2.1 Introduction

In examining earlier discussed constant frequency resonant converter topologies, many topologies offer high power densities and reduced switching losses but only for medium to high power levels [15-20]. The APWM resonant DC/DC converter has been shown to exhibit near-zero switching losses while operating at constant and high frequencies with high power level [21-24].

This chapter presents a new topology that will overcome the previously mentioned drawbacks. In combination with an additional network, a modified APWM resonant DC/DC converter is evolved.

Section 2.2 will present a detailed description of the modified topology as a series of functional blocks. Section 2.3 examines the operating principle by looking at the events occurring in one switching cycle. Section 2.4 performs the steady-state analysis of the converter by examining each functional block separately.

2.2 Circuit Description

Taking basic network as reference, a modified designed APWM resonant DC/DC converter is shown in fig. 2.1. The circuit can be divided into functional blocks including the compensating network ($C_{1a}$, $C_{2a}$ and $L_{a}$), a chopper, a series-resonant tank ($L_a$, $C_j$), a power transformer ($T_x$), a rectifying block ($D_{r1}$, $D_{r2}$), the output filter, $C_a$ and the load $R$. The chopper consists of the two switches ($S_1$, $S_2$), the two snubber capacitors ($C_1$, $C_2$) and two-anti parallel diodes ($D_1$, $D_2$). With the presence of the compensating network, the switch currents are the sum of the resonant current and the auxiliary current.

![Fig.2.1 Asymmetrical Pulse-Width Modulated DC/DC Converter](image)

Two switches, $S_1$ and $S_2$, are controlled by two complementary gating signals, $v_{gs1}$ and $v_{gs2}$ respectively. The gating signal of $S_1$ has a duty cycle of $D$ and that of $S_2$ is $1-D$. 
When $S_1$ is on, the power from the source is transferred to the load and the output of the chopper sees a positive voltage of $V_{in}$ from the source. When $S_2$ turns on, the input side is separated from the rest of the power circuit and the output of the chopper reduces to zero volts. The energy from the resonant components now freewheels through $S_2$ and supplies power to the load. By varying $D$, the output voltage can be controlled.

### 2.3 Principle Operation

There exist four intervals within each switching time period, $T_s$. For each interval, the operation of the converter is described below.

**Interval I:** Prior to this interval, $S_2$ was switched on. At the beginning of this interval the gate drive to $S_2$ is removed and it turns off. The currents flowing through the auxiliary inductor and the resonant branch are negative, thus forcing the discharging of $C_1$ and the charging of $C_2$. This charging-discharging process is complementary, i.e. as $C_1$ discharges from $V_{in}$ to 0, $C_2$ charges from 0 to $V_{in}$. At any given moment throughout the cycle the total charge in both $C_1$ and $C_2$ is $V_{in}$. In order to maintain this, the voltage across $L_a$ changes from $-V_2$ to $F_1$ where $F_1$ is the voltage across $C_1$ and $V_2$ is the voltage across $C_2$. In order to maintain the volt-second balance across the auxiliary inductor, $V_1$ and $V_2$ are given as follows:

\[
V_1 = (1 - D)V_{in} \tag{2.1}
\]

\[
V_2 = DV_{in} \tag{2.2}
\]

Once $C_1$ is fully discharged and $C_2$ is fully charged, the negative currents force the conduction of diode $D_1$. The voltage across $S_1$ is now set at zero volts.

**Interval II:** At the beginning of this interval, gating signal $v_{gs1}$ is applied to the gate of $S_1$ to switch it on. The current previously flowing through $D_1$ now flows through $S_1$. The switch thus turns on under zero voltage. The voltage across $L_a$ remains constant at $F_1$. Since $C_2$ maintains its charge, the voltage across $S_2$ is set at $V_{in}$ during this interval. Power flows from the input DC source to the resonant circuit and to the output load.

