaneous compactification effects, we have investigated the combined role of both five- and six-dimensional operators, which is allowed, in principle, at least, and in the same spirit. Out of, at least, three different possible forms for the six-dimensional operator, we have chosen only one for the sake of simplicity and convenience, and also for obtaining modifications to GUT predictions within the constraint expressed by Eq. (14a). Although our computation in Table II indicates $\sin^{2}\theta_{W} \approx 0.23\pm 0.24$, we have checked that with slight change of $e_{c}$ and $e_{f}$ the allowed range is $\sin^{2}\theta_{W} \approx 0.22 - 0.24$. It seems, at first sight, as if all numerical solutions with $M_{U} \approx 10^{15}-10^{18}$ GeV and $\sin^{2}\theta_{W} \approx 0.22 - 0.24$, as shown in Table II, are allowed. But when we compute the basic parameters $\eta^{11}$ and $\eta^{12}$ and their ratio $|\eta^{11}/\eta^{12}|$, where $|\eta^{11}/\eta^{12}|$ occurs as the coefficient of the five- (six-) dimensional operators, we find that $|\eta^{11}/\eta^{12}| < |\eta^{12}/\eta^{11}|$ for those solutions for which $M_{U} / M_{C} \leq 10^{-2}$, and $M_{C} = 10^{-15}-10^{-17}$ GeV. As the Lagrangian does not make sense with such parameters, the corresponding solutions with low unification masses $M_{U} = 10^{14}$ GeV are ruled out. Thus, it is found that $M_{U} = 10^{15}$ GeV is found to be strongly valid if the compactification occurs at the most generally acceptable scale, $M_{C} = M_{p}$.

The present analysis reveals that the gravity-induced corrections with $d=5$ and 6 operators permit high unification mass, $M_{U} \approx (10^{15} - 10^{18})$ GeV, if $\sin^{2}\theta_{W} \approx 0.22 - 0.24$ for every $M_{U}$. The values of the $\eta$ parameters are never found to be large for such solutions belonging to class (I), and the ratio $|\eta^{11}/\eta^{12}| \approx 0.1$ for others with $\eta^{11}/\eta^{12} \approx 0.1$ for others with $M_{U} \sim M_{p}$. Although such parameters with $|\eta^{11}/\eta^{12}| \approx 0.1$ are generally expected in unified gauge theories, the other values with $|\eta^{11}/\eta^{12}| \approx 0.1$ suggest that the vacuum terms containing higher-dimensional operators might be acting as perturbation upon the renormalizable Lagrangian. Using the most general value, $M_{C} = M_{p}$, we find that solutions with $M_{U} \geq 10^{15}-10^{17}$ GeV are ruled out and the gravity-induced effects permit only $M_{U} \approx 10^{16}-10^{17}$ GeV. For the first time, we find here a grand unified theory with the GUT coupling constant as small as $e_{10} \approx 0.1$. The enhancement of the proton lifetime occurs due to two factors: largeness of $M_{U}$ and smallness of $r_{d}$. Thus, if the addition of five- and six-dimensional operators to the GUT Lagrangian is going to make sense, the predictions of minimal SU(5) with an unstable proton and $\sin^{2}\theta_{W} < 215$ must be modified to be consistent with an extremely stable proton ($r_{d} > 10^{27}$ yr) and $\sin^{2}\theta_{W} \approx 0.22 - 0.24$.

ACKNOWLEDGMENT

One of us (M.K.P.) thanks Professor R. N. Mohapatra for useful discussions.

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Spontaneous compactification effects, low-energy signature of quark-lepton unification, and small neutrino masses in SO(10)

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(Received 15 August 1988)

Signatures of quark-lepton unification can be experimentally verified by rare-kaon-decay processes provided the SU(4)c-breaking scale \( M_\text{c} \approx 10^7 - 10^8 \) GeV. With the single intermediate symmetry, \( \text{SU}(2)_L \times \text{U}(1)_R \times \text{SU}(4)_C \) in SO(10), we find, for the first time, that such a scale and small Majorana neutrino masses are allowed when gravity-induced corrections due to a five-dimensional operator, arising out of spontaneous compactification of extra dimensions, are included. For such lower values of \( M_\text{c} \), the predicted proton lifetime is large depending upon the value of \( \sin^2 \theta_w \) in the range 0.22-0.24. For still larger values of \( M_\text{c} \), neutrino masses and proton lifetime decrease, and the latter saturates the Irvine-Michigan-Brookhaven limit when \( M_\text{c} \approx 10^{13} - 10^{12} \) GeV.

I. INTRODUCTION

Two of the most important theoretical developments in particle physics have been the Kaluza-Klein-type unification with gravity\(^1,2\) and grand unified theories\(^3-5\) (GUTs) of strong, weak, and electromagnetic interactions. Originally proposed to unify gravitation with electromagnetism, the Kaluza-Klein method\(^1\) has been applied with the standard, or grand unifying symmetries,\(^2\) in higher dimensions with a view to unifying all basic forces of nature. In such cases, effective gauge theories in four-dimensional space-time emerge as a result of compactification of extra dimensions. In theories employing spontaneous compactification, nonrenormalizable higher-dimensional operators involving gauge and Higgs fields are always generated and the coefficients of these operators are scaled by the powers of the compactification scale \( M_\text{c} \) (Refs. 2 and 6). Even without invoking the idea of dimensional reduction, such operators scaled by powers of the Planck mass \( (M_\text{pl} = 10^{19} \) GeV) can also be present as the gravity-induced corrections to the GUT Lagrangian.

Compared to many other GUTs, SO(10) has many attractive features.\(^3\) It is the minimal left-right-symmetric extension of SU(5), and contains all known fermions (plus the right-handed neutrino) of one generation in a single spinorial representation. It can attribute the origins of parity \((P)\) and CP violations\(^7\) as arising out of spontaneous symmetry breaking. It can explain neutrino masses over a wide range of values. With the decoupling of parity \((P)\) and SU(2)\(_R\)-breaking scales, the new SO(10) model\(^8\) provides a natural solution to the domain-wall problem.\(^9\) With one or more intermediate symmetries, besides explaining the observed proton stability, SO(10) promises experimental verification of interesting theoretical ideas such as the quark-lepton unification based upon SU(4)\(_C\), and left-right symmetry.\(^10\) To date, possible low-energy signatures of SU(4)\(_C\) breaking in SO(10), SU(16), and SU(8)\(_L\) \times SU(8)\(_R\) GUTs have been predicted,\(^8,10,11\) within the context of decoupling \( P \) and SU(2)\(_R\) breakings, in the presence of the gauge group \( \text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(4)_C = G_{2,4} \) with \( g_L \neq g_R \), as one of the intermediate symmetries, at a scale \( M_\text{c} \approx 10^7 - 10^8 \) GeV, such that both free neutron oscillations\(^12\) and rare-kaon decays are expected to be experimentally observable. Since the first class of experiments are very difficult, because of nonavailability of free neutron sources, it might be useful to search for a grand unified theory where the consequences of SU(4)\(_C\) breaking can be testified by a relatively easier class of experiments, such as the rare-kaon decays only. Even with two intermediate symmetries in the SO(IO) GUT (Ref. 13),

\[
\begin{align*}
\text{SU}(16) & \rightarrow \text{SU}(2)_L \times \text{U}(1)_R \times \text{SU}(4)_C (\equiv G_{2,4}) \\
M_\text{c} & = \text{SU}(2)_L \times \text{U}(1)_R \times \text{U}(1)_B - L \times \text{SU}(3)_C \\
M_\text{r} & = \text{SU}(2)_L \times \text{U}(1)_R \times \text{SU}(3)_C (\equiv G_{2,3})
\end{align*}
\]

\( (1) \)

It has not been possible to obtain \( M_\text{c} \approx 10^7 - 10^8 \) GeV, for the presently accepted values\(^14\) of \( \sin^2 \theta_w = 0.23 \pm 0.005 \). Two or more intermediate symmetries populating the grand desert provide possibilities of richer physical structure; but predictions with a single intermediate symmetry are very appealing because of the minimal nature of the underlying GUT scenario.

In this paper we note that, with the single intermediate symmetry \( \text{SU}(2)_L \times \text{U}(1)_R \times \text{SU}(4)_C (\equiv G_{2,4}) \), the SO(10) GUT is ruled out as it predicts a proton lifetime lower than the Irvine-Michigan-Brookhaven (1MB) limit\(^15\) for the \( p \rightarrow e^+ \pi^0 \) mode. But, when the twin ideas underlying grand unification and Kaluza-Klein theory are combined together, gravity-induced corrections by a five-dimensional operator\(^6,16\) allow the chain

\[
\begin{align*}
\text{SO}(10) & \rightarrow G_{214} \rightarrow G_{213} \\
M_\text{U} & \rightarrow M_\text{c} \rightarrow M_\text{r}
\end{align*}
\]

\( (2) \)
with \( M_C \sim 10^5-10^{11} \) GeV, \( M_U \sim 10^{15-10^{17}} \) GeV, and \( \sin^2 \theta_W = 0.22-0.24 \). For \( M_C \sim 10^5-10^6 \) GeV, corresponding to observable rare-kaon decays, \( \nu_e \) mass is negligible; but \( \nu_x \) masses could be measured in the laboratory. The proton lifetime is significantly larger than the IMB limit depending upon the values of \( \sin^2 \theta_W \) and \( M_C \). For still larger values of \( M_C \sim 10^{-10} \) GeV, corresponding to undetectable rare-kaon decays, \( \nu^2 \) and \( \nu^1 \) masses could be measured in the laboratory. This paper is organized in the following manner. In Sec. II we summarize earlier contributions in SO(10) where gravity-induced corrections have been included and discuss their implications in the context of cosmological domain-wall problem and neutrino masses. In Sec. III our new results are reported. The paper is summarized in Sec. IV.

### II. Modifications in SO(10)

**WITH SU(2)_L × SU(2)_R × SU(4)_C AND SU(2)_L × SU(2)_R × U(1)_{\text{B-L}} × SU(3)_C**

**INTERMEDIATE SYMMETRIES**

In this section we summarize earlier contributions in SO(10) grand unified theory including gravity-induced corrections and discuss their implications in the context of domain walls in the early Universe and neutrino masses. Hill\(^{16}\) and Shafi and Wetterich\(^{6}\) (SW) introduced five-dimensional operators, involving gauge and Higgs fields, and scaled by the Planck mass \( (M_{Pl} = 10^{19} \text{ GeV}) \) (Ref. 16) or the compactification scale \( (M_C < M_{Pl}) \) (Ref. 6), with a view to obtain modified predictions on \( \tau_p \) and \( \sin^2 \theta_W \). Detailed analysis in the minimal GUT has been made by them and others.\(^{17}\) Besides SU(5), SW investigated\(^{6}\) possible changes in SO(10) predictions in the presence of Pati-Salam intermediate symmetry:

\[
SO(10) \rightarrow SU(2)_L \times SU(2)_R \times SU(4)_C \rightarrow G_{2,3} .
\]

In the absence of a \( d=5 \) operator, purely renormalizable interactions permit \( M_C \approx 10^{13} \) GeV, \( M_U \approx 10^{15} \) GeV for \( \sin^2 \theta_W \approx 0.23 \). When a nonrenormalizable term

\[
\mathcal{L}_{NR} = -\frac{\eta}{2M_G} \text{Tr}(F_{\mu\nu} \Phi_{54} F^{\mu\nu})
\]

is added, the following changes were noted:

\[
\sin^2 \theta_W = 0.22, \ \tau_p (p \rightarrow e^+ \nu \nu \nu) \approx (10-100) \tau_p (1 \text{MB}) ,
\]

where \( \tau_p (1 \text{MB}) \) is the IMB limit.\(^{15}\) In Eq. (4),

\[
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig [A_\mu, A_\nu] , \quad (A_\mu)_5^5 = A_\mu^5 (\lambda_i)^5_i ,
\]

\[
\text{Tr}(\lambda_i \lambda_j) = \frac{1}{2} \delta_{ij} ,
\]

and \( A_\mu^s \)'s are the gauge field matrices, \( \lambda_i \)'s are the SO(10) generators, \( \eta \) is an unknown parameter, \( M_G \) is the compactification scale, and \( \Phi_{54} \) is the scalar field \( 54 \subset \text{SO}(10) \). As in SU(5), SW noted that their modifications are consistent with \( M_G \approx 10^{-2} M_{Pl} \), compatible with the parameter \( \epsilon \approx 0.01-0.02 \) where

\[
\epsilon = \frac{1}{\sqrt{30}} \frac{\eta \Phi_0}{M_G}
\]

and \( \Phi_0 \) is related to the vacuum expectation value

\[
\langle \Phi_{54} \rangle = \frac{\Phi_0}{\sqrt{30}} \text{diag}(1, 1, 1, 1, 1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}) .
\]

Without introducing the idea of spontaneous compactification, as in Hill's approach,\(^{16}\) Rizzo\(^{18}\) investigated the possibility of low-mass purity restoration in SO(10),

\[
SO(10) \rightarrow SU(2)_L \times SU(2)_R \times SU(3)_C \rightarrow G_{2,3} .
\]

Using a \( d=5 \) operator scaled by the Planck mass:

\[
\mathcal{L}_{NR} = \frac{C}{2M_{Pl}} \text{Tr}(F_{\mu\nu} \Phi_{210} F^{\mu\nu}) .
\]

With a purely renormalizable Lagrangian, \( M_p \sim M_W \) in the chain requires \( \sin^2 \theta_W \approx 0.27-0.28 \), which are much larger than the accepted world average. Addition of (10) has been found to allow \( M_p \sim M_W \) with \( C \approx -0.5 \) and \( \sin^2 \theta_W \) within acceptable limits, for different combinations of Higgs triplets and doublets.

