Chapter 4

State of Art:
Theoretical Background for Soft computing
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Chapter provides a comprehensive study of the work done by the researchers using soft computing techniques for the design and development of robust observer based control systems. The survey is classified as per the application of techniques used such as Fuzzy, Artificial Neural Network and Neural-Fuzzy techniques.

4.1 Review of work based on Fuzzy Logic

This section deals with the brief description of the foundation and techniques developed and used by the researchers for design, development, simulation and development of fuzzy logic based robust adaptive systems for non linear systems.

4.1.1 State estimation with aid of fuzzy logic

The state variables of a power system (i.e., the operating quantities by which the system may be characterized) are typically chosen to be bus voltage magnitudes and angles. The task of determining the actual values of the state variables is called state estimation. One of the problems in automating a power system is the construction of reliable models of the system whose state variables can be identified sufficiently accurately using available noisy system data.

For the successful operation of large-scale power systems the optimal estimation of the state is required. For this purpose the well-known weighted least squares (WLS) estimator has been widely used. Weighted least squares estimators have been studied extensively and their numerical stability as well as computational efficiency have been greatly improved by various techniques. However, in the presence of gross errors, WLS estimators give poor state estimates [1].

An alternative state estimation approach, the weighted least absolute value (WLAV) has been applied to power system problems. This estimator is more robust than the WLS estimator. Here the estimate is obtained by solving a sequence of linear programming (LP) problems [2, 3]. An important drawback associated with the WLAV estimation is its poor computational efficiency for large problems [4]. A combination of weighted least squares and fuzzy-logic-based techniques to improve the state estimated of power systems.

4.1.2 Adaptive Robust Fuzzy Control of Nonlinear Systems

Uncertainties are inevitable in dynamical systems, and they may arise from errors in system modeling, parameter variations, unknown physical phenomena (e.g., frictions in a mechanical system), and working environments. In addition to the classical feedback control theory, adaptive control and robust control are effective techniques to treat these uncertainties (see, e.g., [5] and [6]). Adaptive control, by online tuning the parameters (of
either the plant or the controller—corresponding to indirect, or direct adaptive control), can deal with large uncertainties, but generally, suffers from the disadvantage of being able to achieve only asymptotical convergence of the tracking error to zero. The online computation burden to update the parameters is also usually heavy. In robust control designs, on the other hand, a fixed control law based on a priori information on the uncertainties (usually bounds on these uncertainties) is designed to compensate for their effects, and exponential convergence of the tracking error to a (small) ball centered at the origin is obtained. But if the uncertainties are larger than the assumed bounds, no stability or performance is guaranteed.

In recent years analytical studies of nonlinear control, using universal approximators such as fuzzy logic system (FLS) [7], [8], [9], [10], [11], [12], [13] is used to approximate the unknown functions involved in the control design. In the class of approximators which are linear in parameters, FLS is much closer in spirit to human thinking and natural language [14], and preferred by control engineering practitioners.

The problem of controlling nonlinear systems expressed in the canonical form to follow a reference trajectory in the presence of uncertainties. FLSs are used to approximate the unknown dynamics of the system. This problem has been extensively investigated, however, most results reported in the literature suffer from at least one of the following drawbacks:

- lack of robustness to unmodeled dynamics and/or external perturbations due to only asymptotical convergence of the tracking error to a residual set of the origin is achieved
- requirement of the knowledge on the nonlinear systems for controller implementation, such as bounding functions on f(x) and b(x);
- requirement of the bound on the norm of the optimal parameter vector of the universal approximator, or a compact set to which the optimal parameter vector of the universal approximator belongs;
- heavy online computation burden due to updating the parameters of the universal approximator.

By combining advantages of FLSs, adaptive, and robust control techniques, an adaptive robust fuzzy control capable of achieving exponential convergence of the tracking error to a small ball of the origin, whose radius can be made arbitrarily small by properly choosing some design parameters.

From a practical point of view, exponential tracking is more desirable for its robustness against unmodeled dynamics and/or external perturbations. To implement the controller only the knowledge of b[a constant lower bound on b(x)], and a nominal parameter vector of the FLS are required. The nominal parameter vector may be obtained either from a priori knowledge of the plant or through offline training, and may be set zero if no a priori knowledge of the plant is available nor offline training is done. The online computation burden is also reduced since only uncertainty bounds are adaptively tuned online.
4.1.3 Integrating membership functions and fuzzy rule sets from multiple knowledge sources

Expert systems have been successfully applied to many fields and have shown excellent performance. Knowledge-base construction remains, however, one of the major costs in building an expert system even though many tools have been developed to help with knowledge acquisition. Building a knowledge-based system usually entails constructing new knowledge bases from scratch. The cost of the effort is high and will become prohibitive as we attempt to build larger and larger systems. Reusing and integrating available knowledge from a variety of sources, such as domain experts, historical documentary evidence, current records, or existing knowledge bases, thus plays an important role in building effective knowledge-based systems.

Especially for complex application problems, related domain knowledge is usually distributed among multiple sites, and no single site may have complete domain knowledge. The use of knowledge integrated from multiple knowledge sources is thus especially important to ensure comprehensive coverage.

Most knowledge sources or actual instances in real-world applications contain fuzzy or ambiguous information. Especially in domains such as medical or control domains, the boundaries of a piece of information used may not be clearly defined. Expressions of the domain knowledge using fuzzy descriptions are thus seen more and more frequently. Several researchers have recently investigated automatic generation of fuzzy classification rules and fuzzy membership functions using evolutionary algorithms [15, 16, 17, 18]. These methods can be categorized into the following four types:

- learning fuzzy membership functions with fixed fuzzy rules [19];
- learning fuzzy rules with fixed fuzzy membership functions [18, 20];
- learning fuzzy rules and membership functions in stages [16] (i.e., first evolving good fuzzy rule sets using fixed membership functions, then tuning membership functions using the derived fuzzy rule sets);
- learning fuzzy rules and membership functions simultaneously [15, 17].

4.1.4 Fuzzy Trees

Machine learning is the essential way to acquire intelligence for any computer system. Learning from examples, i.e. concepts acquisition is one of the most important branches of machine learning. It has been generally regarded as the bottle-neck of expert system development.

The induction of decision trees is an efficient way of learning from examples [21]. Many methods have been developed for constructing decision trees [22] and these methods are very useful in building knowledge-based expert systems. Cognitive uncertainties, such as vagueness and ambiguity, have been incorporated into the knowledge induction process by using fuzzy decision trees [23]. Fuzzy ID3 algorithm and its variants [24-27] are popular and efficient methods of making fuzzy decision trees.
The fuzzy ID3 can generate fuzzy decision trees without much computation. It has the great matching speed and is especially suitable for large-scale learning problems.

Most of existing algorithms for constructing (fuzzy) decision trees focus on the selection of expanded attributes (e.g. [25, 28, 21, 29]). They attempt to obtain a small-scale tree via the expanded attribute selection and to improve the classification accuracy for unknown cases. Generally, the smaller the scale of decision trees, the stronger their generalizing capability. It is possible that the reduction of decision tree scale results in the improvement of the classification accuracy for unknown cases. An important problem is whether or not there exists an exact algorithm for constructing the smallest-scale fuzzy decision tree.

Fuzzy logic systems have been successfully applied in many fields. The issue of how to form a fuzzy rule base directly from a given data set has attracted many researchers. Various different learning (non-constructive) approaches, such as in [30, 31] were applied to the construction of fuzzy rule bases representing a given fuzzy logic system. However, these non-constructive approaches may not guarantee finding proper fuzzy rules for a given squared error. On the other hand, a constructive approach, based on uniform continuity of the process $f$ for constructing a piecewise linear fuzzy logic system, was proposed in [32]. Unfortunately, this constructive approach depends on the process $f$ rather than on the fuzzy logic system itself, and is not effective in constructing a piecewise linear fuzzy logic system with near-optimal or optimal number of fuzzy rules.