**Interval III:** At the beginning of this interval, the gate drive to $S_1$ is removed and this switch is turned off. The currents flowing through the auxiliary inductor and the resonant branch are positive thus forcing the charging of $C_1$ and the discharging of $C_2$. The voltage across $L_a$ changes from $V_1$ to $-V_2$. 

\[
0.5 = (1 - D)\frac{V_{in}^2}{L_aC_1} \tag{2.1}
\]

\[
F_1 = DV_{in} \tag{2.2}
\]
Once $C_i$ is fully charged and $C_2$ is fully discharged, the positive currents force the conduction of diode $D_2$. The voltage across $S_2$ is now set at zero volts.

**Interval IV:** At the beginning of this interval gating signal $v_{gs2}$ is applied to the gate of $S_2$ to switch it on. The current previously flowing through $D_2$ now flows through $S_2$. Therefore, zero voltage switching is achieved at turn-on. The voltage across $L_a$ remains constant at $-V_2$. The voltage across $S_2$ is set at zero volts during this interval since $C_2$ maintains its charge of zero volts. To maintain a constant supply of power to the output load, the energy stored in the resonant components during interval II now flows through $S_2$.

### 2.4 Steady State Analysis

This section presents the steady-state analysis of the circuit shown in Fig.2.1. It is useful to first introduce the assumptions prior to the analysis.

#### 2.4.1 Assumptions

In presenting the analysis, the following assumptions are made:

1) The semiconductor switches and diodes are ideal.
2) The effect of the capacitors across the switches is negligible.
3) The diode rectifier incurs zero losses.
4) An AC equivalent resistance at the primary of the transformer represents the output rectification stage.
5) The delay time between the two switches is neglected.

In order to simplify the analysis, the circuit will be divided into functional blocks as is shown in Fig.2.1. Since the analysis of the resonant tank will depend on the output stage, it is examined first, followed by the analysis of the resonant tank and finally the switch network and the compensating network. Fig.2.2 shows the key waveforms of the proposed topology. These waveforms are based on the equations developed, the details of which are discussed in the next section.
Fig. 2.2 Ideal Waveform of the APWM DC/DC Converter

2.4.2 Output Stage

The purpose of the analysis here is to show how the entire output stage can be modeled as a resistance. This will then help in conducting the analysis of the resonant network. The output stage consists of the rectifier network, the output filter and the load. The rectifier is driven by the nearly sinusoidal resonant tank current $i_{r}(t)$. A large
capacitor $C_0$, then removes the higher harmonics so that only negligible harmonics are present at the load. For this part of the analysis, the power losses through the diode rectifier will be neglected. This way, it can be approximated that the load (represented by a resistance $R_L$) has a DC voltage $V_0$ across it and draws a DC current $I_0$. The input to the output stage is the output of the resonant tank, $v_r(t)$, namely an AC square-wave voltage where the positive and negative rails are proportional to $V_0$. It is also the voltage seen across the primary of the main transformer, $T_X$. This waveform can be expressed using Fourier series:

$$v_r(t) = \frac{4n_r V_0}{\pi} \sum_{m=1,3,5} \frac{1}{m} \sin\left(n\omega_s t - \varphi_s\right)$$

(2.3)

Where $\varphi_s$ represents the phase shift of $i_s(t)$. $n_r$ represents the turn’s ratio and $\omega_s$ is the switching frequency. Now considering only the first harmonic, Eq. (2.3) reduces to:

$$v_{r1}(t) = \frac{4n_r V_0}{\pi} \sin(n\omega_s t - \varphi_s)$$

(2.4)

Likewise current can also be represented using Fourier series

$$i_r(t) = \sum_m \frac{I_m}{m} \sin(n\omega_s t - \varphi_s)$$

(2.5)

Where $\varphi_s$ is the phase shift which takes into account the resonant components as well as the duty cycle $D$. If only the fundamental harmonic is considered, this current can be expressed as:

$$i_{r1}(t) = I_{r1} \sin(n\omega_s t - \varphi_s)$$

(2.6)