In the symmetry-breaking chain (3), the parity-violating scale \( M_p = M_C \approx 10^{13} \) GeV, and in (9) \( M_p = M_R \sim M_W \). Several years before, it was noted by Kibble, Lazarides, and Shafi,\(^{9}\) that in a model such as (3), where parity breaks at a scale lower than \( M_U \), domain walls, bounded by strings, are created in the early Universe. Such domain walls contribute to the mass density of the Universe, much larger than observed values, unless \( M_p = M_C \approx 10^{13} \) GeV. With gravity-induced corrections,\(^{6}\) or otherwise,\(^{9}\) \( M_p = M_C \approx 10^{13} \) GeV, such that problematic domain walls are likely to be absent in chain (3). But in (9), since \( M_p = M_R \sim M_W \ll 10^{11} \) GeV, the domain walls created are supposed to be extremely problematic.

Since the scalar representation \( 126 \subset \text{SO}(10) \) is used in the chains (3) and (9), to break the intermediate gauge symmetry spontaneously to the standard group, Majorana neutrino masses are generated by a seesaw mechanism satisfying the formula\(^{20}\)

\[
m_{\nu} = \frac{m_i}{M_{W_R}^2}, \quad i = e, \mu, \tau ,
\]

where \( m_i \) is the charged-lepton mass of \( i \)th generation and \( M_{W_R} \) is the \( W_R \) gauge-boson mass. In case (3), investigated by SW (Ref. 6) \( M_{W_R} = M_C \approx 10^{13} \) GeV, such that

\[
m_{\nu} \sim 10^{-11} \text{ eV}, \quad m_{\nu} \sim 10^{-6} \text{ eV}, \quad m_{\nu} \sim 10^{-4} \text{ eV} .
\]

Such neutrino masses are too small to be detected by laboratory experiments, although they might be compatible
with the solution to the solar-neutrino puzzle. In model (9), observable low-mass parity-restoration requires $M_{lf^2} \sim M_{ty}$, leading to rather larger values of Majorana neutrino masses:

$$m_\nu \sim 1 \text{ eV}, \quad m_\nu^* \sim 100 \text{ keV}, \quad m_\nu^* \sim 10 \text{ MeV}. \quad (13)$$

In this case, although the masses could be measured in the laboratory, $v_\nu^*$ and $v_\nu^*$ masses might be too large.

In the next section we study the possibility of SO(10) grand unification with single $G_{214}$ intermediate symmetry. Since SU(2)$_L$ breaks at the GUT scale, there is no domain-wall problem in this model. When the effects of the $d=5$ operator are included, the allowed solutions for $M_C$ are such that we obtain neutrino masses larger than (12) but smaller than (13) by 3–9 orders of magnitude.

III. GRAVITY-INDUCED CORRECTIONS
WITH SU(2)$_L \times U(1)_Y \times SU(4)_C$
INTERMEDIATE SYMMETRY

In this section we study the modifications caused by the $d=5$ operator in the predictions of the symmetry-breaking pattern (2), with $G_{214}$ intermediate symmetry. It is usually stated that the vacuum expectation value of the Higgs field $\chi^{(1,0,1)} \subset 45 \subset$ SO(10), where the transformation properties of $\chi$ are under $G_{214}$, might achieve the spontaneous symmetry breaking at the first stage of the chain (2). But, according to observations made by Yasue several years ago, both 54 and 45 are needed to break SO(10) $\rightarrow G_{214}$. As 45 is antisymmetric, it does not contribute to the gravity-induced corrections through the $rf = 5$ operator discussed in Sec. II; but the necessary presence of 54 is sufficient to induce significant modifications to the GUT predictions through the $d = 5$ operator of Eq. (4). Following the techniques of Refs. 6 and 16, and using Eqs. (4)-(8), the Lagrangian $\mathcal{L} = \mathcal{L}_R + \mathcal{L}_{NR}$, with $\mathcal{L}_R = -\frac{1}{4} Tr(F_{\mu
u} F^{\mu
u})$, is at first decomposed into kinetic energies of the SU(4)$_C$, SU(2)$_L$, and U(1)$_Y$ gauge fields:

$$\mathcal{L} = -\frac{1}{4} (1 + \epsilon) Tr(F^{\mu
u}_{\mu
u} F^{\mu
u}_{\mu\nu}) \left[ -\frac{1}{2} (1 - \frac{1}{3} \epsilon) Tr(F_{\mu\nu} F^{\mu\nu}) \right] \quad (14)$$

where the superscripts (C), (L), and (R) stand for the SU(4)$_C$, SU(2)$_L$, and U(1)$_Y$, respectively. Now rescaling the gauge fields changes their coupling constants as

$g_C^2(M_U) \rightarrow g_C^2(M_U)(1 + \epsilon)$, $g_L^2(M_U) \rightarrow g_L^2(M_U)(1 + \epsilon)$, $g_R^2(M_U) \rightarrow g_R^2(M_U)(1 - \frac{1}{3} \epsilon)$,

where $g_C(M_U)$, $g_L(M_U)$, and $g_R(M_U)$ denote the coupling constants of SU(4)$_C$, SU(2)$_L$, and U(1)$_Y$, respectively, without gravity-induced corrections. In order to achieve unification of strong, weak, and electromagnetic interactions for $\mu \geq M_U$, the GUT condition is imposed through the equations

$$g_C^2(M_U)(1 + \epsilon) = g_L^2(M_U)(1 + \epsilon) = g_R^2(M_U)(1 - \frac{1}{3} \epsilon) = g_0^2, \quad (15)$$

where $g_0$ is the bare-GUT coupling constant. With the boundary conditions modified as in (15), we solve one-loop renormalization group equations for the chain (2).

$$\frac{1}{\alpha_i(M_w)} = \frac{1}{\alpha_i(\mu)} + \frac{a_i}{2 \pi} \ln \frac{\mu}{M_w}, \quad i = Y, L, 3C \quad (16)$$

with

$$a_i(\mu) = \frac{\alpha_i(M_w)}{4\pi}, \quad a_Y = \frac{\alpha_Y}{4\pi}, \quad a_L = -\frac{\alpha_L}{4\pi}, \quad a_{3C} = -\frac{\alpha_{3C}}{4\pi}.$$

$$M_C \leq M_U:$$

$$\frac{1}{\alpha_i(M_C)} = \frac{1}{\alpha_i(\mu)} + \frac{a_i}{2 \pi} \ln \frac{\mu}{M_C}, \quad i = R, L, 4C \quad (17)$$

with $\alpha_R = \frac{\alpha_R}{2\pi}$, $\alpha_L = -\frac{\alpha_L}{2\pi}$ and $\alpha_{4C} = -\frac{\alpha_{4C}}{2\pi}$.

Note that we have been confined to the minimal fine-tuning condition and used three fermion generations, and the minimal number of Higgs scalars, needed for spontaneous symmetry breaking. The Higgs scalars used in the two different mass ranges are $M_{w^\prime} \leq M_C$, $\Phi(1,2,1)$ under SU(3)$_C \times SU(2)_L \times U(1)_Y$, $M_C \leq M_U$, $\Phi(2,\frac{1}{2},1)$ and $\Delta_{11,10}$ under SU(2)$_L \times U(1)_Y \times SU(4)_C$. These are present in 10 and 126 of SO(10). Using Eqs. (15)-(17) and the combinations, $e^{-2(M_U)} - \frac{1}{3} g_C^2(M_U)$, $e^{-2(M_U)} - \frac{1}{3} g_L^2(M_U)$, $e^{-2(M_U)} - \frac{1}{3} g_R^2(M_U)$, yields the following constraints on the unification mass, $\sin^2 \theta_W$, and the GUT coupling constant ($\alpha_G = g_0^2/4\pi$):

$$\frac{M_U}{M_w} = \frac{6\pi}{71-74\epsilon} \left[ \frac{1}{\alpha} - \frac{8}{3\alpha_s} + \frac{7}{3\alpha_s} \right], \quad (18)$$

$$\sin^2 \theta_W = \frac{1}{71-74\epsilon} \left[ \frac{2}{\alpha} + (19 - \frac{19}{3\alpha_s}) \frac{\alpha}{\alpha_s} - 53 \right], \quad (19)$$

$$\frac{1}{\alpha_G} = \frac{1}{71-74\epsilon} \left[ \frac{29}{3\alpha_s} + (19 - 19\epsilon) \frac{\alpha}{\alpha_s} \right], \quad (20)$$

In Eqs. (18)-(20), $\alpha_1 = g_1^2(M_w)/4\pi$ and $\alpha_\mu = g_\mu^2(M_w)/4\pi$.

For the chain (3), or (9), where formula (11) is applicable, SU(2)$_L$ breaks along with U(1)$_Y$ at the same scale such that $M_{w^*} = M_{Z^\prime}$. But, in the present case, SU(2)$_L$ breaks at $M_{w^*} = M_U$, keeping U(1)$_Y \times SU(4)_C$ unbroken down to $\mu = M_C$. It is at the second stage of the spontaneous symmetry breaking that the Majorana neutrino mass is generated when $\Delta_{11,10}$ under $G_{214}$ acquires vacuum expectation value. In this case $M_{z^\prime}$ = $M_C << M_{w^*} = M_U$, and the corresponding seesaw mechanism, worked out by Parida and Hazra, yields a different formula for the Majorana neutrino mass.

$$m_\nu \sim m_{\nu^*}^2 = \frac{m_{\nu^*}^2}{M_C}, \quad i = e, \mu, \tau. \quad (21)$$
Using Eqs. (18)-(20), \( \alpha_i = 0.1088 \) (\( \Lambda_{\text{MS}} = 160 \text{ MeV} \), where MS denotes the modified minimal subtraction scheme), \( \alpha^{-1} = 127.54 \), we compute numerically allowed regions for \( M_C = 10^7 \) and \( \epsilon \) within the available experimental constraints (Ref. 14) on \( M_U \) and \( \sin^2 \theta_W \). Note that \( \epsilon = 0 \) corresponds to the absence of gravity-induced effects and such solutions are presented in Table I. It is clear that, with a purely renormalizable Lagrangian, chain (2) is ruled out as it yields a maximum \( M_U = 3 \times 10^{14} \text{ GeV} \) and the corresponding proton lifetime \( \tau_p \sim 10^{23-22} \text{ yr} \) which is significantly less than the IMB limit.\(^{15}\)

Interesting solutions are obtained when \( \epsilon > 0 \) and are presented in Figs. 1–3, and Tables II and III. At first, Fig. 1 is plotted using Eq. (18), and Fig. 2 using Eq. (19). In Fig. 1 the horizontal lines are the IMB and the Planck limits on the unification mass. The projection of the line \( P \bar{Q} \) onto Fig. 2 has been denoted as the IMB limit in the latter. The horizontal lines in Fig. 2 represent the 2\( \sigma \) limits of the world average,\(^{14} \sin^2 \theta_W = 0.230 \pm 0.005 \). The projection of the Planck limit from Fig. 1 onto Fig. 2 does not provide any useful boundary for the allowed region. But, a much better limit exists\(^{19} \) from the experimentally observed bounds on the rare-kaon decay mode, \( K_L \to \mu \nu \), corresponding to \( M_C \geq 3 \times 10^3 \text{ GeV} \). Specifying the four sides of the quadrilateral in Fig. 2 in this fashion, the allowed solutions are shown by the shaded area.

The numerical values of \( M_C, \epsilon, M_U, \sin^2 \theta_W \), and \( \alpha_i^{-1} \) are shown in Table II for \( M_C = 10^3-10^6 \text{ GeV} \) and, in Table III for \( M_C = 10^7-10^{11} \text{ GeV} \).

We find, ignoring the uncertainty in \( \Lambda_{\text{MS}} \) and proton decay matrix elements,\(^{24} \) that the modifications caused by the \( d=5 \) operator permit \( 10^5 \leq M_C \leq 10^{11} \text{ GeV} \). For every \( M_C \), the parameter \( \epsilon \) and the unification mass \( M_U \) are allowed over a wider range depending upon the 2\( \sigma \) or 1\( \sigma \) limit of \( \sin^2 \theta_W \). The solutions with smaller (larger) values of \( \sin^2 \theta_W \) are associated with larger (smaller) values of \( M_U \) and \( \epsilon \). Our solutions include the values of \( M_C \sim 10^7-10^{10} \text{ GeV} \), which predict rare-kaon decays to be observable for any value of \( \sin^2 \theta_W \) in the range of \( 0.22-0.24 \). The highest value of \( M_U \sim 3 \times 10^{17} \text{ GeV} \) is possible for \( M_C = 10^7 \text{ GeV} \) and \( \sin^2 \theta_W = 0.22 \). This has been shown by the point \( R \) in Fig. 1 which has been obtained by the projection of the corresponding point in Fig. 2.