In order to overcome the weaknesses of the constructive approach in [32], a direct constructive approach for constructing a single-input-single-output piecewise linear fuzzy logic system with near-optimal number of fuzzy rules under a fixed squared error can describe more complex patterns than strings do. Therefore, it is necessary to research a series of problems for fuzzy tree grammars. In syntax pattern recognition, the grammatical inference means that a grammar can be generated from a given sample set. Grammatical inference is an important and difficult task, even for inference of fuzzy tree grammars. An inference method for fuzzy tree grammars is described in [33,34]

4.1.5 Analysis and Design of Fuzzy Controller Based on Observer

During the last decade, fuzzy logic control has been attracted great attention from both the academic and industrial communities. Many people have developed a great deal of time and effort to both theoretical research and implementation techniques for fuzzy controllers. To date, fuzzy logic control has been suggested as an alternative approach to conventional control techniques for complex control systems.

Fuzzy logic control techniques suffer from problems:

- the design of the fuzzy logic control is difficult because no theoretical basis is available
- the performance of the fuzzy logic control can be inconsistent because the fuzzy logic control depends mainly on the individual operators' experience.
Therefore, despite the fact that much progress has been made in successfully applying fuzzy logic control to industrial control systems, it has become evident that many basic issues remain to be further addressed.

Stability analysis and systematic design are certainly among the most important issues to fuzzy control systems. Recently, the issue of stability of fuzzy control systems has been considered extensively in nonlinear stability frameworks. The stability analysis and robust fuzzy controllers design methods for a class of uncertain nonlinear systems was discussed in [35], which only considered the uncertainty of fuzzy model without considering the unobservable problem of states of systems. The stabilization of a feedback system containing a fuzzy controller and a fuzzy observer for Fuzzy systems for multi-input and multi-output linear systems was addressed in paper [36, 37,38], which only took into account of the state unobservable problem of the system without considering the uncertainty of fuzzy model. It is well known that the observer design and robust control are very important problems in control systems, the fuzzy observer design is hardly addressed in the present research. A very key problem is that the stability of the whole system must be guaranteed.

4.1.6 Fuzzy Observer-Based Control of Nonlinear Systems

Recently there has been a rapidly growing interest in using Takagi-Sugeno [39] fuzzy models to approximate nonlinear systems. These models consist of fuzzy If...Then rules with linguistic terms in antecedents, and analytic dynamical equations in the consequents. There has been a great deal of effort in trying to find conditions for stability of these type of control systems. The approach proposed by Sugeno and Tanaka [40, 41], uses a common quadratic Lyapunov function. The most important draw back of this method was that finding a common matrix that satisfies all Lyapunov inequalities is not easy. With the emergence of new optimization methods [42] that can solve Linear Matrix Inequalities (LMI's) in polynomial time, this problem has been solved.

Among many results in fuzzy control, Takagi and Sugeno introduced the now so-called T-S Fuzzy model. The T-S fuzzy systems have certain relations to conventional linear models, and therefore many classical control theory methods can be used to the analysis and synthesis of fuzzy systems. Many control strategies are achieved via state feedback control. As a result, Observer design plays a significant role in controller implementations. Fuzzy observer design of a class of T-S type fuzzy systems is discussed [43]. Modeling of such fuzzy systems and construction of their observers are introduced, the quadratic stability and stabilization of observer error dynamics are discussed. The results tare presented via a set of linear matrix inequalities. The observers can be constructed through solving the given linear matrix inequalities, which is comparatively easy.

- **Thau-Luenberger Observers for TS Fuzzy systems**

The development of observers for linear (Luenberger ) and nonlinear (Thau [44]) systems makes the state feedback control of plants possible in the case when only some system states are measurable. For TS fuzzy models, however, most of the design/analysis methods consider the case when the whole state vector is available for measurement. If this is not the case then the problem of designing an appropriate TS observer has to be
solved. Only during the last couple of years this issue has started to receive some attention. The usual approach to the fuzzy observer and controller problem is to describe the original nonlinear system as TS multiple model system, i.e. as a convex combination of linear state space systems. The convex combination can be a function of the states and/or the inputs to the system. A thorough analysis of stability and simultaneous design of fuzzy controllers and fuzzy observers based on such a system description can be found [45]. Linear matrix inequalities (LMI) is used as the main tool both for analysis and design such that certain performance criteria are met. It is however noted that simultaneous design of the gain controller and observer gains is difficult within the LMI framework when the convex combination function depends on the state variables. In [46] the authors consider the so-called separation property for a fuzzy controller and a fuzzy observer given a TS model of the system under control. The separation property guarantees that the fuzzy controller and the fuzzy observer can be separately designed and that the overall system including both of them is still stable. Although they have showed that the separation principle holds for certain conditions it is still lacking an explicit design procedure.

The LMI based design has been extended for guaranteeing observer stability with constraints on the regions of the eigenvalues of the observer error system. The common factor of the mentioned approach is that they rely on as TS model with linear consequents. Although some system may be exactly represented by such a model other nonlinear systems cannot and must in that case be linearized at some equilibrium points to get an approximation of the system in the desired form. The extension of the analysis presented in [47] with a LMI based design procedure for a so called fuzzy Thau-Luenberger observer based on a TS fuzzy model with affine consequents i.e. fuzzy model in consideration can be obtained by linearization at possible off-equilibrium points, which differ from the approaches mentioned above.

- **Sugeno-type Fuzzy Observers**

It is obtained by “fuzzily interconnecting” local linear Luneburger observers. The approach uses techniques of robust and particularly quadratic stabilization to show the global quadratic stability of the fuzzy observer and does not assume the linearity of the nonlinear process with respect to the inputs, which generally assumed for the conventional nonlinear observers [48].

In despite of impressive results on modeling and control the dual and very important problem of fuzzy observers has not yet attracted enough attention. These problem could be addresses from at least two points of views: (i) estimate the membership functions associated with different state variables tacking fuzzy values, (ii) construct in the line of Takagi-Sugeno models, nonlinear global fuzzy observer by fuzzily connecting linear local observer. It is the latter approach that we use in this paper. First the state space is divided into fuzzy subspaces. Then, on each fuzzy subspace a linear or affine local model and the corresponding Luneburger observer are defined. Finally we “fuzzily interconnect” local linear models using the Sugeno standard inference method to obtain the global model. Interconnecting local models, generates linear uncertain models with matched and unmatched nonlinear uncertainties [49,50]. These uncertainties represent interactions between local models and depend on the choice of membership functions. Assuming some bounds on these uncertainties, we use techniques of quadratic
stabilization and piecewise smooth continuous Lyapunov function, to deduce stability of the global error dynamics.

Since fuzzy logic systems are universal approximations (UA), adaptive control schemes for nonlinear systems that incorporate UAs have become increasingly popular. The main advantage of using UAs is to relax the linear-in-the-parameters condition on the uncertain nonlinearities of the controlled system. Thus, the class of nonlinear systems to which the adaptive control scheme can be applied is enlarged by using UAs. Another advantage is that we can use conventional and well-developed adaptive techniques with a robustifying control action which compensates for the reconstruction errors. Observer design has been a very active field during the last decade and has turned out to be much more difficult than the control problem. How to design a robust observer for the unknown nonlinear system is a very challenging problem.

In [51-53], adaptive observers using UAs are proposed. Their schemes need strictly positive real (SPR) conditions on the estimation error dynamics so that they can use the Meyer-Kalman-Yakubovic (MKY) lemma. The need of SPR conditions result is in the filtering of the regressor (or basis) vectors of the UAs, which makes the dynamic order of the observer very large. Moreover, the fixed structure of the UAs causes an unnecessarily larger dimension for the basis vectors. Our goal is to design a robust adaptive observer for a class of unknown nonlinear systems using fuzzy systems. We especially focus on the realization of the minimal dynamic order of the observer. For this purpose, we propose a new method in which no SPR condition is needed and combine a dynamic rule activation scheme with on-line estimation of fuzzy parameters. The update laws for fuzzy parameters and an additional robustifying term provide the Lyapunov stability for the closed-loop system, and guarantee that all signals involved are uniformly ultimately bounded.

