Preface

This Ph.D., thesis entitled "On domination theory and related concepts in graphs" embodies the work done by the author under the guidance and supervision of Dr. J. Paulraj Joseph, Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli.

Graph Theory has become a part of Discrete Mathematics which is indispensable in the field of Computer Science. The earliest ideas of dominating sets are found in the classical problems of covering chess boards with minimum number of chess pieces. The theory of domination was formalized by Claude Berge [3] in 1958. He wrote a book on graph theory, in which he defined for the first time the concept of the domination number of a graph (although he called this number the "coefficient of external stability"). A set \( S \subseteq V \) is said to be a dominating set, if every vertex in \( V-S \) is adjacent to at least one vertex in \( S \). A dominating set \( S \) in \( G \) is a minimal dominating set if no proper subset \( S_1 \subset S \) is a dominating set. The minimum cardinality taken over all minimal dominating set is called the domination number of \( G \) and is denoted by \( \gamma \).

In 1962, Oystein Ore published a book on graph theory and it is believed that Ore was the first person to use the term "domination number". Berge mentions the problem of keeping a number of strategic locations under surveillance by a set of radar stations. The minimum number of radar stations needed to survey all the locations is the domination number of a associated graph. In 1977, Cockayane and Hedetniemi [4] published a survey of the few results known at that time about dominating sets in graphs. They point out that the notion of domination is a standard one in coding
theory. It also boasts a host of applications to Social network theory, Computer Communication Networks, land surveying, game theory etc., Now a days, domination theory ranks top among the most prominent areas of research in Graph Theory and Combinatorics.

By a graph $G = (V, E)$, we mean a finite undirected graph without loops or multiple edges. The order and size of $G$ are denoted by $p$ (or $n$) and $q$ (or $m$ ) respectively. This thesis consist of six chapters.

**In chapter 1**, we collect the basic definitions and theorems on graphs which are needed for the subsequent chapters. For graph theoretic terminology, we refer to Harary [6]. We also collect definitions and theorems on domination theory following the terminologies and notations in Teresa W. Haynes, Stephen T. Hedetniemi and Peter J. Slater [25].

**In chapter 2**, we investigate the relationship between chromatic number and paired domination number of a graph, which was introduced by Haynes, Teresa W [9]. A set $S \subseteq V$ is a **paired - dominating set**, if $S$ is a dominating set of $G$ and the induced sub graph $< S >$ has a perfect matching. The minimum cardinality taken over all paired dominating sets of $G$ is called the paired domination number and is denoted by $\gamma_{pr}$. We find an upper bound for the sum of the paired domination number and chromatic number of a graph and characterize the corresponding extremal graphs.

**In chapter 3**, we investigate the relation between chromatic number and induced - paired domination number of graph which was introduced by Haynes et. al., [10]. A set $S \subseteq V$, is an **induced - paired dominating set**, if
S is a dominating set of G and the induced subgraph $<S>$ is a set of independent edges. The minimum cardinality taken over all induced - paired dominating sets of G is called the induced - paired domination number and is denoted by $\gamma_{ip}$. We find an upper bound for the sum of the induced paired domination number and chromatic number of a graph and characterize the corresponding extremal graphs.

The concept of non-split dominating set was introduced by Kulli [12] and it has been renamed as complementary connected domination number by Tamizh chelvam, T. et. al., [24]. A set $S \subseteq V$, is called a complementary connected dominating set, if $S$ is a dominating set of $G$ and the induced subgraph $<V-S>$ is connected. The minimum cardinality taken overall complementary connected dominating sets of $G$ is called the complementary connected domination number and is denoted by $\gamma_{cc}$.

In chapter 4, we investigate the relation between chromatic number and complementary connected domination number of graph. We find an upper bound for the sum of the complementary connected domination number and chromatic number of a graph and characterize the corresponding extremal graphs. The contents of this chapter have been published in [17].

In chapter 5, we introduce the concept of complementary perfect domination number of a graph. A set $S \subseteq V$ is a complementary perfect dominating set, if $S$ is dominating set and $<V-S>$ has a perfect matching. The minimum cardinality of $S$ is called the complementary perfect domination number and is denoted by $\gamma_{cp}$. 
This concept is likely to have good application in the prevailing situations in developing countries. In India, due to poverty and unemployment problems, people from villages move in large scale towards metropolitan cities, which are already over crowded. Hence the State Governments plan to construct model villages far away from the cities and provide all possible facilities by adopting pairs of villages which are connected to each other by means of a single road. Thus in the corresponding road network, the Governments wish to select a minimum number of cities from which facilities can be extended to all pairs of model cities at a minimum cost. This is nothing but the complementary perfect domination number of a graph associated with this problem.

We proved that any complementary perfect dominating set of $G$ must contain all the pendant vertices of $G$. We also proved that for any connected graph $G$ of order $n \geq 2$, $\gamma_{cp} = n$ if and only if $G \cong K_{1,n}$. We determine the exact value of $\gamma_{cp}$ for paths, cycles and other standard class of graphs. We have also proved that if $L_n$ denotes the $m$-array tree with maximum level $n$. Then for any $n \geq 0$,

(i) $\gamma_{cp}(L_{3n}) = 1/(m^3 - 1) \left\{ m^{3n+2} (m^2 + m - 1) - (m^2 - m + 1) \right\}$

(ii) $\gamma_{cp}(L_{3n+1}) = 1/(m^3 - 1) \left\{ m^{3n+2} (m^2 + m - 1) + (m^2 - m - 1) \right\}$

(iii) $\gamma_{cp}(L_{3n+2}) = \left\{ (m^{3n+3} - 1) / (m^3 - 1) \right\} (m^2 + m - 1)$

We also find the relationship of this new parameter with chromatic number of the graph.
There are several papers in which graphs with equal parameters are investigated. In [29], Volkman studied graphs for which $\gamma = \beta_1$. He also investigated graphs for which $\gamma = \alpha_0$ [28]. In [15], Paulraj Joseph and Arumugam investigated graphs for which $\gamma = \gamma_c$. In [7,8], Harary and Livingston studied trees and caterpillars for which $\gamma = \gamma_t$. In [16], Paulraj Joseph J. analysed graphs for which the chromatic number equals domination parameters.

In chapter 6, we investigate cubic graphs whose domination number equals chromatic number. We observe that for such graphs the maximum number of vertices $p = 8, 10, 12$. This chapter consists of three sections. In section 1, we characterize connected cubic graphs on 8 vertices for which $\gamma = \chi = 3$. In section 2, we characterize all connected cubic graphs on 10 vertices for which $\gamma = \chi = 3$. In section 3, we characterize all connected cubic graphs on 12 vertices for which $\gamma = \chi = 3$. Thus we have characterized all connected cubic graphs for which $\gamma = \chi$.

The contents of section 1 and 2 of this chapter, have been published in [13].