As \( M_C \) increases, the unification mass, for a fixed value of \( \sin^2 \theta_W \), and the proton lifetime for the \( p \to e^+ \pi^0 \) mode decreases. This has been shown in Fig. 3 for the 1\( \sigma \) and 2\( \sigma \) boundaries, and the central value of \( \sin^2 \theta_W = 0.230 \). For \( M_C > 10^8 \text{ GeV} \), the allowed range of \( \tau_p \) also decreases being restricted by the IMB limit form below. The IMB limit is found to be saturated nearly at \( M_C \sim 10^{12} (10^{13}) \text{ GeV} \) if the value of \( \sin^2 \theta_W \) is allowed to be 0.225 (0.220). Including uncertainty in \( \tau_p \) by a factor 10\(^{\pm 2} \), arising out of uncertainties in the proton decay matrix element and the QCD parameter,\(^{24} \) we find that the maximum allowed value of \( M_C \) can be increased by 1 order, (i.e., \( 10^{11}-10^{12} \text{ GeV} \) unless \( \sin^2 \theta_W \) is allowed to be significantly lower than 0.220.

Using Eq. (21) and the allowed range \( M_C \sim 10^7-10^{12} \text{ GeV} \), we now calculate neutrino masses as shown in Fig. 4. They are found to vary over a wider range:

\[
\begin{align*}
{m}_{\nu_e} & \sim \left( 2 \times 10^{-10}-2 \times 10^{-5} \right) \text{ eV} , \\
{m}_{\nu_\mu} & \sim \left( 10^{-5}-100 \right) \text{ eV} , \\
{m}_{\nu_\tau} & \sim \left( 3 \times 10^{-2} \text{ eV}-30 \text{ keV} \right) ,
\end{align*}
\]

where the lower (upper) limit corresponds to \( M_C = 10^{12} \) (\( 10^{13} \) \text{ GeV} ). The observable signatures of SU(4)_C breaking by rare-kaon decay processes predict

\[
\begin{align*}
{m}_{\nu_e} & \sim \left( 2 \times 10^{-4}-2 \times 10^{-3} \right) \text{ eV} , \\
{m}_{\nu_\mu} & \sim \left( 11-110 \right) \text{ eV} , \\
{m}_{\nu_\tau} & \sim 3-30 \text{ keV} .
\end{align*}
\]

### TABLE I. One-loop solutions for SO(10) with single intermediate symmetry, SU(2)_L \times U(1)_Y \times SU(4)_C, in the absence of gravity-induced corrections.

<table>
<thead>
<tr>
<th>( M_C ) (GeV)</th>
<th>( M_U ) (GeV)</th>
<th>( \sin^2 \theta_W )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^3 )</td>
<td>( 9.7 \times 10^4 )</td>
<td>0.273</td>
</tr>
<tr>
<td>( 10^4 )</td>
<td>( 1.2 \times 10^5 )</td>
<td>0.260</td>
</tr>
<tr>
<td>( 10^5 )</td>
<td>( 1.5 \times 10^6 )</td>
<td>0.247</td>
</tr>
<tr>
<td>( 10^{11} )</td>
<td>( 2.0 \times 10^{10} )</td>
<td>0.233</td>
</tr>
<tr>
<td>( 10^{13} )</td>
<td>( 2.57 \times 10^{14} )</td>
<td>0.220</td>
</tr>
<tr>
<td>( 10^{14} )</td>
<td>( 2.95 \times 10^{14} )</td>
<td>0.214</td>
</tr>
</tbody>
</table>
Out of these, $\nu_{\mu}$ and $\nu_{\tau}$ masses are measurable by laboratory experiments. The masses are about 6–7 orders of magnitude larger than those obtained with single Pati-Salam intermediate symmetry, but they are 3–4 orders of magnitude smaller than the models having low-mass $W$ and $Z$ gauge bosons. In theories exploiting Kaluza-Klein-type unification with gravity, nonrenormalizable terms involving higher-dimensional ($d > 4$) operators, scaled by suitable powers of the compactification scale, are usually present. We have investigated the modifications caused by a $d = 5$ operator on the SO(10) GUT with single $G_{14}$ intermediate symmetry. In the absence of gravity-induced corrections, such a model is ruled out as it predicts $\tau_p$ significantly below the IMB limit. Including gravity-induced corrections, the $SU(4)$-breaking scale is found to be permitted over a wide range, $M_C \sim 10^3 - 10^5$ GeV, leading to predictions on neutrino masses:

$$m_{\nu_{\mu}} \sim (10^{-10} - 10^{-9}) \text{ eV}, \quad m_{\nu_{\tau}} \sim (10^{-5} - 100) \text{ eV},$$

$$m_{\nu_{e}} \sim 10^{-2} \text{ eV} - 30 \text{ keV}.$$
TABLE III Same as Table II but for larger values of $M_C$

<table>
<thead>
<tr>
<th>$M_C$ (GeV)</th>
<th>$\epsilon$</th>
<th>$M_U$ (GeV)</th>
<th>$\sin^2\theta_W$</th>
<th>$\alpha_\gamma$</th>
<th>$\tau_\gamma$ (yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^7$</td>
<td>0.07</td>
<td>$1.3 \times 10^{16}$</td>
<td>0.228</td>
<td>51.04</td>
<td>$4.2 \times 10^{16}$</td>
</tr>
<tr>
<td></td>
<td>0.06</td>
<td>$6.7 \times 10^{15}$</td>
<td>0.238</td>
<td>50.48</td>
<td>$2.9 \times 10^{15}$</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>$3.3 \times 10^{15}$</td>
<td>0.238</td>
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<td>45.63</td>
<td>$2.0 \times 10^{10}$</td>
</tr>
</tbody>
</table>

Although $m_{\nu}$ is too small to be detected, $m_{\nu_R}$ and $m_{\nu_L}$ could be measured by laboratory experiments depending upon $M_C$.

For the first time, we have obtained interesting SO(10) predictions, with single intermediate symmetry, for the observable SU(4)$_C$ breaking by rare-kaon decay modes at low energies with $M_C \sim 10^{11}-10^{12}$ GeV, and any value of $\sin^2\theta_W$ in the range $0.22-0.24$. For such lower values of $M_C$, the predicted $\nu$ masses are 6-7 orders of magnitude larger than the SO(10) prediction with Pati-Salam intermediate symmetry, but 3-4 orders smaller than models with low-mass right-handed gauge bosons. For larger values of $M_C > 10^6$ GeV, the allowed range of $\tau_\gamma$ decreases with increasing $M_C$, and the IMB limit is saturated when $M_C \sim 10^{11}-10^{12}$ GeV. The order of magnitude of the compactification scale, estimated in this model, is found to be in the range $10^{11}-10^{12}$ GeV, unless the parameter in the nonrenormalizable term has the value $|\gamma| = 10$, or larger.

With the SU(2)$_L \times U(1)_X \times SU(4)_C$ gauge symmetry, existing at a scale $\mu \geq M_C = M_{\nu_R} \geq 10^6$ GeV, and $M_{\nu_R} = M_U \sim 10^{11}-10^{12}$ GeV, there is negligible contribution to the $\nu + A$ structure of charged and neutral currents, in this model. Similarly, the $K_L - K_S$ mass difference and other CP-violating parameters have, essentially, the same prediction as the standard model. At low energies, this model does not seem to predict any other detectable signatures, except rare kaon decays and neutrino masses.

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1. Th. Kaluza, Sitzungsber Preuss Akad Wiss Math Phys K1, 996 (1921), O Klein, Z Phys 37, 895 (1925)
SPONTANEOUS COMPACTIFICATION EFFECTS IN SO(10) WITH LOW-MASS W^R-GAUGE BOSONS WITHOUT OBSERVABLE PARITY RESTORATION

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Received 19 August 1989, revised manuscript received 23 October 1989

In contrast to the conclusion for an SO(10) model including gravity induced corrections through a five-dimensional operator we note that low-mass right-handed gauge bosons are ruled out as the parity restoring gauge group SU(2)_L x SU(2)_R x U(1)_B-L x SU(3)_C(G_2213) is not allowed at low-mass scales. Using the mechanism of decoupling parity and SU(2)_R breakings the low-mass right-handed gauge bosons (M_R ~ 100 GeV - 10 TeV) without observable parity restoration are found to be permitted when the five-dimensional operator is scaled by the compactification mass.

Experiments being planned with ultrahigh-energy accelerators in the TeV ranges are expected to reveal new physics beyond the standard model that might be associated with new gauge symmetries. One of the promising gauge theories beyond SU(3)_C x SU(2)_L x U(1)_Y (=G_2213) is the left-right model based upon the gauge symmetry SU(2)_L x SU(2)_R x U(1)_B-L x SU(3)_C (=G_2213) which has been the focus of considerable attention during the past years. Ever since the proposal of left-right symmetry [1,2], many attempts have been made to obtain a low right-handed scale (M_R) associated with the spontaneous symmetry breaking (SSB) of G_2213, including the left-right discrete symmetry (P= parity when g_{2L} = g_{2R}), or excluding it (g_{2L} \neq g_{2R}). If the right-handed scale associated with the SSB of the G_2213 model (G_2213 with g_{2L} = g_{2R}) is low, besides observing the V + A structure of weak-charged and neutral currents by low-energy experiments and detecting the right-handed (W^R, Z^R) gauge bosons by the high-energy accelerators, it might also be possible to observe low-mass parity restoration. In addition such a model provides a natural mechanism for neutrino masses and weak CP violation. But when embedded in grand unified theories (GUTs) such models are known to be in serious conflict with cosmology, the well known problems being the presence of undesirable domain walls [3], and inadequate baryon number generation [4].

In the conventional approach emphasizing the G_2213 model, P and SU(2)_R are broken at the same scale. When such a model is embedded in a GUT like SO(10) [5], the right-handed scale is found to be large (M_R > 10^{12} GeV) for sin^2 \theta_w = 0.230 \pm 0.005. This rules out verification of all possible signatures by low-energy experiments, or high-energy accelerators.

When SO(10) breaks with two intermediate symmetries [6],

SO(10) \rightarrow G_1 \rightarrow G_2 \rightarrow \ldots \rightarrow G_3

a low-mass right-handed neutral gauge boson is known to be predicted with M_R \sim 500 GeV whether G = G_2213 or G_2213P. In such cases the predicted value of the W^R-gauge boson mass is large, M_W^R > 10^{10} GeV [6]. It has been noted very recently that, if the superheavy components of Higgs representations used in the SSB of the SO(10) model are permitted to have certain nondegenerate masses different from M_H by one order, the resulting uncertainty in the predicted value of sin^2 \theta_w might permit low-mass right-handed W^R gauge bosons in (1) when G = G_2213 [7]. How-

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ever we are interested in the model calculations exploiting the extended survival hypothesis under the natural assumptions that all superheavy masses are the same as $M_U$.

It has been shown by Rizzo [8] that additional gravitational corrections due to a five-dimensional operator in the nonrenormalizable lagrangian,

$$\alpha_{NR} = \frac{C}{M_C} \Gamma(F_\mu \Phi_{(120)}) F^{\mu\nu}$$

(2)

with $M_C = 2 \times 10^{19}$ GeV permits low-mass right-handed gauge bosons ($M_{R} \sim 100$ GeV) in $SO(10)$ with observable parity restoration. In eq (2) $\Phi_{(120)}$ is the four-index antisymmetric Higgs-scalar representation $210 \in SO(10)$, $F_{\mu\nu} = \partial_\mu H_{\nu} - \partial_\nu H_{\mu} + ig(\sigma^{\mu\nu}) H$, $H_{\mu} = \frac{1}{2} \sum_{i,j=1}^{10} \sigma^{i\mu} H_{\nu}^j$, $C = -\frac{i}{2} \eta$.

(3)

where $\frac{1}{2} \sigma^{i\mu} H_{\nu}^j$, with $i,j = 1, 2, 6, 7, 8, 9, 10$ denote the 45 generators (gauge bosons) [9] of $SO(10)$ and the constant $C$ has been reparametrized. The fact that the addition of such five-dimensional operators could drastically modify the GUT predictions was first proposed by Hill [11] and Shafi and Wetterich [12].

As a distinguishing feature of the two suggestions note that the higher-dimensional operator was emphasized to be originating as an effect of quantum gravity in ref [11] while this was taken to be occurring naturally in some effective lagrangian as a result of compactification of extra dimensions with the compactification scale $M_C \sim 10^{17}$-10$^{18}$ GeV [12]. We follow the convention in which $i,j = 1, 2, 6, 7, 8, 9, 10$ are the $SO(6)$ ($SO(4)$) indices. It is the purpose of the present paper to demonstrate that (i) low-mass right-handed gauge bosons in the $G_{2213F}$ model as envisaged in ref [8] are ruled out, (ii) they are permitted in the $SO(10)$ model with decoupled $P$- and $SU(2)_R$-breakings proposed recently by Chang, Mohapatra and one of us (M K P ) [13], especially when eq (2) emerges as a result of compactification of extra dimensions with $M_C \leq M_W$ [12]. When all superheavy-Higgs scalar masses are degenerate and the same as the unification mass, only through such a model the spontaneous compactification effects on $SO(10)$ predict the possibility of observing $\nu + \tau$ structure of weak currents and the right-handed $W^\pm$ gauge bosons, but no observable parity restoration.