4.1.7 Design and analysis of a fuzzy logic controller

People very often make decisions in their daily lives based on qualitative information. Zadeh's fuzzy sets theory was thus proposed to enable people to describe and formulate the linguistic mental models apparent in daily life behaviour. Mamdani and his coworkers (Mamdani 1974[54], Mamdani and Assilian 1975[55], King and Mamdani 1976 [56]) were pioneers in applying fuzzy techniques to process control. Their results, as well as those of many other researchers, have demonstrated the potential value of the fuzzy logic control system on simple process dynamics. Practical fuzzy logic control applications on industrial process plants have also been reported in turn. A comprehensive review of the classical design and implementation of the fuzzy logic controller can be found in (Lee 1990[57]). More advanced design techniques have also been reported in the literature, such as the adaptive fuzzy logic control (Batur and Kasparian 1991[58]), the hierarchical fuzzy logic control (Raju et al. 1991[59] and the fuzzy logic controller with multiple inputs (Wong et al. 1993[60]).

The superior performance of fuzzy logic controllers reported in the literature usually conjectured to have its origin in their switching nature (Tang and Mulholland 1987[61]), where the magnitude of the rate of change in controller output is greater for
larger error and small for process output close to set point. The nonlinearity and complexity of fuzzy control responses, however, causes the analysis and systematic tuning of fuzzy logic controllers to remain a difficult research problem.

The relations between fuzzy and conventional three-mode (proportional-integral-derivative, PID) controllers have been studied by several authors (Siler and Ying 1989[62], Ying et al 1990[63]). Some specialized fuzzy logic controllers, such as the simplest possible structure (Ying et al. 1990[63]), have been proven to be equivalent to a nonlinear two-mode PI controller with state-dependent variable controller gains. A simplified structure of the fuzzy logic controller proposed by Siler and Ying (1989[62]) is investigated in [63]. A uniformly distributed triangular family of membership functions is applied for linguistic members of each fuzzy variable, and all control laws are expressed in a simple form in such a controller. Such a simplified design makes the number of undetermined fuzzy members the sole design variable, and the input/output scalars along the most relevant tuning variables in the controller. Some characteristic properties of such an FLC are derived and explored in this paper. Such an FLC, no matter the number of fuzzy members it uses, is proved to be functionally equivalent to a non-fuzzy nonlinear PI or PD controller with state-dependent controller gains and a value of integral (derivative) time dependent on input scalars only. An investigation of its servo-control performance follows analysis. Its best achievable performance, in the sense of minimum integral of absolute error (IAE), is found in general on the same level as that of the conventional PI controller. Several extensions of the basic FLC, including the FLC with dual control laws (PI and PD forms) by including one switching factor, and the FLC with varying gains according to the discrepancy between process variables and set point, are thus proposed to enhance servo-control performance. The superior results obtained employing various extensions to the FLC are illustrated by several numerical examples. A neutralization process is also employed to demonstrate the potential applicability of the fuzzy logic control method on industrial process control problems.

4.1.8 Sliding Mode Observers for Takagi–Sugeno Fuzzy Systems

A dynamic TS [64] fuzzy system is composed of multiple local affine dynamic linear models. These local models, for the purpose of this paper are related to local linearizations, via Taylor series expansion, of the original nonlinear system at off-equilibrium points [65]. Thus, the local models have no equilibrium point within their regions of validity, i.e., they are off-equilibrium local models (for detailed exposition see [66]). As shown in [67], incorporating off-equilibrium linearization in the model can significantly improve the transient response in closed-loop control.

In [366] it is proved that a TS fuzzy system where the local affine dynamic models are off-equilibrium local linearizations leads to an arbitrary close approximation of the linear time varying (LTV) dynamic system resulting from dynamic linearization of the original nonlinear system about an arbitrary trajectory (see also [68] for related results). Thus, the results concerning observers for TS fuzzy systems are also relevant to systems such as linear parameter varying (LPV) systems, piecewise linear systems, and conventional gain-scheduled systems.
TS fuzzy system is subject to observation and defines a Luenberger type of observer, which may be realized in terms of a parallel distributed compensation scheme incorporating an interpolation between local Luenberger observers. This is the type of nonlinear observer that has received most attention in the fuzzy control literature, but all results reported assume no matched/unmatched uncertainties.

A sliding mode fuzzy observer that is related to the so-called min-max observer described by Zak and Walkot [69] utilizes interpolation between local observer gains which has one major negative effect: large number of local models will give a large number of linear matrix inequalities (LMI) in the stability analysis and design, which may prohibit the use of existing LMI tools.

TS fuzzy system may be reduced to a dominant linear one, that is one local model is chosen and the effect of the rest are incorporated in it in terms of known deviations interpreted as uncertainties. This avoids the use of LMIs for analysis and design and instead a direct sliding mode observer design is possible. The model mismatches are taken into account as known upper bounds of matched and unmatched uncertainties. The results are comparable to the ones produced by a fuzzy Thau–Luenberger observer for the same system.

4.1.9 LMI-Based Design of T-S Fuzzy Estimator based Controllers

Tanaka and Sugeno [71] proposed a theorem on the stability analysis of T-S fuzzy model. Later Wang et al. [70] proposed the so-called PDC as a design framework and also modified the Tanaka’s stability theorem to include the effect of control. An important observation in the paper is that the stability problem is a standard feasibility problem with several LMIs when the feedback gains are pre-determined and can be solved numerically using an algorithm named interior-point method.

They are, however, NMIs (Nonlinear Matrix Inequalities) when the feedback gains are treated as unknowns. Recently Joh et al.'s [72] (B) converted the NMIs to LMIs for both of continuous and discrete T-S fuzzy controllers by applying the Schur complements (3. page 7) to the Wang et al.’s stability criterion and named it as stability LMIs. And they proposed a systematic design method based on the stability LMIs for T-S fuzzy controllers which guarantees global asymptotic stability and satisfies desired performance of the closed-loop system.

Joh et al.'s (~ [73] assumed, however, that all the state variables are accessible. The fuzzy state estimator is proposed as a T-S type fuzzy rules using Wang et al.’s PDC structure to estimate the inaccessible states. In particular, a systematic design method for PDC - Fuzzy controllers with inaccessible states is obtained by combining LMIs for fuzzy state estimator and LMIs for control.

4.1.10 Fuzzy Regulators and Fuzzy Observers: Relaxed Stability Conditions and LMI-Based Designs

The issue of stability of fuzzy control systems has been considered extensively in nonlinear stability frameworks [74]–[78], [79], [80]–[83]. Specially, the stabilization of a
feedback system containing a fuzzy regulator and a fuzzy observer for discrete fuzzy systems was discussed in [79]. The work of [79] and, more importantly, new relaxed stability conditions and LMI- (linear matrix inequality) based design procedures are obtained for both continuous and discrete fuzzy systems. The stability analysis and design procedures proposed here are straightforward and natural, although the nonlinear regulator and observer design is difficult in general. Linear regulators and linear observers [84] play an important role in modern control theory and practice. We envision that a systematic design method of fuzzy regulators and fuzzy observers would be important for fuzzy control as well. To begin with, Takagi–Sugeno (T–S) fuzzy models and previous stability results are recalled. To design fuzzy regulators and fuzzy observers, nonlinear systems are represented by T–S fuzzy models. The concept of parallel distributed compensation (PDC) [74], [75], [76] is used to design fuzzy regulators and fuzzy observers from the T–S fuzzy models. [85] derive new stability conditions by relaxing the ones in our previous works [74], [77]. LMI-based design procedures for fuzzy regulators and fuzzy observers are constructed using the PDC and the relaxed stability conditions. Other LMI’s with respect to decay rate and constraints on control input and output are also derived and utilized in the design procedures. Design examples for nonlinear systems demonstrate the utility of the relaxed stability conditions and the LMI-based design procedures are described in [85].

4.1.11 Fuzzy Model Predictive Control

MODEL predictive control (MPC) has emerged as one of the most attractive control techniques in the chemical and petrochemical industries during the past decade. In MPC, a process dynamic model is used to predict future outputs over a prescribed period [86], [87]. Dynamic matrix control [89], model algorithmic control [88], and simplified model predictive control [90] are excellent examples that have been applied to various industrial processes [91], [92]. Continuous and batch processes in chemical and petrochemical plants are inherently nonlinear and many of them are highly nonlinear. For a highly nonlinear system, a linear MPC algorithm may not give rise to satisfactory dynamic performance.