Where $I_{r1}$ is the peak of the fundamental component of the resonant current. At the output stage this current is half-wave rectified and then averaged by $C_o$ to produce an essentially DC current. This is shown by equ. (2.7) where only the first harmonic is considered:

$$I_o = \frac{2n_r}{T_s} \int_0^{T_s} I_{r1} \sin(n\omega_s t - \varphi_s) dt = \frac{2n_r}{\pi} I_{r1}$$

(2.7)
This current is proportional to $I_u$. Substituting of Eq. (2.7) into Eq. (2.6) and then dividing of Eq. (2.4) by Eq. (2.6) gives an expression for an equivalent resistance that is proportional to the output load

$$R_{eq} = \frac{V_s(t)}{I_s(t)} = \frac{8n_{r_s}V_0}{\pi^2 I_o} = \frac{8n_{r_s}^2}{\pi^2} R_L$$  \hspace{1cm} (2.8)

This result leads to the conclusion that the tank network is damped by an effective load resistance $R_{eq}$ that is equal to approximately 81% of the actual load resistance $R_L$. The above analysis can be summarized by Fig. 2.3.

![Fig. 2.3 Equivalent Model of the Ideal Output Stage](image)

The DC component of output voltage can be written as:

$$V_o = I_o R_L = \frac{2n_{r_s} I_s R_L}{\pi}$$ \hspace{1cm} (2.9)

This result demonstrates that the DC output voltage $V_o$ is directly proportional to the peak of the resonant current $I_s$, and therefore can be regulated against load and line variations by varying $I_s$ in such a way that the product of $I_s$ and $R_L$ remains constant. It will be seen in the following section that the resonant current is dependent on the duty cycle $D$ therefore the output voltage $V_o$ is also a function of $D$.

### 2.4.3 Resonant Tank Circuit

The two gating functions $v_{g1}$ and $v_{g2}$, as shown in Fig. 2.2, control the switch network. As explained earlier in section 2.3, when $S_1$ is turned on, the input DC source is seen across $S_2$ and acts as a power source for the resonant circuit. At time $t = DT_{S}$,
however, \(S_1\) turns off and \(S_2\) turns on. Therefore, \(V_s\) can be represented by its Fourier series in one switching cycle \(T_s\) as:

\[
V_s(t) = V_m D + \sum_{n=1,3,5...} \sqrt{2} \frac{V_m}{n\pi} \left[ \sqrt{1 - \cos n\pi D} \cdot \sin(n\omega_s t + \theta_n) \right]
\]

(2.10)

Where

\[
\theta_n = \tan^{-1} \left[ \frac{\sin 2n\pi D}{1 - \cos 2n\pi D} \right]
\]

(2.11)

It is seen that Eq. (2.10) includes both the DC and AC components where \(V_mD\) represents the DC component & \(\sum_{n=1,3,5...} \sqrt{2} \frac{V_m}{n\pi} \left[ \sqrt{1 - \cos n\pi D} \cdot \sin(n\omega_s t + \theta_n) \right]\) represents a.c. component. The resonant capacitor \(C\) will block the DC component from the transformer so that only the AC component will pass through the transformer. Fig.2.4 shows the equivalent model that will be used to find the resonant current. Eqs. (2.8) and (2.10) have already defined \(R_{eq}\) and \(V_s\) respectively. It is also useful to define some terms given below [Appendix - I]:

\[
f_r = \frac{1}{2\pi\sqrt{L_sC_s}}
\]

(2.12)

\[
f_r = \frac{f_0}{f_r}
\]

(2.13)

\[
Q = \frac{2\pi f_r L_s}{R_{eq}} = \frac{1}{2\pi f_r C_s R_{eq}}
\]

(2.14)
Eq. (2.13) uses the resonant frequency $f_r$ as a base value. The resonant current is essentially the voltage divided by the tank impedance.