Before the works of ref [13], a number of authors used the vacuum expectation value (VEV) of the neutral component $\chi_0$ of $\chi_0 = 1, 1, 15 \in 45 \in SO(10)$ to break

$$SO(10) \rightarrow \langle 210 \rangle \neq 0 \quad G_{2213F}$$

where $(1, 1, 15)$ denotes the transformation properties of $\gamma$ under $SU(2)_L \times SU(2)_R \times SU(4)_C$ and it was found in ref [13] that, under the constraint of minimal finetuning, it is not possible to keep parity ($P$) unbroken below the GUT scale when the VEV of $\chi_0$ is used. It was noted that an element of $SO(10)$, called $D = \delta_{210}g_0$, acts like $P$ when all couplings in the $SO(10)$-invariant lagrangian are real. The neutral components $\chi_0$ and $\eta_0 (1, 1, 1) \in 210 \in SO(10)$ are odd under $D$ and, hence, under $P$. But the corresponding components of $\xi_0 (1, 1, 15) \in 210$ and $\eta_0 (1, 1, 1) \in 54$ are even. Thus it is possible to descend down to $G_{2213F}$ ($G_{2224}$) with $P$ unbroken through the Higgs representation $210$ but not through $45$ (210). Similarly when a VEV is assigned along directions odd under $P$, it is possible to break $P$ at the GUT scale without breaking $SU(2)_R$. In such cases, $P$- and $SU(2)_R$-breakings are decoupled, and it is possible to descend down to $G_{22134}$ ($G_{2224}$) with $P$ broken at the GUT scale through the Higgs representation $45$ (210). In this paper while reexamining the gravity-induced effects in the $SO(10)$ scenario considered by Rizzo [8],

$$SO(10) \rightarrow \langle 210 \rangle \neq 0 \quad G_{2213F} \quad \langle 210 \rangle \neq 0 \quad G_{2224}$$

we discuss yet another possibility of decoupling $P$- and $SU(2)_R$-breakings through $210$ that predicts the existence of low-mass $W^\pm$ gauge bosons when compactification effects are included. When $210$ acquires the VEV as

$$\langle \Phi_{(120)} \rangle = \frac{\theta_0}{4\sqrt{6}} \left( -\Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 + \Gamma_1 \Gamma_2 \Gamma_3 \Gamma_6 + \Gamma_3 \Gamma_4 \Gamma_5 \Gamma_6 \right)$$

(4)

with $M_R \sim M_W$, we discuss yet another possibility of decoupling $P$- and $SU(2)_R$-breakings through $210$ that predicts the existence of low-mass $W^\pm$ gauge bosons when compactification effects are included.

In this notation [9] every gauge boson appears in more than one element of the $16 \times 16$ matrix $W_\mu$, and the kinetic energy term is $\frac{1}{2} \Gamma(F_\mu F^{\mu\nu})$. Note the presence of $\frac{1}{2} \Gamma$ instead of $\frac{1}{2}$ For details in $SO(2N)$ models see ref [10].

\[46\]
SO(10) → G_{2213P}

which has been used in the analysis of ref [8] by allowing G_{2213P} to survive down to the W_L-scale. But since \( \eta(1,1,1) \leq 210 \) is odd under \( P \), when VEV is assigned such that

\[
\langle \Phi_{(210)} \rangle = \frac{\phi_0}{\sqrt{2}} (1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}) + \frac{1}{2} (1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3})
\]

\( P \) breaks at the GUT scale. Thus, while eq (5) gives the parity invariant vacuum with \( G_{2213P} \) gauge symmetry, addition of (5) and (6) with

\[
\langle \Phi_{(210)} \rangle = \frac{\phi_0}{\sqrt{2}} (1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}) + \frac{1}{2} (1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3})
\]

yields the parity-violating vacuum having \( G_{2213P} \) gauge symmetry. In eqs (5)-(7) \( \langle 0 \rangle' \langle 0 \rangle' = \langle 0 \rangle' \langle 0 \rangle' = \langle 0 \rangle' \langle 0 \rangle' \), where \( \langle 0 \rangle' \) being antisymmetric in \( y_A \). For a symmetry-breaking pattern of the type

\[
SU(3)_c \times U(1)_L
\]

the renormalization group equations (RGEs) for the gauge coupling constants \( a_j(M) = g_j^2(M)/4\pi \) in the various ranges of the mass scales are

\[
M_L = \mu < M_R \quad \frac{1}{a_j(M)} = \frac{1}{a_j(M_L)} + \frac{a_j}{2\pi} \ln \frac{M_R}{M_L} \quad \tau = L, 2L, 3C \quad (9)
\]

\[
M_L < \mu < M_U \quad \frac{1}{a_j(M)} = \frac{1}{a_j(M_U)} + \frac{a_j}{2\pi} \ln \frac{M_L}{M_U} \quad \tau = L, 2L, 2R, 3C \quad (10)
\]

Introduction of the five-dimensional operator in eq (2) is to modify the boundary conditions in the general form,

\[
\alpha_{BL}(M_U)(1 + \epsilon_{BL}) = \alpha_{2L}(M_U)(1 + \epsilon_{2L}) = \alpha_{2R}(M_U)(1 + \epsilon_{2R}) = \alpha_{SC}(M_U)(1 + \epsilon_{SC}) = \alpha_G \quad (11)
\]

where \( \alpha_G = g_3^2/4\pi \), \( g_3 \) being the bare coupling of the GUT lagrangian. Using suitable combinations of the coupling constants, the matching relation 1/

\[
\alpha_Y(M_R) = \frac{\alpha_{2R}(M_R)}{\alpha_{2L}(M_R)} \quad \text{and the boundary conditions (11), the following equations for } \ln \frac{M_U}{M_L}, \text{ and the coupling constant } \alpha_G \text{ are obtained in a straightforward manner.}
\]

\[
\ln \frac{M_U}{M_L} = 2\pi \left( \frac{\frac{3}{2} + \epsilon_{2L} + \epsilon_{2R} + \frac{1}{2} \epsilon_{BL}}{\alpha_{2L}(M_U)} - \frac{1 + \epsilon_{SC}}{\alpha_{2L}(M_U)} a_{SC} \right)
\]

\[
\sin^2 \theta_w = \frac{1}{2\pi} \left( (1 + \epsilon_{2L}) a_{2L} - \frac{a_{2L} a_{SC}}{4} (1 + \epsilon_{SC}) \right)
\]

\[
\frac{1}{\alpha_G} = \frac{1}{D} \left( a_{2L} + a_{2R} + \frac{1}{2} a_{BL} \right)
\]

where

\[
D = a_{2L}(\frac{3}{2} + \epsilon_{2L} + \epsilon_{2R} + \frac{1}{2} \epsilon_{BL}) - \frac{1 + \epsilon_{SC}}{2\pi} (a_{2L} + a_{2R} + \frac{1}{2} a_{BL}) \quad (12)
\]

The case considered by Rizzo [8] corresponds to \( M_R = M_L \) with \( a_{2L} = a_{2R} = \frac{1}{2} a_{BL} = \frac{1}{2} \eta_3 + \frac{1}{2} (D + T) \), where \( \eta_3 \) is the fermion generation number and \( D + T \) is the number of light Higgs doublets (triplets). In ref [8]
the following values of the coefficients occurring in
eqs (12)-(15) have been derived

\[ \epsilon_{2L} = \epsilon_{2R} = \frac{154}{\sqrt{2}}, \quad \epsilon_{BL} = \frac{A}{\sqrt{2}}, \]

\[ \epsilon_{SC} = -\frac{204}{\sqrt{2}}, \quad A = \frac{C_{mu}}{(64\pi\alpha_G)^{1/2}M_{pl}^2}, \]  

(16)
such that all the gauge coupling constants at the
boundary \( \mu = M_u \) get modified according to eq (11)
Noting that

\[ -\Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 + \Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 + \Gamma_3 \Gamma_4 \Gamma_5 \Gamma_6 \]

\[ = \text{diag}(-1, -1, -1, 3, -1, -1, 3, -1, -1, 1, 3, -1, -1, 3), \]  

(17)
we use the VEV given by (5) in eq (2) After a suitable rescaling of the gauge fields in the usual fashion
we find that (2) gets absorbed in the kinetic energies
terms for SU(3)_c, SU(2)_L, SU(2)_R, and U(1)_N-L

gauge fields resulting in

\[ \epsilon_{2L} = \epsilon_{2R} = 0, \quad \frac{1}{\sqrt{2}} \epsilon_{BL} = -\epsilon_{SC} = \epsilon, \]

(18)
Thus we find that the modification to the boundary condition occurs only for the coupling constants
\( \alpha_{2C}(M_u), \) and \( \alpha_{BL}(M_u), \) but not for \( \alpha_{2L}(M_u) \) or \( \alpha_{SN}(M_u). \) That eq (2) does not contribute to the modifications of SU(2)_L-R kinetic energies can be further checked by using \( i, j = 7, 8, 9, 10 \) in eq (3) and verifying that \( \text{Tr} \left( F^{\mu L \nu R} \right) \left( \Phi^{(10)} \right)^{\mu L \nu R} = 0 \)
As we find the boundary condition (18) to be substantially different from that used in ref [8],
we computed numerical solutions to eqs (12)-(15) under the condition (18), and with \( M_R = M_w, \)
\( M_C = 2M_{pl} = 2 \times 10^{19} \) GeV, \( \alpha_{2C}(M_w) = 0.1, \) and \( \alpha_{SN}(M_u) = 128 \) as has been used in ref [8]. In contrast to ref [8] we note that, in all cases of \( D = T, \)
\( \sin^2 \theta_w \) given by eq (13) is independent of \( \epsilon. \) This happens due to the fact that the \( \epsilon \)-dependence occurring in the first factor \( (D = 1) \) in eq (13) is exactly cancelled by the same dependence in the numerator of the second factor. For other combinations of \( D \neq T \) the \( \epsilon \)-dependence of \( \sin^2 \theta_w \) is weak. Our numerical solutions for \( D = T = 1, 2, D = 2, T = 1, \) and \( D = 1, T = 2 \) are shown in table 1. The lowest value of \( \sin^2 \theta_w \) is
obtained for the case \( D = 1, T = 2 \) and is found to be
0.266 with \( \epsilon = -0.208 \) for which \( M_U = 2M_P. \) For all other values of \( M_U < 2M_P, \)
\( \sin^2 \theta_w > 0.266 \) when we attempted to decrease \( \sin^2 \theta_w \) further with \( \epsilon < -0.208, \)
\( M_U > 2M_P \) making the solutions unacceptable. This, the possibility of low-mass right-handed
gauge bosons with \( V_R \sim M_w \) accompanied by observable
low-mass parity restoration through \( G_{213}, \) intermediate symmetry needs \( \sin^2 \theta_w \approx 0.266 \) which is
far too large as compared to the present world average \( \sin^2 \theta_w \approx 0.230 \pm 0.005 \)
To find the lowest allowed value of \( M_R \) under the boundary conditions (11) and (18) we allowed
\( M_R \gg M_w \) in eqs (12)-(14) Some of our solutions with the same input parameters are shown in table 2
where the presence of \( T > 1 \) has been taken between the scales \( M_R \sim M_U \) In the case \( D = T = 1 \) \( (D = 1, \)
\( T = 2) \) whenever we attempted to decrease \( \sin^2 \theta_w \) by decreasing \( \epsilon < -0.12 \) \((\epsilon < -0.2), \)
\( M_U > 2M_P \) which ruled out the possibility of \( M_R < M_P \) below \( 10^8 \) GeV
We find that \( \sin^2 \theta_w < 0.235 \) constrains \( M_R > 10^8 \) GeV
Thus, the low-mass right-handed gauge bosons and observable parity restoration are ruled out in this
model Even with such high values of \( M_R < 10^{12} \) GeV
the model gives rise to stable domain walls [3] and negligible baryon asymmetry of the universe [4] unaccepta­
table to the modern big-bang cosmology
In contrast to the conventional models the new
SO(10) model [13] with separate \( P = \) and SU(2)_R-
breaking scales does not suffer from the domain wall
problem In such a model since the baryon generation
in the early universe is related to the \( P \)-breaking
scale \( (M_P) \) [14], it is possible to generate the ob­
served baryon asymmetry of the universe with
\( M_R \approx M_P < M_U \) But the renormalization group con­
straints up to two-loop level have been found to per­
mit \( M_R > 10^9 \) GeV for \( \sin^2 \theta_w < 0.235 \) with \( G_{213} \) as
the single intermediate symmetry To investigate the impact of the gravity induced corrections on this
model we used the VEV given by eq (7) and computed the \( \epsilon \)-parameters contributing to the modifi­
cation of the boundary conditions,

\[ \epsilon_{2L} = -\epsilon_{2R} = -\epsilon_{SC} = \frac{1}{2} \epsilon_{BL} = \epsilon, \]