Recently, several researchers[93] have developed nonlinear model predictive control (NMPC) algorithms that accept various kinds of nonlinear models such as nonlinear ordinary differential/algebraic equations, partial differential/algebraic equations, integrodifferential equations, and delay equation models. Such models can be accurate over a wide range of operating conditions. However, these models, usually based on the first principles, are very difficult to develop for many industrial cases. Moreover, an NMPC incorporating a nonlinear model may require tremendous computational effort for optimization; this may disqualify it for on-line applications. If a nonlinear process can be precisely described by a set of linear submodels in someway, then the design of a model predictive controller can be greatly simplified. Reference [94] introduced a novel fuzzy logic-based modeling methodology, where a nonlinear system is divided into a number of linear or nearly linear subsystems. A quasi-linear empirical model is then developed by means of fuzzy logic for each subsystem. The model is a rule-based fuzzy implication (FI).
The whole process behavior is characterized by a weighted sum of the outputs from all quasi-linear FIs. The methodology facilitates the development of a nonlinear model that is essentially a collection of a number of quasi-linear models regulated by fuzzy logic. It also provides an opportunity to simplify the design of model predictive controllers. Reference [95] developed an MPC algorithm using a Takagi–Sugeno (T–S) type model. However, tremendous difficulties have been found in tuning controller parameters since the algorithm requires frequent model updating in control. More recently, [96] proposed an approach for designing a fuzzy model-based state–space feedback controller. A T–S type model is the basis of their fuzzy model. However, they essentially treated the fuzzy model as a set of conventional piecewise linear models. Thus, the uniqueness of a Takagi–Sugeno type model exhibiting soft transition through any operating regions is lost, causing deterioration in the closed-loop dynamic performance of a system.

In [97] a fuzzy model predictive control (FMPC) approach is introduced to design a control system for a highly non-linear process system. The approach utilizes the Takagi–Sugeno modeling methodology to generate a fuzzy convolution model. With this model, a novel hierarchical control design approach is described. Three case studies are provided to demonstrate the attractiveness of the FMPC.

4.2 Review of work based on ANN

This section reviews the foundation and techniques developed and used by the researchers for design, development, simulation and development of ANN based robust adaptive observers.

4.2.1 ANN Based Adaptive Extended Luenberger Observers

The potential of powerful mapping and representational capabilities of artificial neural network architectures has long been recognized in the neural network community [98]. The introduction of back propagation algorithm by Rumelhart et.al. enabled real-time applications of neural networks as adaptive systems [99]. Neural networks (NN) are being recognized in other fields of research including signal processing, communications, and control as valuable tools that offer simple solutions to difficult problems [100-102].

Especially in the area of adaptive controls, neural networks have experienced an increased interest in the last decade, due to their inherent adaptability and universal approximation properties. This interest was mainly ignited by the early works of Werbos, Shoureshi et.al. Narendra and Parthasarathy, Gupta and Rao, and Miller et.al. [103-107]. although these first attempts were mostly heuristic in nature, there has been a stream of publications inspired by the idea, involving deeper analyses. These later works have mostly focused on the application of recurrent neural networks for system identification and observer design, as well as adaptive and robust controllers for general nonlinear systems.
In [108], Puskorius and Feldkamp investigate the application of recurrent multiplayer perceptrons (MLP) to the control of nonlinear dynamical systems and propose an alternative training algorithm to update the dynamic weights of the network based on parameter-based extended Kalman filter (EKF) estimates.

Their simulation results with a number of nonlinear systems favor the use of EKF-based training algorithm over the conventional backpropagation. Zhu et al. focus on the application of dynamic recurrent neural networks (DRNN) as observers for nonlinear systems [109]. They consider a class of single-input-single-output (SISO) nonlinear time-varying systems in their work, where they prove the boundedness of the observer error and the DRNN weights during adaptation using Lyapunov stability theory and the well-known universal approximation theorem for neural networks. Wang and Wu exploit the multiplayer recurrent neural networks as matrix equation solvers and utilize this scheme to synthesize linear state observers in real-time by solving the Sylvester's equation for pole placement [110].

There are also examples of static feedforward neural network applications to observer and controller design. Ahmed and Riyaz consider an off-line training scheme for a MLP based observer design for nonlinear systems. They note that although the NN observer requires more computation in the training phase, it is more computationally efficient compared to the EKF in the implementation phase [111].

An interesting approach is presented in [112], employ linearly parametrized neural networks (LPNN) for the design of an adaptive observer for general nonlinear systems. LPNN include a wide class of networks including radial basis function (RBF) networks, adaptive fuzzy systems (with specific choices of rules and membership functions it can be shown that fuzzy systems are equivalent to RBF networks), and wavelet networks. They use Lyapunov stability theory to prove the stability of the observer and the neural network weights and demonstrate the performance of the designed observer on a single-link robot manipulator model. Fretheim et al. on the other hand, utilize the feedforward MLP in the observer design problem with a little twist.

They formulate the problem as a multi-step prediction, and exploit the extrapolation capabilities of the MLP to obtain the state estimates [113]. Ge et al., in their contribution to neurocontrol, use RBF networks in an adaptive output feedback NN controller scheme, for which they provide proofs and simulation studies for bounded tracking errors given sufficiently large networks [114]. Hovakimyan et al. utilize a feedforward MLP in the prediction-framework to obtain a model of the unknown nonlinear dynamical system.

On the other side, some researchers followed a more conservative approach to nonlinear observer design. Mostly inspired by the idea of extended Kalman filtering, the Luenberger observer was extended to nonlinear systems and its convergence properties were studied [115-116].

These extensions, under the influence of the classical observer design theory, were focused on analytical design techniques of the nonlinear observer. The main approach followed in this line of practice is to choose the observer gains such that the
overall linearized error dynamics matrix consisting of the gain vector and the Jacobians of the state dynamics, and the output mapping has stable eigenvalues over a closed subset \( c_0 \) of the state space.

For convergence, the state trajectory is constrained to remain in this subset at all times. This procedure of analytic ELO design has been applied successfully to realistic nonlinear systems. It has been utilized in designing nonlinear state feedback stabilizers for nonlinear systems [117-118].

4.2.2 Neuro-Observer Controller Design for Nonlinear Dynamical Systems

The control of systems with complex, unknown and nonlinear dynamics has become a topic of considerable importance. The controller designs for these control systems can be sliding mode control, adaptive control, fuzzy control and neural network control.

Neural networks are capable of approximating any nonlinear continuous functions to arbitrary accuracy and may be good candidates for implementing a real-time controller for a class of complex, unknown and nonlinear dynamical systems. Although the neural networks are capable of approximating any continuous nonlinear functions to an arbitrary accuracy, there still has mismatch between the neural networks model and the controlled system (i.e., the molding error).

From the consideration of on-line control, the order (or structure) of control law should be as small (or easy) as possible to reduce the computational burden and to increase the convergent rate of estimated parameters. Because the neural networks suffer from the problem of slow learning, their use in on-line control should be made with caution [119-121]. Hence, the concept of robust sliding mode control law, including a time-varying switching gain, a time-varying boundary layer and an integration of tracking error in the dynamics of the sliding surface, is plunged into the proposed control scheme to fulfill an acceptable tracking performance for the complex, unknown and nonlinear dynamical systems.

It is well-known that the controls of neural network require the strong assumption that all the inputs of their input layer are available. Hence, an adaptive nonlinear observer is suggested to attain the estimated states which are required for the inputs of the neural network. [121]In this paper, a single-layered radial basis function network (RBFN) is applied to approximate the nonlinear dynamical systems [120-122]. Because the RBFN results in nonlinear maps in which the parameters occur linearly the on-line control law for adjusting are substantially simplified, its learning of weighting parameters is expected to be faster, and the local stability of the overall control system can be demonstrated by the Lyapunov stability criteria. Although the condition of persistent excitation (PE) in the radial basis function network identification of the nonlinear systems is accomplished by Gorinevsky [122], it is difficult to guarantee the condition of PE in the (proposed) closed-loop control system. Hence, a pretraining of the nonlinear dynamical systems, via a random reference input or an extra dither input combined with the control input, is employed to ensure a quick convergence of the weighting parameters and then to improve the system performance.
4.2.3 A General Back propagation Algorithm

Feedforward neural networks (FNN) have been widely used for various tasks, such as pattern recognition, function approximation, dynamical modeling, data mining, and time series forecasting. The training of FNN is mainly undertaken using the back propagation (BP)-based learning algorithms.