$$I_s(t) = \sum_n \sqrt{2} \frac{V_{in}}{n\pi |Z_m|} \sqrt{1 - \cos2n\pi D} \cdot \sin(n\omega t + \theta_n - \phi_n)$$  

(2.15)

Where,

$$|Z_m| = R_o \sqrt{1 + Q^2 \left(\frac{n\omega - \frac{1}{n\omega}}{1}\right)^2}$$  

(2.16)

$$\phi_n = \tan^{-1} \left[ Q \left(\frac{n\pi - \frac{1}{n\pi}}{n\pi}\right) \right]$$  

(2.17)

From eq. (2.15) the peak value of fundamental peak resonant current $I_{ni}$ can be given as

$$I_{ni} = \frac{\sqrt{2} V_{in}}{\pi R_o} \left(1 - \cos(2\pi D)\right) \sqrt{1 + Q^2 \left(\frac{1}{\omega} - \frac{1}{\omega}\right)^2}$$  

(2.18)

The time varying voltages across the resonant capacitor and the resonant inductor are given by:

$$V_c(t) = -\sum_n \frac{\sqrt{2} Q}{n^2 \omega n |Z_m|} \sqrt{1 - \cos2n\pi D} \cdot \cos(n\omega t + \theta_n - \phi_n)$$  

(2.19)

$$V_{Li}(t) = \sum_n \frac{\sqrt{2} Q \omega}{n \pi |Z_m|} \sqrt{1 - \cos2n\pi D} \cdot \sin(n\omega t + \theta_n - \phi_n)$$  

(2.20)

### 2.4.4 Switch Compensating Network

The switch currents are functions of both the resonant and auxiliary currents. The auxiliary inductor generates the auxiliary current. To find the value of this current, the voltage across the auxiliary inductor $V_{La}$ must first be derived. As explained earlier, when $S_i$ is on, $V_{Li}$ is set at $V_i$ and when $S_1$ is on, $V_{Li}$ is set at $-V_2$. The Fourier series of this waveform is given as:

$$V_{Li}(t) = \sum_n \frac{\sqrt{2} V_{in}}{\pi} \sqrt{1 - \cos2n\pi D} \cdot \sin(n\omega t + \theta_n)$$  

(2.21)

As expected, this result is same as the AC component of $v_k$. By integration of Eq. (2.21) the auxiliary inductor current $i_{La}$ can be found:
\[ i_{a} = \sum_{n} \frac{\sqrt{2}}{\pi n \pi} \frac{I_{a}}{D(1-D)} \sqrt{1-\cos(2\pi n D)} \cdot \sin(n\omega_{a} t + \delta_{a}) \]

(2.22)

Where,
\[ I_{a} = \frac{D(1-D)w_{i}}{2f_{s}L_{a}} \]

(2.23)

\[ \delta_{a} = \tan^{-1}\left( \frac{\cos(2\pi n D) - 1}{\sin(2\pi n D)} \right) \]

(2.24)

\( I_{a} \) represents the peak of the auxiliary current. The current through the switches is then the sum of the resonant current and the auxiliary current:

\[ i_{a}(t) = \sum_{n} \frac{\sqrt{2}}{n \pi} \frac{V_{m}}{\pi} \sqrt{1-\cos(2\pi n D)} \left[ \frac{1}{|Z_{m}|} \sin(n \omega_{f} t + \theta_{a} - \phi_{a}) + \frac{1}{2n \pi f_{s}L_{a}} \sin(n \omega_{a} t + \delta_{a}) \right] \]

(2.25)

It should be noted that Eq. (2.25) is valid only when the switches are on (during intervals II and IV for \( S_{1} \) and \( S_{2} \) respectively). When the switches are off, the current is zero. To complete the analysis, the input current \( i_{d}(t) \) is also being presented. The value of this current depends on which switch is on. During interval II when \( S_{1} \) is on \((0 < t < DT_{S})\), the input current is:

\[ i_{d}(t) = i_{a}(t) + i_{c_{a}}(t) \]

