To show that a SRGD design with parameters \( b = v - m + r \), \( v = 2k \), where \( k \) is an odd integer, cannot exist.

Proof. As \( v = 2k \), we have \( b = 2r \). Hence substituting \( b = 2r \) in \( b = v - m + r \), we get \( v = m + r \). But for a SRGD design \( v = mn \). Hence \( r = m(n - 1) \). Now \( rk - v \alpha_2 = 0 \). Hence \( \alpha_2 = rk/v = \frac{m(n-1)k}{2k} = \frac{m(n-1)}{2} \).

Now \( v = mn = 2k \). As \( k \) is an odd integer, therefore \( m \) and \( n \) cannot both be even simultaneously. Hence \( m \) or \( n \) must be a multiple of \( 2 \), but not of \( 2^\alpha \), where \( \alpha \geq 2 \); while the other must be odd. Thus, we have two alternatives:

(i) \( m = 2t \), where \( t \) is an odd positive integer and \( n \) is odd, or

(ii) \( n = 2s \), where \( s \) is an odd positive integer and \( m \) is odd.
Suppose (i) holds. Now \( k = cm = mn/2 = tn \), so that \( c = n/2 \). Hence \( n \) must be an even integer, which contradicts the requirement in (i) that \( n \) be odd. Hence the given design cannot exist.

Next, suppose that (ii) holds. Substituting the value of \( n = 2s \) in \( \lambda_2 = m(n-1)/2 \), we have \( \lambda_2 = ms - (m/2) \). As \( m \) is odd, \( \lambda_2 \) is fractional which is impossible. Hence the given design cannot exist.
APPENDIX 4.2

To show that a triangular design with parameters satisfying the relations \(rk - v \lambda_1 = n\frac{(r - \lambda_1)}{2}\), \(b = v - n + r\) and \(v = 2k\) cannot exist.

Proof. As \(v = 2k\), we have \(b = 2r\). Substituting \(b = 2r\) in \(b = v - n + r\), we get \(r = v - n = n(n-1)\frac{2}{2} - n = n(n-3)/2\). Also \(v = 2k = n(n-1)/2\) gives \(k = n(n-1)/4\).

Putting the values of \(v\), \(r\) and \(k\) in terms of \(n\) in \(rk - v \lambda_1 = n\frac{(r - \lambda_1)}{2}\), and solving it for \(\lambda_1\), we get \(\lambda_1 = n(n-3)^2/4(n-2)\).

Now from \(k = n(n-1)/4\), we see that if \(n\) is even, it must be of the form \(4t\), (\(t\) a positive integer); while if \(n\) is odd, \(n-1\) must be of the form \(4t\), (\(t\) a positive integer). Thus, we consider two alternatives for \(k\):

(i) \(n\) is even and of form \(n = 4t\), or

(ii) \(n\) is odd and of form \(n = 4t + 1\),

where \(t\) is a positive integer.
If (i) holds, then substituting \( n = 4t \), in
\[
\lambda_1 = \frac{n(n-3)^2}{4(n-2)},
\]
we get
\[
\lambda_1 = 4t^2 - 4t + \frac{t}{(4t-2)},
\]
which is clearly fractional for all positive integral values of \( t \). Hence the given design cannot exist.

Next, suppose (ii) holds. Then substituting \( n = 4t + 1 \) in
\[
\lambda_1 = \frac{n(n-3)^2}{4(n-2)},
\]
we get
\[
\lambda_1 = 4t^2 - 2t - \frac{(2t-1)}{(4t-1)},
\]
which is again fractional for all positive integral values of \( t \). Hence the given design cannot exist.
To show that a $L_2$ design with parameters satisfying the relation $v = 2k$, where $k$ is an odd integer, cannot exist.

Proof. As $k = v/2 = s^2/2$, it follows that $s$ must be even. Then, $k$ is also even, which contradicts the fact that $k$ is an odd integer. Hence the given design cannot exist.
To show that a rectangular design with parameters satisfying relations $\theta_1 = 0 = \theta_2$, $b = p + r$ and $v = 2k$, where $k$ is an odd integer, cannot exist.

Proof. As $v = 2k$, therefore $b = 2r$. But $b = p + r$, hence $r = p$. Also $k = v/2 = v_1 v_2/2$. Using this information, we have from Section 2.5 of Chapter 2,

$$\lambda_1 = (v_1 - 1)(v_2 - 2)/2, \quad \lambda_2 = (v_1 - 2)(v_2 - 1)/2 \quad \text{and} \quad \lambda_3 = (v_1 v_2 - v_1 - v_2 + 2)/2.$$

As $k = v_1 v_2/2$ and $k$ is an odd integer, it follows that $v_1$ and $v_2$ cannot both be even. Also for the same reason $v_1$ nor $v_2$ can contain a factor $2^{a}$ where $a \geq 2$. Hence, the following two alternatives are possible:

(i) $v_1$ is even and $v_1 = 2v_1'$, where $v_1'$ and $v_2$ are both odd integers, or

(ii) $v_2$ is even and $v_2 = 2v_2'$, where $v_2'$ and $v_1$ are both odd integers.
Under alternative (i), $\lambda_2$ is integral but $\lambda_1$ and $\lambda_2$ are fractional, which is an impossible situation; under alternative (ii), $\lambda_1$ is integral but $\lambda_2$ and $\lambda_3$ are fractional, which is also an impossible situation. Hence, the given rectangular design cannot exist.
REFERENCES


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