A number of different kinds of BP learning algorithms have been proposed, such as an on-line neural-network learning algorithm for dealing with time varying inputs [123], fast learning algorithms based on gradient descent of neuron space [124], and the Levenberg–Marquardt algorithm [125-126], [127] In this letter, we will develop a general BP learning algorithm for FNN with time varying inputs. This algorithm unifies variations of the BP learning algorithms.

The Lyapunov function approach, which has been widely used in analyzing the stability of self-organizing neural networks such as Kohonen and Hopfield types of networks [128-130] are used to derive conditions to guarantee the convergence of weights. In [127] it is shown that trapping into local minima is inherent with the learning algorithms based on the BP principle as they may only enable the weights to converge to global minima if it happens that either the initial weights are near a global minimum or the geometric distribution of weights enables the weights to converge to a global minimum. We will also show that major classes of BP learning algorithms are special cases of the developed learning algorithm.

4.2.4 ANN based Adaptive Extended Kalman Filter

In developing many feedback control schemes, e.g., Linear Quadratic Gaussian (LQG) and pole placement, a state-estimation scheme is often employed. One of the most common techniques for state estimation of nonlinear systems is the extended Kalman filter (EKF). However, in order to implement a standard EKF, a priori knowledge of the plant model is required to compute both the prediction of the state estimate and its Jacobian.

Often, the plant model is not completely known due to mismodeling, extreme nonlinearities, or changes in the system's parameters resulting from wear or damage. To perform state estimation when such conditions occur, we propose to implement and EKF whose plant model is augmented by an artificial neural network (ANN). The ANN will capture the unmodeled dynamics by "learning," on-line, the function that describes the difference between the true plant and the best a priori guess of the model chosen for the standard EKF implementation. Unlike many neuro-identification techniques, which train off-line [131-132], the neuro-observer described in this paper trains on-line while control is being applied.

The ANN in the neuro-observer trains using an EKF training paradigm. In [133] an EKF for state estimation and an EKF for ANN training Two EKFs are coupled so that
a single EKF will be used that will pull double duty by simultaneously estimating the states and training the weights of the ANN.

The state observation and the control problems are one of the essential points in the modern control theory. For plants with unknown parameters the nonlinear adaptive observer was proposed. When we have no complete modeling information, a model-free nonlinear observer is required. If the nonlinear system is given in the normal linearized form, the high-gain observers may estimate the derivative of the output. They do their job well even if the plant is being considered as "a black box", but such observers loose their capability in presence of output unmeasured disturbances.

Neural networks can be considered as an alternative approach to high-gain method since they offer potential benefits for nonlinear modeling and control of a wide class of unknown system Neuro-control, based on the neuro identifiers, turns out to be very useful for designing model-free controllers. Dynamic neural networks were also applied to design a Luenberger-like observer. The stability of this observer with on-line updating of neural network weights was analyzed, but several restrictive assumptions were required: the nonlinear plant should contain a known linear part and verify a strictly positive real (SPR) condition to proof observation error stability.

4.2.5 Neural Adaptive Observer for General Nonlinear Systems

NN-based adaptive observers are being investigated. This approach allows the replacement of parts of the unknown plant dynamics by NN (plus small approximation error terms), that have known structure and unknown weights, hence the parameterization problem is simplified and the adaptive observer design boils down to the choice of appropriate robust adaptation laws for the weights. However, most of the results reported only can be applied for some classes of SISO nonlinear systems. A neural adaptive observer for SISO systems with a single unknown static nonlinearity is proposed. SISO systems in the Brunovsky canonical form are considered and it is supposed that the output error equation is SPR, which are restrictive assumptions that do not hold in general for nonlinear systems.

Recently in [134] the authors proposed a neural adaptive observer for a fairly large class of MIMO nonlinear systems, in which an unknown general state equation is considered. Furthermore, no SPR condition on output error equation was required. However, it was assumed that the output equation is a linear function of the states. [134] extends the result in order to tackle more general nonlinear systems. Two linearly parameterized neural networks are used to capture the unknown dynamics of the system, where their weights are adjusted via robust adaptation laws (defined by using the estimated states, inputs and measured outputs) in order to guarantee the stability of the overall scheme.

4.2.6 ANN Based Adaptive Observer Back-stepping Control

During the past three decades, parametric adaptive control theory [135]–[137] has been extensively developed for linear systems with parametric uncertainty. This research has provided fundamental techniques for the design and analysis of adaptive systems.
Recently, novel adaptive control design methods have been proposed for several classes of nonlinear systems [138]. These remarkable design methods such as backstepping design [139], tuning function design [140], and modular design with swapping identifiers [141] provide useful techniques for research on nonlinear adaptive control systems with parametric uncertainties. Nonparametric adaptive control methods have also been actively studied using neural networks such as sigmoidal neural networks [142] and radial basis function networks [143]. In early research, the feasibility of neural networks for the identification and control of nonlinear dynamical systems was investigated in offline environments.

More recently, theoretical frameworks for on-line adaptive control using neural networks have been developed for a class of nonlinear systems. These nonparametric adaptive schemes guarantee the stability of the systems by using Lyapunov design methods. Most of these neural control methods have been applied to a class of nonlinear system diffeomorphic to the standard form where the unknown nonlinearities appear in the same equation as the control input. This condition is usually referred to as a matching condition. In addition, the neuro-control literature assumes that the states of the system are directly measured. In practice, the states might not be directly measured.

Recently, a new neural adaptive control method has been suggested for a system not satisfying a matching condition, but this method still requires the state to be directly measured. As for the similar issues, neural-network output feedback control schemes have been proposed for robot manipulators described by second-order systems [144] and for the nonlinear systems diffeomorphic to the standard form in [145].

[146] addresses the adaptive output feedback control problem. In such problems, the system does not satisfy a matching condition and elements of the state vector are not directly measured. Recently the same problem has been researched independently [146], [147]. In [146], the uncertainty is described by a weighted norm and the uncertainty level should be known. The approach developed in [148], uses a backstepping approach with nonlinear damping to accommodate unmodeled nonlinear effects, a dynamic bounding signal to accommodate unmodeled dynamics, and adaptation to accommodate parameter uncertainty. The approach requires that the uncertain nonlinearities be bounded by a linear combination of two known functions with unknown positive coefficients. The parameter adaptive laws are derived during the backstepping control design procedure. The problem addressed in [149], is more general than the problem that we discuss herein, as the present article does not include the analysis of unmodeled dynamics. The main focus of [148] is on the development of new methods and insight for addressing unmodeled nonlinear effects in output feedback control problems. In this paper, a different approach is proposed for functional uncertainty in the space and knowledge of the bound on the uncertainty is not required. The proposed neural-based adaptive observer identifies unknown functions in the system and estimates the unmeasured states.

The adaptive observer is designed so that the state estimation accuracy increases as the function approximation accuracy increases. The same estimated functions are used to improve the performance of a backstepping control law. The basic structure of the adaptive observer follows the nonlinear model estimation method for automated fault diagnosis presented in [149]. [148] modifies the method so that it can be applied to adaptive control for nonlinear systems diffeomorphic to the output feedback form [138].
The semiglobal stability of the overall systems including the observer-based controller is analyzed by Lyapunov stability theory. The feasibility and performance of the proposed approach is investigated by an illustrative numerical simulation example.

4.3 Neuro-Fuzzy Systems

This section deals with the brief description of the foundation and techniques developed and used by the researchers to combine the fuzzy logic and ANN i.e. Neural Fuzzy or Fuzzy Neural and Hybrid set up and structures for the training, design, development, simulation and development of robust adaptive systems for non linear systems.

4.3.1 Neuro-fuzzy learning algorithms by gradient descent method

In recent fuzzy applications, it is becoming more important to consider how to design optimal fuzzy rules from training data, in order to construct a reasonable and suitable fuzzy system model for identifying the corresponding practical systems. Normally, fuzzy rules are decided by experts or operators according to their knowledge or experiences, in general. However, when a fuzzy system model is designed, it is sometimes too hard or impossible for human beings to give desired fuzzy rules or MBF due to the ambiguity, uncertainty or complexity of the identifying system. Due to the above reasons, it is natural and necessary to generate or tune fuzzy rules by some learning technique.