(2.26)

\[ i_{d}(t) = \sum_{n} \frac{\sqrt{2}}{n \pi} \frac{V_{m}}{\pi} \sqrt{1-\cos(2\pi n D)} \left[ \frac{1}{|Z_{m}|} \sin(n \omega_{f} t + \theta_{a} - \phi_{a}) + \frac{1}{4n f_{s}L_{a}} \sin(n \omega_{a} t + \delta_{a}) \right] \]

(2.27)

During interval IV when \( S_{2} \) is on \((DT_{S} < t < T_{S})\), the input current is:

\[ i_{d}(t) = i_{a_{0}}(t) = \sum_{n} \frac{\sqrt{2}}{4(n \pi)^{2}} \frac{V_{m}}{f_{s}L_{a}} \cdot \sqrt{1-\cos(2\pi n D)} \cdot \sin(n \omega_{a} t + \delta_{a}) \]

(2.28)

### 2.4.5 Modified Series Resonant Converter

Fig.2.5 shows the equivalent circuit of the entire modified APWM series resonant converter. It is a combination of Figs.2.3 and 2.4.
From Eq. (2.9) and (2.18) the voltage conversion ratio $M$ can be found out. This is the ratio of output voltage to the input voltage.

\[
M = \frac{V_o}{V_{in}} = \frac{2\sqrt{2} \cdot n_T \cdot R_L}{\pi^2} \cdot \frac{\sqrt{1 - \cos(2\pi D)}}{\sqrt{R_q^2 + Q^2 \left( \frac{\omega - \frac{1}{\omega}}{\omega} \right)}}
\]  

(2.29)
The plot of $M$ v/s $D$ is shown below (fig.2.6).

For a loss less converter the input power is equal to the output power, $V_oI_o = VinI_d$

$$\frac{V_o}{V_{in}} = \frac{I_d}{I_o} = M \quad (2.30)$$

With Eqs. (2.7) and (2.29) the output power of the converter is given by:

$$P_o = V_oI_o = VinM \frac{2\eta_r}{\pi} \cdot I_s \quad (2.31)$$

The current drawn from the inverter is given by,

$$I_s = \frac{P_o}{Vin} \frac{\pi}{2\eta_r} \cdot \frac{1}{M} \quad (2.32)$$
2.5 Simulation Result:

The simulation ckt is as shown in fig.2.7 and simulation results are shown in fig.2.8. The topology selected for the converter is the series-loaded resonant. In this configuration the switches and resonant components $L$ and $C$ are connected to the low voltage side of the transformer. Only the rectifiers on the transformer secondary must have high voltage ratings. By closing the switches in pairs and in proper order, pulses of alternate polarity are applied to the transformer, with the high turn ratio transformer, high voltage pulses are generated at the secondary.

The characteristic impedance $Z_0$ of the bridge load is given by

$$Z_0 = \sqrt{\frac{L}{C_{eq}}}$$  \hspace{1cm} (2.33)

And its resonance frequency $f_0$ by:

$$f_0 = \frac{1}{2\pi\sqrt{LC_{eq}}}$$  \hspace{1cm} (2.34)

$C_{eq}$ is the series combination of capacitors $C$ and $C'c$, where $C'c$ is the equivalent capacitance $C_c$ reflected to the primary:

$$C'c = \left(\frac{N_2}{N_1}\right)^2C_c = 81\cdot C_c$$  \hspace{1cm} (2.35)

Usually in high-voltage applications, $C_c$ is at least order of magnitude greater than $C$. While it’s contribution is negligible:

$$C_{eq} = \frac{1}{\frac{1}{C} + \frac{1}{C_c}} = C$$  \hspace{1cm} (2.36)