(19)
As explained earlier [13] within the constraint of
Table 1
Prediction of the SO(10) model with parity-restoring left-right gauge group at low-mass scales, \( M_C = 2 \times 10^{18} \) GeV, and \( M_R \sim 100 \) GeV

<table>
<thead>
<tr>
<th>Number of ( D ) and ( T )</th>
<th>( \epsilon )</th>
<th>( M_D ) (GeV)</th>
<th>( M_U ) (GeV)</th>
<th>( \sin^2 \theta_w )</th>
<th>( \alpha_D )</th>
<th>( C = -\frac{1}{2} \eta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D = T = 1 )</td>
<td>-0.05</td>
<td>( 2.6 \times 10^9 )</td>
<td>0.274</td>
<td>49.0</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.08</td>
<td>( 2.0 \times 10^9 )</td>
<td>0.274</td>
<td>49.8</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>( D = 2, T = 1 )</td>
<td>-0.08</td>
<td>( 4.4 \times 10^9 )</td>
<td>0.282</td>
<td>48.2</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.10</td>
<td>( 2.0 \times 10^9 )</td>
<td>0.282</td>
<td>48.7</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>( D = 1, T = 2 )</td>
<td>-0.18</td>
<td>( 4.4 \times 10^9 )</td>
<td>0.266</td>
<td>44.4</td>
<td>0.31</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.208</td>
<td>( 2.0 \times 10^9 )</td>
<td>0.266</td>
<td>44.0</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>( D = T = 2 )</td>
<td>-0.20</td>
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<td>43.0</td>
<td>0.51</td>
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</tr>
<tr>
<td></td>
<td>-0.238</td>
<td>( 2.0 \times 10^9 )</td>
<td>0.274</td>
<td>43.4</td>
<td>0.09</td>
<td></td>
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</tbody>
</table>

Table 2
Prediction of the SO(10) model with the parity-restoring gauge group as an intermediate symmetry and \( M_C = 2 \times 10^{18} \) GeV

<table>
<thead>
<tr>
<th>Number of ( D ) and ( T )</th>
<th>( \epsilon )</th>
<th>( M_D ) (GeV)</th>
<th>( M_U ) (GeV)</th>
<th>( \sin^2 \theta_w )</th>
<th>( \alpha_D )</th>
<th>( C = -\frac{1}{2} \eta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D = T = 1 )</td>
<td>-0.12</td>
<td>( \times 10^9 )</td>
<td>4.6 \times 10^9</td>
<td>0.233</td>
<td>46.5</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>-0.12</td>
<td>( 7.9 \times 10^9 )</td>
<td>0.238</td>
<td>47.1</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>( D = 1, T = 2 )</td>
<td>-0.2</td>
<td>( 9.3 \times 10^9 )</td>
<td>9.7 \times 10^9</td>
<td>0.238</td>
<td>44.1</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>-0.2</td>
<td>( 10^9 )</td>
<td>9.7 \times 10^9</td>
<td>0.238</td>
<td>44.1</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Minimal finetuning of parameters, the left-handed triplet is made superheavy with its mass \( \sim M_D \) and does not contribute to the RGEs of the coupling constants. At first, confining to the minimal number of Higgs particles needed for the SSB of the gauge symmetries, \( D = T = 1 \) corresponds to including the following Higgs contributions in the two different mass ranges: \( D = T = 2 \). In the minimal case the coefficients occurring in eqs (12)-(14) are

\[
\begin{align*}
\alpha_{2L} &= -\frac{3}{8}, \\
\alpha_Y &= \frac{1}{10}, \\
\alpha_{3C} &= -\frac{7}{3}.
\end{align*}
\]

(20)

Modifying the boundary conditions as in eqs (11) and (19) we solved eqs (12)-(14) to obtain values of \( M_U, \sin^2 \theta_w \), and \( \alpha_D \) for certain values of \( M_R \) as a function of \( \epsilon \). Some of our solutions are presented in table 3 where the entry in the last column is the parameter \( C = -\frac{1}{2} \eta \) that has been computed using formula (18) and different values for the compactification scale \( (M_C) \). It is clear that low-mass right-handed gauge bosons with \( M_R \sim 100 \) GeV–10 TeV are permitted with \( |C| |\eta| \sim 0.2–3 \) provided the compactification scale \( M_C \) is in the range of \( \sim 10^{17} \)–\( 10^{18} \) GeV [12]. It may be noted that the compactification of the fifth dimension on a circle in the Kaluza-Klein model corresponds to \( \frac{M_C}{\sqrt{\pi}} \sim 10^{16} \) GeV = 1.6 \times 10^{19} \) GeV for which the value of the parameter \( C \) has also been calculated. It has been shown by Freund [15] that in Kaluza-Klein theories \( M_C \) could be easily made two orders of magnitude smaller than \( M_R \). If we use \( M_C = M_R \), the parameter \( C \) increases by a factor 10–100 making it unacceptably large for the minimal choice of Higgs representations \( (D = T = 1) \). Thus, low-mass \( W_R \) bosons are favoured in the SO(10) model which might be appearing as an effective gauge theory in four dimensions resulting from compactification of extra dimensions in some basic higher-dimensional theory [16].

We have carried out a similar analysis in the case \( D = 1 \) and \( T = 2 \) corresponding to the nonminimal
Table 8
Prediction of the SO(10) model with parity-violating left-right gauge group at lower mass scales ($M_R \sim 10^{-10} \text{GeV}$)

<table>
<thead>
<tr>
<th>Number of D and $T$</th>
<th>$\epsilon$</th>
<th>$M_{R}$ (GeV)</th>
<th>$M_{L}$ (GeV)</th>
<th>$\sin^2 \theta_L$</th>
<th>$\alpha_D$</th>
<th>$V_{R}$ (GeV)</th>
<th>$C = \frac{1}{\sqrt{\lambda}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D = T = 1$</td>
<td>0.05</td>
<td>$10^6$</td>
<td>$1.6 \times 10^{16}$</td>
<td>0.234</td>
<td>48.3</td>
<td>$1.6 \times 10^{16}$</td>
<td>$-2.1$</td>
</tr>
<tr>
<td></td>
<td>0.06</td>
<td>$10^6$</td>
<td>$10^{16}$</td>
<td>$0.232$</td>
<td>48.1</td>
<td>$10^{16}$</td>
<td>$-0.28$</td>
</tr>
<tr>
<td></td>
<td>0.08</td>
<td>$10^7$</td>
<td>$4.4 \times 10^{13}$</td>
<td>0.229</td>
<td>48.2</td>
<td>$10^{17}$</td>
<td>$-0.76$</td>
</tr>
<tr>
<td></td>
<td>0.08</td>
<td>$10^7$</td>
<td>$8.2 \times 10^{13}$</td>
<td>0.233</td>
<td>49.0</td>
<td>$10^{17}$</td>
<td>$-0.40$</td>
</tr>
<tr>
<td></td>
<td>0.07</td>
<td>$10^7$</td>
<td>$1.6 \times 10^{14}$</td>
<td>0.235</td>
<td>49.2</td>
<td>$1.6 \times 10^{14}$</td>
<td>$-2.8$</td>
</tr>
<tr>
<td>$D = 1 ~ T = 2$</td>
<td>0.01</td>
<td>$10^7$</td>
<td>$2 \times 10^{14}$</td>
<td>0.236</td>
<td>46.8</td>
<td>$1.6 \times 10^{14}$</td>
<td>$-0.34$</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>$10^7$</td>
<td>$1.7 \times 10^{14}$</td>
<td>0.236</td>
<td>46.8</td>
<td>$1.6 \times 10^{14}$</td>
<td>$-0.81$</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>$10^7$</td>
<td>$7.4 \times 10^{13}$</td>
<td>0.232</td>
<td>46.9</td>
<td>$10^{17}$</td>
<td>$-0.23$</td>
</tr>
</tbody>
</table>

choice of Higgs representations, the results are also reported in Table 3. It is clear that low-mass $W_R$-bosons are favoured with $M_{R} \sim 10^{12}-10^{18}$ GeV when $|C| = 0.2-0.8$. In this case, however, $M_{L} = V_{R}$ can be permitted provided $C$ is allowed to be in the range 3-10.

Although the renormalization group constraints including spontaneous compactification effects are found to permit a low scale like $M_{R} \sim 100-1000$ GeV, there are several phenomenological constraints on the $W_R$-mass [17]. The most stringent constraint from the $K_{L} - K_{S}$ mass difference sets a lower bound on $M_{R}$ close to 25 TeV in manifestly left-right symmetric models where the two gauge coupling constants, and the fermion mixing angles in the left- and the right-handed sectors are equal ($g_L = g_R$, $\theta_L = \theta_R$), but this lower bound can be decreased substantially in the asymmetric models ($g_L \neq g_R$, $\theta_L \neq \theta_R$) [18]. One of the promising low-energy processes investigated extensively and expected to provide signatures of $V+A$ charged currents is the neutrinoless double $\beta$-decay. With the Majorana neutrino mass $m_{\nu} \sim 1-2$ eV available experimental data are consistent with a low $W_R$-mass $\sim 3-4$ TeV [19]. If $M_{R} \sim 10-20$ TeV the electric dipole moment of the neutron, a manifestation of $CP$ and $P$ violations, has been predicted to be close to the experimental limit, $d_n \lesssim 10^{-26}$ e cm [17,20]. The low $M_{R}$-scale also allows the neutral and charged Higgs scalar components of the right-handed triplet $\Delta_R(1,3,2,1)$ to have low masses. The charged components can mediate neutrinoless double $\beta$-decay, muon decay, $\mu \rightarrow 3e$, and muonium-antimuonium transitions. Improved experimental measurements for these processes would set limits on the Higgs masses in the near future [17]. One of the spectacular signatures of low-mass $W_R$-bosons at SSC energy would be through the decay modes

$W_R \rightarrow e^+ N_R \rightarrow e^+ (\text{jets})$, where $N_R$ is the right-handed Majorana neutrino.

Detailed investigations have shown that the detection limit in this case is nearly $M_{R} \approx 8-6$ TeV [21]. It has been shown [22] that the seesaw mechanism for generating Majorana neutrino mass operates in a profound manner when the mechanism of decoupling $P$- and $SU(2)_R$-breakings is employed, as compared to the conventional method [17]. With $M_{R} \sim 1$ TeV the $G_{2213}$ model predicts $m_{\nu} \sim eV$, $m_{\nu} \sim 10$ keV, and $m_{\nu} \sim 4$ MeV. Such masses, if testified by laboratory measurements, would be in conflict with the cosmological bound according to which the sum of stable neutrino masses should not exceed $\sim 40-100$ eV. Out of several mechanisms proposed to satisfy the cosmological bound with low $M_{R}$, the one that applies here is the decay of unstable heavier neutrinos to the stable light neutrinos like $\nu_e$ by the emission of a majoron [23] that arises as a result of spontaneous breaking of a global lepton number associated with the $G_{2213}$ gauge symmetry. The lightest Majorana neutrino mass $m_{\nu} \sim eV$ compatible with the observable decay rate for the neutrinoless double $\beta$-process, is far too large compared to the mass needed to solve the solar neutrino puzzle using the MSW [24] conjecture.

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82 For phenomenological constraints on the $W_R$- and $\Delta_R$-boson masses see the recent review in ref [17] and references therein
References


Models with natural seesaw mechanism for neutrino masses with identical parity-
and SU(2)\textsubscript{R} -breaking scales

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(Received 17 September 1990)

Chang and Mohapatra have observed that the implementation of the seesaw mechanism explaining small neutrino masses in left-right symmetric or SO(10) models requires the parity (P) and SU(2)\textsubscript{R} breaking scales to be widely separated (M\textsubscript{P} > M\textsubscript{R}). In this paper we show how the mechanism operates naturally even though the two scales are identical. The gauge group immediately preceding the standard model emerges to be its minimal extension based upon SU(3) \times SU(2) \times U(1) with a second neutral Z\textsubscript{2} boson mass M\textsubscript{Z} > 3 \times 10^{10} GeV. An embedding in the partial unification scheme leads to observable rare kaon decays. In the two symmetry-breaking chains investigated in SO(10) with parity broken either at the unification scale (M\textsubscript{P} = M\textsubscript{R}, or at an intermediate scale (M\textsubscript{P} \approx 10^{10} GeV), proton decay is predicted with lifetime near the observable limit, but, in the former case, rare kaon decays are also predicted near the observable limit when an intermediate gauge group SU(2) \times SU(2) \times U(1) survives down to the scale M\textsubscript{R} \approx 10^{10} GeV provided \sin \theta_{\text{L}} \approx 0.24. The criterion for naturalness turns out to be wide separation between P and U(1)\textsubscript{R} -breaking scales.