By means of the back propagation algorithm of a neural network the so-called neuro-fuzzy learning algorithms which are widely used in recent fuzzy applications for generating or tuning optimal fuzzy system models, have been proposed by Ichihashi [150, 151], Nomura et al. [152, 153], Wang and Mendel [154, 155], independently. By one of the neuro-fuzzy learning algorithms, the fuzzy rules can be generated or tuned for constructing an optimal fuzzy system model, which is used to identify the corresponding practical system [156, 157].

we may review the approaches of conventional neuro-fuzzy learning algorithms, and then try to analyze their characteristics and give a summarization of the properties in detail. Some of their properties show that the uses of the conventional neuro-fuzzy learning algorithms are difficult or inconvenient sometimes for learning a fuzzy system model in practical fuzzy applications.

4.3.2 FUNCOM: A constrained learning algorithm for FNN

It is recognized that neural networks and fuzzy logic systems can be regarded as model-free approximators. In particular, both multilayer neural networks [158] and fuzzy inference systems [159] can approximate any real nonlinear function to an arbitrary degree of accuracy. Hence, they can be employed in complex processes of the real world, where a great amount of imprecision and uncertainty is encountered.

Recently, the fuzzy reasoning suggested by Sugeno et al. [160] (TSK fuzzy reasoning) has been gaining increasing interest in several practical applications of fuzzy
modeling and control. The so-called quasinonlinear fuzzy models (QNFM) and quasilinear fuzzy models (QLFM) [161] are based on this approach. An attempt to combine the computational power of neural networks with the inference attributes of the fuzzy logic systems has led to the fuzzy neural network (FNN), an intelligent tool where expert knowledge is incorporated in the form of IF\{THEN rules.

The fuzzy model identification, involves two issues:
- establishing a suitable model structure
- Tuning the model parameters such that a desired input/output mapping is accomplished.

Thus, for an effective training scheme two major issues have to be properly addressed: the structure identification and the parameter identification. The structure identification (structure learning) task consists of decomposing the premise space into fuzzy regions (rules). Then, having determined a certain input partition, parameter identification (parameter learning) is performed to adjust the premise / consequent parameters such that a prescribed error measure is minimized.

The popular Back Propagation (BP) method is a gradient based algorithm which is usually used to perform parameter learning of both neural networks and fuzzy neural systems. BP is a simple, well established and easily applicable optimization method; the learning task is accomplished by minimizing a single objective function, E, representing the degree of input/desired output matching. Despite its simplicity, BP exhibits certain disadvantages, such as:
- it shows a low speed of convergence,
- most often it becomes trapped to local minima of the error surface,
- its learning performance is seriously affected by the network complexity.

In this context, a number of standard BP variants have been suggested in literature [162-165] which exhibit improved performance characteristics by remedying the drawbacks inherent to gradient descent learning. Most of these algorithms perform unconstrained optimization of a single cost function, that is, no additional information is taken into account apart from the input/output error measure and the architectural constraints.

A novel algorithm with enhanced learning capabilities is suggested in [170] for training fuzzy neural networks (FNN). The learning task is formulated here as a constrained optimization problem which has been solved in [166], based on optimal control theory concepts. Apart from the usual error measure function E, an additional nonlinear functional \( \phi \) is introduced, called the pay-off function. At each epoch, the suggested algorithm calculates the weight changes in such a way that the following goals are achieved, simultaneously: Minimize the cost function E to establish the proper input/output mapping while at the same time optimizing the pay-off function \( \phi \) such that some additional objectives are met.

Optimization is carried out in terms of the network weights and the state variables; the former are regarded as control or decision variables while the latter describe certain internal functionalities and are appropriately selected according to the particular FNN structure under consideration. The state and control variables are related...
through a system of state equations which represent the architectural constraints of the problem. Finally, a fuzzy adaptation scheme is designed, which continuously modifies the step size during training. This mechanism facilitates the searching procedure and prevents them from being trapped to local minima of the error surface.

Formulation of the pay-off function $\varphi$ is a key issue in the suggested algorithm. $\varphi$ is generally expressed in terms of the system input/output vectors and the model parameters and can serve as a mean for the introduction of certain identification and control objectives into the learning algorithm, according to the design requirements.

$\varphi$ can also implement other relations which permit the formulation of suitable internal representations, such as the partition quality of the QLFM or QNFM [167], the correlation between the parameters and the system outputs [168,169], etc. In view of the above considerations, the suggested algorithm has been named FUNCOM which stands for: fuzzy neural constrained optimization method. FUNCOM is a robust, fast learning algorithm, suitable for training FNN constructs to accomplish several tasks, such as identification and control of complex plants.

4.3.3 A fuzzy back propagation algorithm

Recently, the interest in neural networks has grown dramatically: it is expected that neural network models will be useful both as models of real brain functions and as computational devices. One of the most popular neural networks is the layered feedforward neural network with a backpropagation (BP) least mean-square learning algorithm.

For the purpose of multicriteria analysis [171-173] a hierarchy of criteria is used to determine an overall pattern evaluation. The hierarchy can be encoded into a hierarchical neural network where each neuron corresponds to a criterion. The input neurons of the network corresponds to single criterion. The hidden and output neurons correspond to complex criteria. As evaluation function it can be used as the net function of the neurons. However, the criteria can be combined linearly when it is assumed that they are independent. But in practice the criteria are correlated to some degree. The linear evaluation function is unable to capture relationship between the criteria.

In order to overcome this drawback of standard backpropagation (SBP) algorithm [174] propose a fuzzy extension called fuzzy backpropagation (FBP) algorithm. It determines the net value through the fuzzy integral of Sugeno [175] and thus does not assume independence between the criteria.

Another advantage of FBP algorithm is that it reaches always forward to the target value without oscillations and there is no possibility to fall into local minimum. Necessary and sufficient conditions for convergence of FBP algorithm for single-output networks in case of single- and multiple-training patterns are proved. The results of computer simulation are reported and analyzed: FBP algorithm shows considerably greater convergence rate compared to SBP algorithm.
4.3.4 Neuro-fuzzy learning algorithms for tuning fuzzy rules

Fuzzy rules are decided by experts or operators according to their knowledge or experiences, in general. However, when a fuzzy system model is designed, it is sometimes too hard or impossible for human beings to give the desired fuzzy rules or membership functions, due to the ambiguity, uncertainty or complexity of the identifying system. Due to the above reasons, it is natural and necessary to generate or tune fuzzy rules by some learning technique. Neuro-fuzzy learning algorithms are widely used in recent fuzzy applications for generating or tuning optimal fuzzy system models. They have been proposed by Ichihashi [176-177], Nomura et al. [178-179], Wang and Mendel [180,181], independently. By one of the neuro-fuzzy learning algorithms, the fuzzy rules can be generated or tuned for constructing an optimal fuzzy system model, which is used to identify the corresponding practical system.

As discussed and illustrated in [182,183], while these neuro-fuzzy techniques have high generative ability for tuning fuzzy rules, it is sometimes not reasonable and suitable for practical fuzzy applications, due to some of their disadvantages such as the weak-firing or non-firing and the difficulty of the representation of fuzzy rules in the form of fuzzy rule table used widely in fuzzy logic controls, and so on.

In order to improve the problems of the conventional neuro-fuzzy learning algorithms [184] in this paper we develop a new approach of neuro-fuzzy learning algorithm. The main advantages of this approach are that, firstly, the tuning parameters in the fuzzy rules can be learned without changing the fuzzy rule table form, and, secondly, the case of the weak-firing or non-firing can be well avoided, which are different from the conventional neuro-fuzzy learning algorithms. Moreover, some properties of the developed approaches are also discussed corresponding to one of the conventional neuro-fuzzy learning algorithms. Furthermore, the so-called Gaussian-type neuro-fuzzy learning algorithm and Triangular-type neuro-fuzzy learning algorithm are formulated. Finally, the efficiency of the developed approach is illustrated by means of identifying non-linear functions.

4.3.5 Fuzzy Modeling Using FNN with the Back-Propagation Algorithm

Fuzzy modeling is the method of describing the characteristics of a system using fuzzy inference rules. The method has a distinguishing feature in that it can express complex nonlinear systems linguistically. It is, however, very hard to identify the fuzzy rules and tune the membership functions of the fuzzy reasoning. A great deal of research on neural networks (NN’s) utilizing their learning capability for automatic identification/tuning has been done [185-191].