Therefore $Z_0$ and $f_0$ are uniquely defined by $C$ and $L$. If the circuit operates at a frequency $f_s < f_0/2$, all switches and anti-parallel diodes turn on and off at zero current. Using a frequency $f_0/2 < f_s < f_0$ the diodes turn off and the switches turn on happens at a current greater than zero. Conversely, for $f_s > f_0$, the diodes turn on and the switches turn off at a current greater than zero.
The maximum peak reverse current flowing through the diodes when one switch pair is open is given by:

\[ I_{OFF\, max} = \frac{V_{in}}{Z_0} \]  

(2.37)

The above value is assumed at the beginning of charging and drops down to zero at the end of charging. This is due to the voltage reversal effect across the load terminals caused by the voltage present on the capacitor under charging. Conversely, the maximum peak current flowing in the load \( C_2 \) and \( L_1 \), when a switch pair is closed is given by:

\[ I_{ON\, max} = 2\frac{V_{in}}{Z_0} = -2I_{OFF\, max} \]  

(2.38)

This value is reached only at the end of the charging and is due to the energy accumulated in the resonant load \( C_2 \) and \( L_1 \). At the beginning of charging \( I_{ON} = -I_{OFF\, max} \). Therefore, the converter switches must be capable of withstanding \( I_{ON\, max} \), while the diodes can be sized to operate at \( I_{OFF\, max} \) only.

\[ I_{OFF\, max} = \frac{V_{in}}{Z_0} = \frac{-V_{in}}{Z_0} = \frac{-560}{\sqrt{\frac{L_1}{C_2}} \sqrt{\frac{0.07 \times 10^{-3}}{0.037 \times 10^{-6}}}} = -13\, A \]  

(2.39)
\[ I_{ON_{\text{max}}} = -2I_{OFF_{\text{max}}} \quad (2.40) \]

The capacitor charging current can be calculated directly from the capacitor charging rate.

\[ I_{Crms} = \frac{C_1 \Delta V}{\Delta t} = \frac{0.1 \times 10^{-6} \times 5000}{600 \times 10^{-6}} = 832mA \quad (2.41) \]

Where \( \Delta V \) is the difference between output and input of the transformer voltage. The RMS current in the resonant load and transformer primary \( I_{\text{rms}} \) can be calculated by balancing the average power between capacitor \( (P_{\text{Cave}}) \) and load \( (P_{\text{Lave}}) \):

\[ P_{\text{Cave}} = \frac{I_{Crms} \cdot \Delta V}{2} = \frac{832 \times 10^{-3} \times 5000}{2} = 2080W \]

\[ I_{\text{rms}} = \frac{P_{\text{Lave}}}{V_{in}} = \frac{P_{\text{Cave}}}{V_{in}} = \frac{2080}{560} = 3.714A \quad (2.42) \]

\[ P = \frac{0.5 \cdot C_1 \cdot (\Delta V)^2}{\Delta t} = \frac{0.5 \times 0.1 \times 10^{-6} \times 5000^2}{600 \times 10^{-6}} = 2083W \quad (2.43) \]

The theoretical value of voltage is

\[ N_1 = 15 \text{ turns} \]

\[ N_2 = N_1 \frac{V_2}{V_1} = 15 \frac{5000}{560} \approx 135 \text{ turns} \quad (2.44) \]

\[ N_1 = 15 \text{ turns} \]

\[ V_2 = N_1 \frac{N_2}{N_1} = 560 \frac{135}{15} = 5040\text{volts} \quad (2.45) \]

The accuracy reduces as the switching frequency is away from resonance frequency (as in variable-frequency-controlled converters) or as the duty ratio decreases (as in fixed-frequency-controlled converters).
Modeling and Simulation of Pulse-Width Modulated Resonant DC/DC Converter

(a)

(b)

(c)
The input voltage is shown in Fig. 2.8 (e) which is 560 volt and the output voltage is 5 kV as shown in Fig. 2.8 (f).
2.6 Conclusion

The resonant converter has been analyzed using standard electronics software ORCAD 9.2. The circuit has input voltage of 560 V and output voltage of 5 kV. The circuit has been simulated successfully as per requirement.