1 INTRODUCTION

Out of several methods proposed to explain small neutrino masses, the seesaw mechanism has been widely explored in partially unified or grand unified theories (GUT's) of strong, weak, and electromagnetic interactions. Recently Chang and Mohapatra have made an important observation on the general validity of the mechanism as a viable theory for neutrino masses. They found that the implementation of the mechanism in the left-right symmetric (LRS) model or SO(10) GUT needs a wide separation of parity (P) and SU(2)\textsubscript{R} breaking scales. Starting with LRS models or GUT's such as SO(10), SU(16), or SU(16) \times SU(16), it has been demonstrated earlier that a wide separation between P- and SU(2)\textsubscript{R} breaking scales can be realized in the presence of suitable Higgs representations, or specific spontaneous symmetry-breaking patterns. In the case when the P- and SU(2)\textsubscript{R} breaking scales are identical, the present bound on neutrino masses does not permit the right-handed gauge bosons to be light (M\textsubscript{P} = M\textsubscript{R} \approx M = M > 10^{8} - 10^{9} GeV), thus leaving no other testable signatures at lower energies. In the latter situation the mechanism has no role in explaining neutrino masses. The main objective of this paper is to demonstrate that the seesaw mechanism is natural in certain models even if the two scales are identical. This is achieved in spontaneous symmetry breaking (SSB) of a LRS gauge group or a GUT to the standard model in several steps such that SU(2) \times U(1) \times GUT \rightarrow SU(2) \times U(1)

This paper is organized in the following manner. In Sec II we review the work of Chang and Mohapatra illustrating the naturalness of the seesaw mechanism in gauge models with a wide separation between P- and SU(2)\textsubscript{R} -breaking scales. In Sec III we show how the naturalness criterion operates with identical P- and SU(2)\textsubscript{R} -breaking scales using the LRS model and partial unification scheme. In Secs IV and V we show how such models can be embedded in two different scenarios of SO(10) grand unification. Section VI is devoted to summary and discussions.
II NATURAL SEESAW MECHANISM WITH SEPARATE P- AND SU(2)E-BREAKING SCALES

In this section we summarize the work of Chang and Mohapatra establishing the naturalness of the seesaw mechanism in left-right gauge models and GUT's with a wide separation between P- and SU(2)E-breaking scales. For convenience we discuss the conventional mechanism in the context of LRS models based upon the gauge group

\[ SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c \times P \]

where the quarks (Q, Q', Q'') and leptons (\psi_L, \psi_R) of each generation, and Higgs scalars (\phi, \Delta_L, \Delta_R), possess the following transformation properties under \( G_{211} \):

\[ \psi_L \rightarrow \psi_L, \quad \psi_R \rightarrow \psi_R, \quad \phi \rightarrow \phi, \quad \Delta_L \rightarrow \Delta_L, \quad \Delta_R \rightarrow \Delta_R. \]

In order to define the SSD, and implement the seesaw mechanism in the chain

\[ G_{211} \supset G_{12} \supset G_{10} \]

the scalars are assigned the following vacuum expectation values (VEVs)

\[ \langle \Delta_L \rangle = \begin{pmatrix} 0 \\ V_L \end{pmatrix}, \langle \Delta_R \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \phi = \begin{pmatrix} k \\ 0 \end{pmatrix}, \]

leading to the neutrino mass term in the Lagrangian

\[ L_{\nu_{(R)}} = \left( \nu^T \Gamma \right) \begin{pmatrix} m_{LL} & m_{LE} & m_{LR} \\ m_{EL} & m_{EE} & m_{ER} \\ m_{RL} & m_{RE} & m_{RR} \end{pmatrix} \nu \]

where \( \nu \) is the lepton mass, \( N \) is the Higgs potential, and \( m_{\nu} \) is the neutrino mass.

For the maximum values of \( M_\Delta = M_\phi = M_\Delta \), obtained for \( V_L = 0 \) using extended survival hypothesis, the fine-tuning needed to satisfy (7), or \( \lambda \ll h_1/h_2 \), is arbitrary since the standard-model Yukawa coupling \( h_1 = 10^{-5} \), and there is no reason for \( h_2 \) to be small. Without arbitrary fine-tuning, (6) dominates over (4) and the bound on neutrino masses needs \( M_\Delta = M_\phi = M_\Delta \approx 10^{16} \) GeV consistent with \( m_\nu \approx 1-10 \) eV. In such a situation the proposed mechanism does not explain neutrino masses.

Introducing \( P \)-odd singlets of scalars in LRS, or using \( D \)-odd singlets already present in SO(10) (D = a discrete symmetry defined in Ref. 7) in the representations 45 and 210, P and SU(2)_R breaking were decoupled earlier with the possibility \( M_\Delta = M_P = M_\Delta \). Then \( m_{\nu} \) in Eq. (6) is made negligible compared to Eq. (4) and the seesaw mechanism provides a natural explanation for neutrino masses even for low values of \( M_\phi \).

A number of symmetry-breaking patterns including two-loop effects have been worked out in SO(10) and found to be consistent with \( M_\phi = M_\Delta \). In these new SO(10) models the seesaw mechanism is natural. In GUT's of higher rank such as SU(8)_L \times SU(8)_R, or SU(16) specific SU(-)

\[ m_\nu \approx 10^{-11} \text{eV}, \quad m_\nu \approx 10^{-9} \text{eV}, \quad \text{and} \quad m_\nu \approx 10^{-7} \text{eV} \]

Such a feature of the mechanism as obtaining small \( m_\nu \) masses simultaneously with small mixing angles was considered very natural until Chang and Mohapatra observed that the presence of the terms

\[ V_\nu = \sum_{i, j} \lambda_i \bar{\nu}_i \phi \Delta_L \phi^* \nu_j \]

in the Higgs potential, where \( \lambda_i \phi \Delta_L \phi^* \nu_j \) leads to much larger induced values of \( \langle \Delta_R \rangle \) and the left-handed Majorana mass through Fig. 1.

\[ m_{\nu} = \frac{\lambda \nu^2}{M_\Delta^2} \]

even though one has \( \langle \Delta_R \rangle = 0 \) to start with. Here \( \lambda \) is a function of scalar couplings and \( M_\Delta \) is the mass of \( \Delta_R \). Thus, the seesaw mechanism suggests to explain small neutrino masses holds provided values given by (4) dominate over those in (6), which requires

\[ \frac{\lambda V_\nu}{M_\Delta^2} \ll \frac{h_1}{h_2} \]

For the maximum values of \( M_\Delta = M_\phi = M_\Delta \), obtained for \( V_L = 0 \) using extended survival hypothesis, the fine-tuning needed to satisfy (7), or \( \lambda \ll h_1/h_2 \), is arbitrary since the standard-model Yukawa coupling \( h_1 = 10^{-5} \), and there is no reason for \( h_2 \) to be small. Without arbitrary fine-tuning, (6) dominates over (4) and the bound on neutrino masses needs \( M_\Delta = M_\phi = M_\Delta \approx 10^{16} \) GeV consistent with \( m_\nu \approx 1-10 \) eV. In such a situation the proposed mechanism does not explain neutrino masses.

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terms were found that generate asymmetry in the G_{211} gauge group with partial unification scheme with SU(2) \times SU(2) (Ref. 8) and wide separation between P and SU(2)i breaking. Similar arguments permit the seesaw mechanism to be natural in these GUT's.

III. NATURAL SEESAW MECHANISM

In this section we demonstrate how the seesaw mechanism is natural in other gauge models with identical P- and SU(2)i breaking scales. In popular models LRS is associated with the gauge group G_{211}p or

\[ SU(2)p \times SU(2)i \times SU(4)c \equiv G_{211}p, \quad G_{211}p = G_{211}i. \]

GUT's such as SU(10), SU(8) \times SU(10), or SU(16) contain these as subgroups. B - L forms a diagonal generator of SU(2)c. In the alternate class of models exhibiting a natural seesaw mechanism, although P and SU(2)i break at the same scale, SU(2)p \times U(1)p = U(1)p or SU(2)i \times SU(4)c breaks to U(1)i in more than one steps. In the first step U(1)i or SU(4)c must remain unbroken but SU(2)i - U(1)i to generate wide separation between P and SU(2)i-

- In the second stage G_{211}i - G_{211}p proceeds in the same manner as in case (i) through the VEV (\Delta \phi^p \neq 0). In the cases (i) and (ii) the final stage of SSB is achieved by the standard doublet of Higgs scalars. Thus compared to the seesaw mechanism envisaged in Sec. II new types of Higgs scalars are needed in certain cases to drive the SSB in the models. In the light Higgs scalar case there are two neutral particles, the standard Higgs scalars with mass \( M_h \sim M_A \) and the neutral component of the right-handed triplet with mass \( M_{h}^\prime \sim M_{A} \). Since LRS is maintained at scales \( \mu \sim M_{h} \), the Higgs sector must be left-right symmetric for such values of \( \mu \). Using extended survival hypothesis, the charged components \( A_\pm, A_\pm^\prime \), and \( A_\pm^\prime \) in the triplet \( A_\pm, (1,1,0,1) \) under \( G_{211}p \) in case (i) acquire masses of order \( M_{h} \). The left handed counterpart of \( \delta_\pm = A_\pm^{(3,1,2,1)} \) under \( G_{221}p \) does not contribute to the SSB at any stage. Its role is to maintain LRS and, according to extended survival hypothesis, masses of all the components in \( \delta_\pm \) is of the order \( M_{h} \). In the cases (i) and (ii) the right-handed triplet is contained in the \( G_{211}i \) representation \( (1,1,0,1) \), whereas the left-handed triplet is contained in \( (1,1,1,1) \). Only the neutral component of \( A_\pm^\prime \) acquires a mass \( M_{h} \) but all other components of \( A_\pm^\prime \) have masses of order \( M_{h} \). In the case (ii) all other components of \( G_{211}i \) acquire masses of order \( M_{h} \) except the neutral component which acquires a mass \( M_{h} \). All the components of \( (1,1,1) \) under \( G_{211}i \) have masses \( M_{h} \). In the case (ii) all the components of \( (1,1,1) \) under \( G_{211}i \) have masses \( M_{h} \).

Now using the seesaw mechanism and adding the induced mass term due to Fig. 1, we obtain, for the neutrino mass of n generation,

\[ m_n = \frac{\lambda h_{n1}^2 M_{h}^2 M_{h}^0}{g^2 M_{h}^2 M_{h}^0} \frac{m_n h_{n1}^0}{M_{h}^2}, \quad n = e, \mu, \tau. \quad (8) \]

where the first (second) term is the induced seesaw mechanism contribution and \( g \) is the appropriate coupling constant. The dominance of the second term over the first, desired by the naturalness criterion, requires

\[ m_n^0 \gg \frac{R M_{h}^2 M_{h}^0}{M_{h}^2}, \quad \text{or} \]

\[ R m_n^0 \gg M_{h}^2, \quad i = e, \mu, \tau. \quad (9) \]

where we have used \( \lambda h^0_{n1} h_{n1}^0 \gg 1 \) and \( i = e, \mu, \tau \). Inequality (9) is our new naturalness condition in order that the seesaw mechanism provides a meaningful solution to the Majorana neutrino mass. If we use the charged lepton masses for \( m_\mu^0 \), then \( R \gg 10^3 \) for the first generation. This automatically guarantees naturalness for the second and third generations since \( m_\tau^0 \gg m_\mu^0 \gg m_e^0 \). If \( V A \) structure of neutral currents

[\text{Nature of the Higgs component of the left-handed triplet transforming as } (1,3,1) \text{ under } G_{211}p \text{ with } M_{h} \gg M_{h} \text{. In this case } (i) \text{ and } (ii) \text{ the final stage of SSB is achieved by the standard doublet of Higgs scalars. Thus compared to the seesaw mechanism envisaged in Sec. II new types of Higgs scalars are needed in certain cases to drive the SSB in the models. In the light Higgs scalar case there are two neutral particles, the standard Higgs scalars with mass } M_h \sim M_A \text{ and the neutral component of the right-handed triplet with mass } M_{h}^\prime \sim M_{A} \text{. Since LRS is maintained at scales } \mu \sim M_{h} \text{, the Higgs sector must be left-right symmetric for such values of } \mu \text{. Using extended survival hypothesis, the charged components } A_\pm, A_\pm^\prime \text{, and } A_\pm^\prime \text{ in the triplet } A_\pm, (1,1,0,1) \text{ under } G_{211}p \text{ in case (i) acquire masses of order } M_{h} \text{. The left handed counterpart of } \delta_\pm = A_\pm^{(3,1,2,1)} \text{ under } G_{221}p \text{ does not contribute to the SSB at any stage. Its role is to maintain LRS and, according to extended survival hypothesis, masses of all the components in } \delta_\pm \text{ is of the order } M_{h} \text{. In the cases (i) and (ii) the right-handed triplet is contained in the } G_{211}i \text{ representation } (1,1,0,1) \text{, whereas the left-handed triplet is contained in } (1,1,1,1) \text{. Only the neutral component of } A_\pm^\prime \text{ acquires a mass } M_{h} \text{ but all other components of } A_\pm^\prime \text{ have masses of order } M_{h} \text{. In the case (ii) all other components of } G_{211}i \text{ acquire masses of order } M_{h} \text{ except the neutral component which acquires a mass } M_{h} \text{. All the components of } (1,1,1) \text{ under } G_{211}i \text{ have masses } M_{h} \text{. In the case (ii) all the components of } (1,1,1) \text{ under } G_{211}i \text{ have masses } M_{h} \text{. Now using the seesaw mechanism and adding the induced mass term due to Fig. 1, we obtain, for the neutrino mass of n generation,}