The NN’s in [185-187] have the following problems:
- It is difficult to identify the fuzzy inference rules and tune the membership functions simultaneously. The learning of the networks is time consuming.
- The fuzzy rules are implicitly acquired in the networks.
In [193] three types of NN structures are proposed, of which the connection weights have particular meanings for acquiring fuzzy inference rules and for tuning membership functions [188-191]. The new networks are based on the use of the back-propagation (BP) algorithm [192], and they can acquire the fuzzy inference rules and tune the membership functions simultaneously through learning from experts’ inference data. The new networks can be categorized into fuzzy neural networks (FNN’s). The rules identified by the FNN’s are very easy to comprehend.

[193] presents a fuzzy modeling method using FNN’s with the Bp algorithm. The new method can identify the fuzzy model of a nonlinear system automatically. The three types of models identified by the three types of FNN’s are very easy to understand. A new criterion of r identifying the structures of the models is also introduced in this paper. The new one is effective in evaluating the generalities as well as the accuracies of the models. The feasibility for the methods is examined through modeling of a nonlinear system.

4.4 Separation principle: for the analysis & design of fuzzy controller & observer

The objective of this section is to develop a concept of separation property for the design of Fuzzy logic controller and observer, which helps in designing both of them separately as in normal control system. Analysis & Design of the Fuzzy Controller and Observer based on Takagi-Sugeno (T-S) fuzzy model is also discussed. A numerical simulation is also carried out to illustrate performance of the fuzzy controller & the fuzzy observer.

Since the last decade fuzzy logic based systems are gaining more attention from the scientific and industrial community. The fuzzy control departs significantly from the traditional control theory, which is essentially based on the mathematical models of the controlled process. Instead of deriving controller via modeling the controlled process quantitatively and mathematically, the fuzzy control methodology tries to establish a controller directly from the domain experts or operators, who are controlling the process manually and successfully. Other way it can be said that it’s a expert system where primary attention is paid to the human’s behaviour and experience rather than to the process to be controlled. It is this distinctive feature that makes fuzzy control applicable and attractive for dealing with those problems where the process is so complex and ill-defined that is either impossible or too expensive to derive a mathematical model, which is accurate and simple enough to be used by the traditional control methods but the process may be controlled satisfactorily by the operators. This in turn leads to the fact that is has lack of theoretical basis and also that its performance is inconsistent.

Based on the above concept some fuzzy models based on the fuzzy control system design methods have appeared in fuzzy control field [194-196]. Linear feedback control methods can be utilized as in the case of feedback stabilization.

The procedure for that is as follows:
First, the non-linear plant is represented by its T-S type fuzzy model. In this type of fuzzy 
model local dynamics in different state space regions are represented by linear models. The 
overall model of the system is obtained by combining these linear models using non-linear 
fuzzy membership functions. The controller design is carried out using the parallel 
distributed compensation scheme. The resulting overall controller is non-linear in general 
and is again a fuzzy combination of each individual linear controller. The same procedure is 
used to design a fuzzy observer. The important point in the paper is the separation property, 
which helps in designing the fuzzy controller and the fuzzy observer independently.

- **Plant Definition:** Obtaining mathematical models of the complex physical systems 
can be difficult or sometimes impossible too. But many of these systems can be 
expressed in some form of the mathematical model locally. Takagi and Sugeno have 
proposed a fuzzy model to describe the complex system [197]. In this paper we have 
consider a dynamic model to represent a complex MIMO system, which has both 
local analytic linear model and fuzzy membership functions. T-S dynamic model is 
described by the fuzzy IF-THEN rules, which locally represents linear input-output 
relations of nonlinear systems.

The $i^{th}$ rule of the fuzzy model is:

**Plant Rule 1:**

**IF** $m_1(t)$ is $F_i$ and ... and $m_g(t)$ is $F_g$

**THEN**

$$x(t) = A_i x(t) + B_i u(t)$$

$$y(t) = C_i x(t)$$

(4.1)

Where $F_g (j = 1, 2, \ldots, g)$ are fuzzy sets, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the 
input vector and $y_i(t) \in \mathbb{R}^p$ is the output vector. $(A_i \in \mathbb{R}^{n \times n}, B_i \in \mathbb{R}^{m \times n}, C_i \in \mathbb{R}^{p \times n})$ is 
the matrix triplet, $r$ is the number of IF-THEN rules and $m_1 \sim m_g$ are some measurable 
system variables.

Considering the T-S model and using a standard fuzzy inference method the final 
state of the fuzzy system is inferred as ...

$$x(t) = \sum_{i=1}^{r} \mu_i[m(t)][A_i x(t) + B_i u(t)]$$

(4.2)

Where, $\mu_i[m(t)] = \mu_i$ is weighted average of fuzzy membership function at each 
local level and $m(t) = [m_1(t) m_2(t) \ldots m_g(t)]$. It is assumed in this paper that fuzzy 
membership at each level is positive and hence

$$\mu_i[m(t)] \geq 0, i = 1, 2, \ldots, r;$$

$$\sum_{i=1}^{r} \mu_i[m(t)] = 1 \quad \forall t$$

The final state of the fuzzy system can be represented as

$$x(t) = \sum_{i=1}^{r} \mu_i A_i x(t) + \sum_{i=1}^{r} \mu_i B_i u(t)$$

(4.3)

The final output of the fuzzy system is inferred as follows...
y(t) = \sum_{i=1}^{r} \mu_i C_i x(t)  \quad (4.4)

**Design of Fuzzy Controller**

From the plant definition given in section II one can say that if the pairs \((A_i, B_i), i = 1, 2, ..., r\) are controllable, the fuzzy system is called locally controllable. For the design of fuzzy controller it is assumed that the fuzzy system \((4.1)\) is locally controllable.

First, the local state feedback controllers are designed based on the controller rules for each pairs \((A_i, B_i)\):

**Controller Rule i:**

IF \(m_j(t)\) is \(F_{ij}\) and ... and \(m_g(t)\) is \(F_{ig}\)  
THEN \(u(t) = -K_i x(t), i = 1, 2, ..., r\)  \quad (4.5)

Then, the final output of the fuzzy controller is given as

\[ u(t) = -\sum_{i=1}^{r} \mu_i K_i x(t) \]  \quad (4.6)

Where \(\mu_i\) is the same weight as \(i^{th}\) rule of the fuzzy system \((2)\). The parameters of the controller are \(K_i\) in each rule. Substituting \((4.6)\) into \((4.3)\) we get

\[ x(t) = -\sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{ij} (A_i - B_i K_i) x(t) \]  \quad (4.7)

A sufficient condition that guarantees the stability of the fuzzy system is obtained in terms on Lyapunov's direct method. The equation \((4.7)\) is said to be asymptotically stable if there exists a positive semi definite matrix \(P_i\) such that

\[ (A_i - B_i K_i)^T P_i + P_i (A_i - B_i K_i) < 0 \]

**Design of Fuzzy Observer**

As we know in practice all the states of the systems are not fully measurable. Hence it is necessary to design the fuzzy observer in order to implement the fuzzy controller of \((4.6)\).

It can be said that if pairs \((A_i, C_i), i = 1, 2, ..., r\) are observable, the fuzzy system is locally observable. First, the local state observers are designed based on the triplets \((A_i, B_i, C_i)\):

**Observer Rule i:**

IF \(m_j(t)\) is \(F_{ij}\) and ... and \(m_g(t)\) is \(F_{ig}\)  
THEN
\[ x(t) = A \cdot x(t) + B \cdot u(t) + G_i [y(t) - \hat{y}(t)] \]

\[ \hat{y}(t) = C \cdot x(t), \quad i = 1, 2, \ldots, r \quad (8) \]

Where, \( G_i \) (\( i=1,2,\ldots, r \)) are observation error matrices. \( y(t) \) and \( \hat{y}(t) \) are the final output of the fuzzy system and the fuzzy observer, respectively. Then the final estimated state of the fuzzy observer is

\[ x(t) = \sum_{i=1}^{r} \mu_i A \cdot \hat{x}(t) + \sum_{i=1}^{r} \mu_i B_i u(t) \]

\[ + \sum_{i=1}^{r} \mu_i G_i [y(t) - \hat{y}(t)] \quad (4.9) \]

the final output of the fuzzy observer is

\[ \hat{y}(t) = \sum_{i=1}^{r} \mu_i C_i \hat{x}(t) \quad (4.10) \]

where we use the same weight \( \mu \), as the weight of \( i^{th} \) rule of the fuzzy system (2). The observer parameter for each rule is \( G_i \). Substituting (4.4) & (4.10) into (4.9) we get

\[ x(t) = \sum_{i=1}^{r} \mu_i A_i \cdot \hat{x}(t) + \sum_{i=1}^{r} \mu_i B_i u(t) \]

\[ + \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j G_{ij} [x(t) - \hat{x}(t)] \quad (4.11) \]

Using the final estimated state \( x(t) \), in (4.5) and (4.6), we get the following fuzzy controller:

**Controller Rule i:**

*IF* \( m_i(t) \) is \( F_{ij} \) and \( \ldots \) and \( m_g(t) \) is \( F_{ig} \)

*THEN* \( u(t) = -K \cdot x(t) \quad i = 1, 2, \ldots, r \quad (4.12) \)

Then, the final output of the fuzzy controller is given as

\[ u(t) = -\sum_{i=1}^{r} \mu_i K_i \cdot \hat{x}(t) \quad (4.13) \]

Now, substituting (4.13) into (4.3) and (4.11),

\[ x(t) = \sum_{i=1}^{r} \mu_i A_i \cdot x(t) - \]

\[ \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j B_{ij} \cdot K_j \cdot \hat{x}(t) \quad (4.14) \]

\[ x(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j (A_i \cdot B_i \cdot K_j) \cdot \hat{x}(t) \]

\[ + \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j G_{ij} \cdot C_{ij} \left[ x(t) - \hat{x}(t) \right] \quad (4.15) \]
let, $x(t) = x(t) - \hat{x}(t)$, this gives

\[ \dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j (A_i - G_i C_j) \dot{x}(t) \]  

(4.16)

The equation (4.16) is asymptotically stable if there exists a positive definite matrix $P_2$ such that

\[ (A_i - G_i C_j)^T P_2 + P_2 (A_i - G_i C_j) < 0 \]  

(4.17)

### Separation Property

The Separation principle of Estimation and Control in traditional control theory is [198]

"When the control law $u = -K x$ is used in conjunction with either a full order or reduced order state observer for

$x(t) = Ax + Bu$

$y(t) = Cx$

the controller gain $K$ does not influence the eigenvalues of the state observer and the choice of the observer gain $G$ does not influence the remaining eigenvalues".

The state-space transformation allows us to look at the system from a different but possibly more informative way. Let's look at the fact that under such transformations, the matrix “A” becomes “$P^T A P$”. Representing the equations (4.14) and (4.16) in matrix form as below...

\[ \begin{bmatrix} & x^* \\ x \end{bmatrix} = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j \begin{bmatrix} A_i - B_i K_j & B_i K_j \\ 0 & A_i - G_i C_j \end{bmatrix} \begin{bmatrix} x^* \\ x \end{bmatrix} \]  

(4.18)

Note that in this new realization, the matrix is block-triangular. According to the matrix algebra [199] “Eigenvalues of a block triangular matrix are equal to the eigenvalues of the matrices along the diagonal blocks”.

Using this fact and the knowledge that system eigenvalues remain invariant under state transformations, we can say that the closed loop poles of the fuzzy observer based control system are union of the fuzzy observer poles and the fuzzy controller poles.

As controllability allows us to place the eigenvalues of

\[ \sum_{i=1}^{r} \sum_{j=1}^{r} (\mu_i A_i - \mu_i B_i K_j) \]

arbitrarily and the observability does the same for the eigenvalues of

\[ \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i (A_i - G_i C_j) \]

From the above two conditions we can say that designer has complete freedom in fuzzy controller and fuzzy observer pole selection as in the case of traditional control theory. The numerical simulations can be carried out using MATLAB.
4.5 Advanced Fuzzy systems: Genetic Algorithms

Genetic Algorithms are reliable and robust methods for searching solution spaces. They are inspired by the biological theory of evolution through natural selection and much of the terminology is similar. Genetic algorithm (GA) is an adaptive search technique based on natural selection and genetic rules. GA takes place in various scientific applications. The steps of GA are:

1. A chromosome is an encoded string of possible values for the parameters to be optimized. These chromosomes can be made up of real-valued or binary strings. Often one of the main challenges in designing a genetic algorithm to find a solution to a problem is finding a suitable way to encode the parameters.

2. A set of potential solutions, called a population, is created. Each member of this set is referred to as an individual and they are evaluated by decoding the parameter values from the chromosomes and applying them to the problem to see how well they perform the task at hand (the objective that is to be optimized). The score that an individual achieves at performing the required task is called its fitness.

3. After the fitness of each individual has been calculated, a procedure known as selection is performed. Individuals are selected to contribute towards creating the next generation, the probability of selection being related to the individual’s fitness.

4. Once selection has occurred, crossover takes place between pairs of selected individuals. The strings of two individuals are mixed. In this way, new individuals are created that contain characteristics that come from different hereto relatively successful individuals.

5. The next operation is mutation, the random changing of bits in the chromosome. It is generally performed with a relatively low probability. Mutation ensures that the probability of searching a given part of the solution space is never zero.

There are many ways in which these different operations can be applied. Different algorithms can be used for each and they can also be applied with varying degrees of probability. Some of the more popular algorithms for each of these operations are now examined and their effects on the GA’s performance are investigated. Using GA optimization process in tuning of MBF is often leads to advanced fuzzy systems.

GA can be also used to determine the membership functions in a fuzzy system. The method has been implemented using a developed GA program for a single input and output fuzzy system to adjust the shape of membership functions and a novice aspect of the study is to determine the kind of membership functions.

A fuzzy set is fully defined by its membership functions. For most control applications, the sets that have to be defined are easily identifiable. However, for other applications they have to be determined by knowledge acquisition from an expert or group of experts. Once the fuzzy sets have been established, one must consider their associated membership functions. How best to determine the membership function is the first question that has to be tackled.
The approach adopted for acquiring the shape of any particular membership function often depends on the application. For most fuzzy logic control problems, the membership functions are assumed to be linear and usually triangular in shape. So, typically the sets that describe various factors of importance in the application and the issues to be determined are the parameters that define the triangles. These parameters are usually based on the control engineer's experience and/or are generated automatically. However for many other applications, triangular membership functions are not appropriate as they do not represent accurately the linguistic terms being modeled and the shape of the membership functions have to be elicited directly from the expert, by a statistical approach or by automatic generation of the shapes.

GA methods, which are basically random search techniques have been applied to many different problems like function optimization, routing problem, scheduling, design of neural networks, system identification, digital signal processing, computer vision, control and machine learning [200-202]. GA was used first by Karr [203] in determination of membership functions. Karr applies GAs to design of fuzzy logic controller (FLC) for the cart pole problem. He presents two examples: a non-adaptive GA designed FLC and a GA designed adaptive FLC where the membership functions are adapted in real time.

The membership functions here are Gaussian in nature and the objective is to minimize an objective function (fitness function in GA terminology) that minimizes the squared difference between the cart and centre of the track that the cart is on, whilst at the same time keeping the pole balanced.

Meredith et al. [204] have applied GAs to the fine tuning of membership functions in a FLC for a helicopter. Initial guesses for the membership functions are made by the control engineer and the GAs that adjust the parameters that define them by using them minimize the movement of a hovering helicopter. Again, triangular membership functions are used.

Lee and Takagi [205] also tackle the cart problem. They take a holistic approach by using GAs to design the whole system (determine the optimal number of rules as well as the membership functions). The membership functions are triangular.

GAs offer a convenient way to model membership functions. However, there is no obligation of membership functions to be triangular shape. By using GAs, the suggested method is relevant for any membership function whose mathematical model is known. [206] describes how the membership functions that are in a given shape can be suitably placed was discussed. A GA program in C++ for a single input/output fuzzy system and a discussion of the results has also been given. As a result of GA process, the shape of the membership functions has been determined.

The theoretical background of soft computing techniques such as fuzzy logic, ANN, GA and Neuro-Fuzzy systems developed by the researches in the field of control system is summarized and described in the chapter. The applications will be described in subsequent chapters.