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where the first (second) term is the induced seesaw mechanism contribution and \( g \) is the appropriate coupling constant. The dominance of the second term over the first, desired by the naturalness criterion, requires

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where we have used \( \lambda h^0_{n1} h_{n1}^0 \gg 1 \) and \( i = e, \mu, \tau \). Inequality (9) is our new naturalness condition in order that the seesaw mechanism provides a meaningful solution to the Majorana neutrino mass. If we use the charged lepton masses for \( m_\mu^0 \), then \( R \gg 10^3 \) for the first generation. This automatically guarantees naturalness for the second and third generations since \( m_\tau^0 \gg m_\mu^0 \gg m_e^0 \). If \( V A \) structure of neutral currents
are desirable at low energies along with the detection of $\Lambda_R$ at the supercolliders, it is necessary that $M_{\nu} \geq 1$ TeV, which implies that the mechanism is natural if $M_{\nu}^2 = M_{\nu}^2 \gg 10^8$ GeV. A very interesting feature of the minimal extended gauge group $G_{\text{1111}}$ at lower energies Using renormalization group equations (RGEs) it is easy to verify that the condition $M_{\nu} \gg M_{\nu}^2$ in the case (ii) and the constraint arising out of $K_{ij} - K_{ij}^2$ mass difference as observed in Ref 9. In fact, RGEs do not constrain $M_{\nu} = M_{\nu}^2$ as there are three unknown gauge coupling constants $K_{ij} = g_{ij} = g_{ij}^2$, and $g_{ij}^2$ for $\mu \geq M_{\nu}^2 = M_{\nu}^2$. But in cases (i) and (ii) there are two unknown gauge coupling constants, $g_{ij} = g_{ij} = g_{ij}^2$, for $\mu \geq M_{\nu}^2 = M_{\nu}^2$ one of which can be eliminated using the one structure constant matching at $\mu = M_{\nu}$. For the case (i) and the relation between $\sin^2 \theta_W$ and the mass scales can be expressed in closing one loop corrections as

$$\sin^2 \theta_W = \frac{1}{2} \frac{\alpha}{\Lambda \pi} \frac{g_{ij}}{3} \frac{M_{\nu}^2}{M_{\nu}^2}$$

(10)

where $\alpha(M_{\nu}) = \pi(M_{\nu}/4\pi)$. The corresponding equation for case (ii) is obtained from Eq (10) by using $M_{\nu}^2 = M_{\nu}^2 = M_{\nu}^2$. Solutions to the RGEs in case (i) and (ii) have been obtained with the QCD parameter $\Lambda_{\text{MS}} = 0.2$ GeV, where MS denotes the modified minimal subtraction scheme, $\sin^2 \theta_W = 0.22$ at $\Lambda = 10^8$ GeV, and for values of $M_{\nu}^2 = (3 \times 10^7 - 10^8)$ GeV, some of which are presented in Tables I and II. For the case (i) we found $7 \times 10^7$ GeV $\leq M_{\nu}^2 \leq 2 \times 10^8$ GeV for $10^8$ GeV $\gg M_{\nu}^2 \gg 10^8$ GeV. In this case, in addition to predicting the low energy lepton group to be $G_{\text{1111}}$, beyond the standard model, the rare kaon decays are also predicted to be observable, corresponding to $M_{\nu}^2 = 10^8$ GeV. In the case (ii) the solutions are consistent $M_{\nu}^2 = M_{\nu}^2 = M_{\nu}^2 = 10^8$ GeV $- 5 \times 10^7$ GeV with a low mass Z boson. The parameter $R = M_{\nu}^2 / M_{\nu}^2$ $\geq 10^{-5}$ in both cases and is found to guarantee the naturalness condition. The neutrino mass spectrum for lower values of $M_{\nu}^2 = 300$ GeV $- 1$ TeV is of the type $\nu = \nu = \nu = M_{\nu}^2$ for the three generations. In such cases, $m_\nu$ and $m_\nu$, would violate the cosmological bound. One method of evading the cosmological bound is to make $\nu_\nu$ and $\nu_\nu$, unstable against Majoron emission as discussed in Sec VI. In the next section we examine embeddings of models (i) and (ii) in SO(10) grand unification.

IV IMPLEMENTATION IN SO(10) WITH $G_{\text{1111}}$ AS AN INTERMEDIATE SYMMETRY

In this section we show how the new seesaw mechanism operates in an SO(10) scenario where $G_{\text{1111}}$ and $G_{\text{1111}}$ occur as the two intermediate symmetries. The well known problem in such a GUT scenario is the presence of undesirable domain walls and inadequate baryon asymmetry of the Universe if $M_{\nu}^2 = M_{\nu}^2 < 10^{-11}$ GeV. On the other hand if $M_{\nu}^2 = M_{\nu}^2 > 10^{-11}$ GeV the baryon asymmetry is compatible with the observed value and the domain walls created in the early Universe might have been removed by inflation. Our analyses in this paper demonstrate that the RGE's permit such solutions. We discuss the embeddings of these groups in SO(10) and find solutions to the unification mass $M_{\nu}^2$, $M_{\nu}^2$, and intermediate scales including renormalization effects on the gauge coupling constants up to two loops and superheavy Higgs scalar effects. The case (i) mentioned in Sec III can be embedded in SO(10) grand unification as follows

\begin{equation}
\text{SO}(10) \rightarrow \frac{310}{164} G_{\text{1111}} \rightarrow G_{\text{1111}} \rightarrow G_{\text{1111}} \rightarrow G_{\text{1111}} \\
\end{equation}

where the Higgs scalars mentioned in (i) are contained in the respective SO(10) representations $\bar{16}, 10, 126$.
If the component masses are taken to be arbitrarily nondegenerate, the model loses its predictive power on the proton lifetime ($\tau_p$) and sin $\theta_W$. We examine their impact on GUT predictions by assuming the masses to be (a) degenerate, (b) nondegenerate but not arbitrary as they are constrained by a Coleman Weinberg-type condition in that a nondegeneracy factor up to 10 might be generated among different component masses in a single GUT representation. In all cases $\tau_p (p \rightarrow e\nu\nu)$ is predicted near the observable limit. In order to constrain masses under condition (b), we maximize $\tau_p$ using the RG equation for $\ln(M^2/M_w)$, which leads to

$$M_{H^+} = M_{H^0} = M_{A^0} = M_{H^+} = M_{H^0} = M_{A^0} = M_{H^+} = M_{H^0} = M_{A^0} = M_{H^+} = M_{H^0} = M_{A^0} = M_{H^+} = M_{H^0} = M_{A^0}.$$  

Using a minimal number of Higgs scalars and three fermion generations we have computed the one and two-loop coefficients in the equations for $\ln(M^2/M_w)$ and sin $\theta_W$ given below

$$\sin^2\theta_W = \frac{1}{2} \left[ \frac{\alpha}{\alpha(M^2)} + \frac{\alpha(M^2)}{\alpha(M^2)} \right] + \frac{\alpha}{\alpha(M^2)} \left( \frac{\alpha}{\alpha(M^2)} \right) - \frac{\alpha}{\alpha(M^2)} \left( \frac{\alpha}{\alpha(M^2)} \right)$$

where

$$\chi_f = \frac{\alpha(M_f)}{\alpha(M^2)}, \quad \chi_f^* = \frac{\alpha(M_f^*)}{\alpha(M_f)}, \quad \chi_f^2 = \frac{\alpha(M_f^2)}{\alpha(M_f)}, \quad \text{and} \quad \alpha(M^2) = \frac{g^2(\mu)}{4\pi}.$$

Using an iterative convergence procedure that ensures fine structure constant matching at $\mu = M_W$ we have computed $M_f$, $\tau_p$, and sin $\theta_W$ as a function of $M^2$ for the degenerate and nondegenerate cases as shown in Figs. 2 and 3, respectively, while keeping $Z_R$ light ($M_Z^2 \approx 1$ TeV), where $\eta = \ln(M^2/M_W)$ Some interesting solutions are summarized in Table III.
FIG 2 Predictions of the symmetry breaking pattern SO(10) → G_{16} → G_{213} as described in the text with \( M_p = \nu < 1 \) TeV with and without degenerate superheavy Higgs scalar contributions. The dot-dashed curve is for \( \Lambda_{G1} = 0.25 \) GeV, others are for \( \Lambda_{G1} = 0 \) 10 GeV.

FIG 3 Same as Fig 2 but for nondegenerate superheavy scalar masses under a Coleman Weinberg type constraint and \( \Lambda_{G1} = 0 \) 10 GeV.

out of uncertainties in the estimation of the proton decay matrix elements, branching ratios, and \( \Lambda_{G1} \) in Eq. 17. Including contributions up to two loops and no superheavy Higgs scalar effects, \( \tau_p \) decreases by two orders corresponding to the curve \( y_U^{-1} = y_U^{-1} = 0 \) in Fig. 2 for which \( \Lambda_{G1} = 160 \) MeV. Including the superheavy Higgs scalars lighter than \( M_U \) by a factor 10 (50) increases the two loop computation of \( \tau_p \) by 2 (3) orders for \( \Lambda_{G1} = 160 \) MeV, and the decrease in \( \sin^2 \theta_U \) is only 0.0015. Allowing the possibility of \( \Lambda_{G1} = 250 \) MeV and the superheavy scalars lighter by a factor 50 from \( M_U \), we find \( \tau_p \) decreases by \( 10^{34} - 10^{15} \) yr, with \( M_p = M_U \geq 10^{14} \) GeV and \( \sin^2 \theta_U = 0.22 - 0.23 \) as shown in Fig. 2 and Table III. Increasing \( \Lambda_{G1} \) from 1 TeV to 100 TeV does not have a significant impact on the GUT predictions. In the case of nondegenerate superheavy components restricting \( M_p = M_U < 10^{11} \) GeV and \( \sin^2 \theta_p = 0.22 - 0.23 \), \( \tau_p \) is found to increase over the one loop predictions by nearly 2 orders if \( M_p = M_U \) and \( M_p = M_U / 5 \). In this case \( \tau_p \approx 10^{32} - 10^{33} \) yr, with \( M_p = M_U = 10^{12} \) GeV and \( \sin^2 \theta_U = 0.22 - 0.23 \). For larger values of nondegeneracy factor, \( \tau_p \) could be larger as shown in Fig. 3. The allowed values of the low mass of the \( Z_R \) boson \( (300 \) GeV), \( 1 \) TeV) are consistent with the eV to MeV type of mass spectrum for the neutrinos of the three generations when we choose \( m_1 = m_2 = 0.001 \) MeV, \( m_3 = m_4 = m_5, \) and \( m_6 = m_7, \) at a

TABLE III Some predictions of the model \( \text{SO}(10) \rightarrow G_{16} \rightarrow G_{213} \) on \( \sin^2 \theta_U \) and \( \tau_p \) with \( \nu \nu = 1 \) TeV, \( \Lambda_{G1} = 0 \) 16 GeV and different values of the parity violating scale \( \Lambda_{G1} \) including superheavy Higgs scalar effects.

<table>
<thead>
<tr>
<th>( y_U^{-1} )</th>
<th>( y_U^{-1} )</th>
<th>( M_p^* = M_U ) (GeV)</th>
<th>( M_U ) (GeV)</th>
<th>( \sin^2 \theta_U )</th>
<th>( \alpha_U^{-1} )</th>
<th>( \tau_p ) (yr)</th>
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<tr>
<td>-2.30</td>
<td>-2.30</td>
<td>8.4 \times 10^8</td>
<td>2.8 \times 10^{13}</td>
<td>0.240</td>
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<tr>
<td>3.65 \times 10^8</td>
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<td>0.235</td>
<td>35.7</td>
<td>3.6 \times 10^{13}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5 \times 10^8</td>
<td>7 \times 10^{13}</td>
<td>0.233</td>
<td>32.0</td>
<td>1.3 \times 10^{12}</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>2 \times 10^{13}</td>
<td>0.235</td>
<td>32.6</td>
<td>10^{12}</td>
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<tr>
<td>10^{13}</td>
<td>1.2 \times 10^{13}</td>
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<td>31.6</td>
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<td></td>
</tr>
<tr>
<td>1.6 \times 10^{11}</td>
<td>2 \times 10^{13}</td>
<td>0.236</td>
<td>31.1</td>
<td>10^{12}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10^{12}</td>
<td>1 \times 10^{13}</td>
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<td>10^{12}</td>
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</tr>
<tr>
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<td>0.221</td>
<td>30.4</td>
<td>10^{12}</td>
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<td></td>
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</tbody>
</table>
in the violation of the cosmological bound. The difficulty is removed by making them unstable with respect to decay into s, by the emission of a Majorana which is obtained by introducing an additional global $\mathbb{Z}_2$ (Majorana number) symmetry in the theory and breaking it spontaneously at a scale $\mathcal{M} \gg M^*_s$. The RGE's also permit solutions with larger values of $M^*_s = M^*_p \sim 10^{-10} \text{ GeV}$ when $M = M^*_p \sim 10^{11} - 10^{12} \text{ GeV}$, for such larger values of $M^*_s$, which satisfies the naturalness criterion. In this case $m < m_s < \sim 10^{-4} \text{ eV}$ and there is no conflict with the cosmological bound. The weak-interaction phenomenology with such large $L^*_s$ mass is indistinguishable from the standard-model predictions.

The Higgs scalars mentioned in Sec. III for case (i) are contained in various SO(10) representations. The transformation properties mentioned are those under $G_{14}$. Note that in both 54 and 45, there are candidates for the $S$SB at $\mu < M^*_s$. The masses of superheavy components of different Higgs representations needed for SSB in the case (i) are noted below with their transformation properties under $G_{14}$.

- $10 \oplus M^*_s(2, - 1, 1) + M^*_s(1, 0, 6)$
- $16 \oplus M^*_s(1, 0, 6) + M^*_s(3, 0, 10) + M^*_s(1, 1, 1) + M^*_s(1, - 1, 10) + M^*_s(2, 1, 15) + M^*_s(2, - 1, 15)$
- $45 \oplus M^*_s(3, 0, 11) + M^*_s(1, 0, 1) + M^*_s(1, - 1, 10) + M^*_s(2, 1, 6) + M^*_s(2, - 1, 6)$
- $54 \oplus M^*_s(3, 1, 11) + M^*_s(3, 0, 1) + M^*_s(3, - 1, 1) + M^*_s(1, 0, 20) + M^*_s(2, 5, 6) + M^*_s(2, - 5, 6)$

Maximization of $\rho$ leads to the following constraint on the superheavy-component masses

$$M^*_s = M^*_p = M^*_s = M^*_s = M^*_s = M^*_s = M^*_s = M^*_s = M^*_s = M^*_s$$

Using three generations of fermions with masses $\mu < M^*_s$, minimal number of Higgs scalars at various stages of $S$SB, and the superheavy Higgs scalar effects near $\mu = M^*_s$ we compute $\ln M^*_s / M^*_s$ and $\sin \theta_{h_3}$ up to two loops as

$$\ln \frac{M^*_s}{M^*_s} = \frac{3}{2} \frac{1}{\alpha} + \frac{8}{3} \alpha_{1,2}$$

$$\sin \theta_{h_3} = \frac{\sqrt{2} \ln \frac{M^*_s}{M^*_s} + \frac{2}{9} \ln \frac{M^*_s}{M^*_s} - \frac{3}{28} \ln \frac{M^*_s}{M^*_s}}{\frac{1}{2}}$$

In this section we show how the weak-machinoon can be implemented naturally with $G_{14}$ as one of the two intermediate symmetries occurring in an SO(10) scenario. Compared to the case discussed in Sec. IV this has the novel feature that both $SU(2)$ and $P$ break the GUT scale that is experimentally constrained to $M_s \leq 10^{13} \text{ GeV}$. The question of domain wall problem does not arise in this case in addition the proton decay rate could be close to the observable limit if RGE's permit $M_s \sim 10^{15} \text{ GeV}$. For case (ii) we have found the unification mass too low to be allowed by proton-lifetime measurements unless additional fine tuning is permitted. On the other hand case (i) is promising in the context of the GUT scenario.
where

\[ X^i = \frac{\langle R \rangle}{\langle W \rangle} \]

Following the iterative convergence approach to solve two-loop renormalization group equations and using plausible values of superheavy component masses, our solutions for the intermediate scale and \( M_\gamma \) for \( M_\gamma \approx 1 \) TeV are shown in Figs 4 and 5 for \( \Lambda_{\text{SO}} = 0 \) 160 GeV, and \( 0 \) 350 GeV, respectively (Ref 17) where \( \bar{m}^2 = \ln(A\gamma^2/M_\gamma^2) \) Some of the interesting solutions are also presented in Table IV. At the one-loop level with \( \Lambda_{\text{SO}} = 0 \) 350 GeV the predicted value of \( \tau^* \) is found to be very close to the observed experimental limit for \( M_\gamma = 10^2 \) GeV and \( \sin^2\theta_\mu = 0.235 \), but \( \tau^* \) is found to be 1-2 orders less than the experimental limit for \( \Lambda_{\text{SO}} = 0 \) 160 GeV. When superheavy-Higgs-scalar effects are included in two-loop calculations, we find \( \tau^* = 10^{8+1} \) yr, \( \sin^2\theta_\mu = 0.230 \), \( M_\gamma = 10^2 \) GeV with \( \Lambda_{\text{SO}} = 160 \) GeV if the heavier (lighter) components differ by a factor 10 from the unification mass. For larger values of \( \Lambda_{\text{SO}} \) or nondegeneracy factors, \( \tau^* \) is found to increase further We find that this SO(10) model permits observable rare \( \mu \) decays corresponding to \( M_\gamma = 10^5 \) GeV provided \( \sin^2\theta_\mu = 0.24 \) In all allowed solutions in this model \( M_\gamma = M_\mu = M_\tau = 10^2 \) GeV With \( M_\gamma = 1\) TeV, \( R \approx 10^4 \), and the naturalness criterion is easily satisfied As in Sec II, the low-mass \( Z_\gamma \) boson yields the neutrino-mass spectrum as eV-keV-MeV for the three generations. The violation of the cosmological bound by the \( \nu_\mu \) and \( \nu_\tau \) masses is avoided by making these neutrinos unstable against Majoron emission through the introduction of an additional global lepton-number symmetry U(1) (Ref 18) But the RGE's also permit \( M_\gamma = 10^2-10^3 \) GeV as the \( Z_\gamma \) boson mass for which \( m_\nu > m_\nu > m_\nu > 1-10 \) eV as a consequence of the natural seesaw mechanism with \( R = 10^3-10^4 \), and this is consistent with the cosmological bound with stable neutrinos. In this case the predicted weak-interaction phenomenology at low energy cannot be distinguished from the standard model predictions.

### Table IV

<table>
<thead>
<tr>
<th>( \Lambda_{\text{SO}} ) (GeV)</th>
<th>( \nu_\mu )</th>
<th>( \nu_\tau )</th>
<th>( M_\mu ) (GeV)</th>
<th>( M_\tau ) (GeV)</th>
<th>( \sin^2\theta_\mu )</th>
<th>( \alpha_\mu )</th>
<th>( \tau^* ) (yr)</th>
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<td>48.3</td>
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<tr>
<td></td>
<td>0</td>
<td>-46</td>
<td>( 3 \times 10^9 )</td>
<td>( 10^9 )</td>
<td>0.235</td>
<td>47.6</td>
<td>1.4 \times 10^{11.2}</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>-46</td>
<td>( 1.6 \times 10^9 )</td>
<td>( 10^9 )</td>
<td>0.240</td>
<td>42.5</td>
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</tr>
<tr>
<td>0.35</td>
<td>23</td>
<td>-23</td>
<td>( 10^7 )</td>
<td>( 2.4 \times 10^8 )</td>
<td>0.240</td>
<td>48.8</td>
<td>4.3 \times 10^{11.2}</td>
</tr>
<tr>
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<td>-46</td>
<td>( 5.6 \times 10^9 )</td>
<td>( 2 \times 10^9 )</td>
<td>0.240</td>
<td>47.4</td>
<td>2.9 \times 10^{11.2}</td>
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<td>( 2 \times 10^9 )</td>
<td>0.240</td>
<td>41.7</td>
<td>5 \times 10^{11.2}</td>
</tr>
</tbody>
</table>
VI. SUMMARY AND DISCUSSION

In this paper we have suggested the new possibility that the seesaw mechanism for neutrino masses could be natural in the context of the left-right-symmetric gauge group, partial unification scheme, and GUT's even if the scales of $P$ and SU(2)$_L$ breaking are identical. In these models, the $P$-breaking scale is the same as the $W$-boson mass ($M_P = M_W$) and the SU(2)$_L$-breaking scale is the same as the Z-boson mass ($M_Z$). The criterion which guarantees naturalness has been derived and is found to depend upon the largeness of the ratio $R = M_Z^2 / M_P^2 > 10^3$. At the critical value of the ratio $R > 10^3$, the induced and seesaw mechanism contributions are comparable, but for larger values of $R$ the induced neutrino mass becomes smaller.

In the LRS model based upon the gauge group $SU(2)^R$, it is very easy to implement the mechanism as there is no much restriction on $M_P = M_R$. In the partial-unification scheme with one intermediate symmetry $SU(10)$, the RGE permits $M_R = M_Z^2 = 10^{-2} - 10^{-3}$ GeV with $M_Z^2 = 300$ GeV$^2$ and $10^3$ GeV$^2$ [case (ii)]. However, with two intermediate symmetries $SU(14)$ and $SU(16)$, there is only the solutions allow $M_R = M_Z^2 = 7 \times 10^4 - 10^5$ GeV for $10^9$ GeV $> M_P > 10^6$ GeV, predicting rare kaon decays to be observable by low-energy experiments besides a low-mass $Z_R$ boson.

In the SO(10) model, implementation of the natural seesaw mechanism has been found to be possible with parity $(P)$ surviving down to an intermediate scale $M_P = M_Z^2 > 10^4$ GeV or broken at the GUT scale $M_P = M_Z^2 > 10^6$ GeV. With $G_{SU(5)}$ and $G_{SU(7)}$ intermediate symmetries, RGE's up to two loops, with superheavy-Higgs-scalar masses lighter than $M_P$ by a factor of 10-50, are found to allow the intermediate $P$-breaking scale $M_P = 10^7 - 10^{10}$ GeV, observable proton decay by the second generation of experiments with $\tau > 10^{15} - 10^{16}$ yr, and a low-mass $Z_R$ boson ($M_R^2 = 300 - 10^8$ GeV). In this case there is the possibility that the domain walls created in the early Universe might have been removed by inflation. In this context it is to be noted that the large $P$-violating scale can be associated with the breaking of Peccei-Quinn symmetries invoked to solve the strong CP problem and can be generated by the principle of geometric hierarchy from $M_{P} \approx 10^{18}$ GeV and $M_{Z} \approx 10^{12}$ GeV or $M_{P}$. Further, it has been observed that a LRS gauge group as an intermediate symmetry in SO(10), the generation of an adequate baryon asymmetry of the Universe needs such a large $P$-violating scale. In the other interesting SO(10) scenario with $G_{14}$ and $G_{113}$ as the two intermediate symmetries, superheavy-Higgs-scalar masses differing by a factor 10-50 lighter or in each from the unification mass allow $\tau > 10^{15} - 10^{16}$ yr, with the possibility of observing rare kaon decays and a low-mass $Z_R$ boson.

In the two SO(10) models discussed here, $G_{14}$, is allowed to be the gauge symmetry beyond the standard model with the permitted values of a $Z_R$-boson mass varying over a wide range $10^{9} - 10^{10}$ GeV.

The weak-interaction phenomenology at low energy does permit a low-mass $Z_R$ boson ($M_R^2 > 300$ GeV$^2$) in the $G_{14}$ model which yields fits to the neutral- and charged-current data similar to the standard-model predictions. When such values of $M_R^2$ are used in the natural seesaw mechanism, the neutrino masses are of the order $10^{-5}$ eV, and the MeV for the first, second, and third-generation neutrinos, respectively, out of which the latter two violate the cosmological bound. The cosmological bound can still be respected with low-$Z_R$ masses by making $\delta$ and $\lambda$ unstable with respect to the expansion of a Mazaron which is a massless scalar carrying 2 units of lepton number, and it is created when an additional global symmetry $U(1)'$, is broken spontaneously. With the other allowed possibility, $M_R^2 > 10^3$ GeV$^2$, $m_\nu < 10^{-1} - 10^{-2}$ eV, there is no violation of the cosmological bound. The weak-interaction phenomenology in lower energies is then indistinguishable from the standard-model predictions within the available experimental accuracies. However, one novel feature in the partial-unification scheme and SO(10) model with $G_{14}$ and $G_{113}$ intermediate symmetries is the prediction of observable rare kaon decays such as $K^0 \to \pi^-$ $\nu\bar{\nu}$. The analysis carried out here in SO(10) can be easily implemented in other GUT's such as $SO(21)$, $SO(18)$, and $SU(16)$ with similar predictions. However, in SU(10) $\times SU(7)$, while all other low-energy predictions are similar, it is possible to have a more stable proton since the gauge-boson-mediated interaction corresponding to the proton decay is absent.

Finally, from the investigations carried out in this paper we conclude that scenarios different from those discussed by Chang and Mohapatra in Ref 3 and work out earlier do exist in LRS models, partial unification schemes and GUT's in which the seesaw mechanism can provide a natural explanation for small Majorana neutrino masses even if the $P$- and SU(2)$_L$ breaking scales are identical.

ACKNOWLEDGMENTS

One of us (M. K. P.) thanks Professor C. K. Majumdar, Professor Partha Ghose and Professor D. P. Roy for initiating the CP-Violation Meeting and High Energy Physics Workshop at the S. N. Bose National Centre, Calcutta, where a part of this work was reported.

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In Figs. 2-5, we meant log \( 30 \) (or \( 10 \)) not \( \ln \). This may be corrected.