CHAPTER 1
INTRODUCTION

1.1 GENERAL

More than two third of the earth's surface is covered with water. Submersibles are primarily employed to observe and explore the subsea environment. Submarine is a submersible which operates in deep waters and can be defined as hydrodynamically designed one atmos. pressure chamber, and which maintains its structural integrity at the chosen diving depth and functions as a floating vessel on surfacing.

Besides the submarines for warfare there are commercial submarines, which are used in the offshore industry for underwater exploration, repair and maintenance. For the functional environment for the crew, submarines are essentially designed as atmospheric pressure chambers and consequently the hull has to withstand safely the hydrostatic pressure prevailing at the operational depth.

1.2 HULL GEOMETRY

High hydrostatic pressure is best withstood by axisymmetric structural forms (Jackson, 1983). The pressure hull of a submarine is often constructed from various combinations of cylinders, cones and domes. The pressure hull is mainly a cylindrical pressure vessel and the changes in hull diameter are accomplished through conical sections. The fore and aft ends of the hull consist of domed and/or conical end closures. These hull forms are hydrodynamically efficient and possess better overall strength. Usually the cylinders are stiffened with rings and/or stringers (Burcher and Rydill, 1994).

1.3 STRUCTURAL BEHAVIOUR

Stiffened cylindrical shells are essential components in various hydrospace, aerospace and terrestrial structures. Cylindrical shell structures by virtue of their
shell geometry carry the applied loads primarily by direct stresses lying in their plane
accompanied by a little or no bending. External hydrostatic pressure induces
compressive stress resultants in the cylindrical shells and may cause buckling at a
pressure, much lower than the axisymmetric yield. Subsequently analytical
investigation on buckling of such shell forms is the major problem to be addressed.
The introduction of stiffeners considerably increases the buckling strength of the
shell and is a satisfactory solution for increasing the strength of the shell.

The primary modes of failure of a stiffened cylindrical shell are considered
to be buckling of shell between ring stiffeners identified by dimples or lobes around
the periphery of shell plating; yielding of shell between ring stiffeners usually
appearing as axisymmetric accordion pleats and general instability characterized by
large dished-in portions of stiffened cylinder wherein the shell and the stiffeners
deflect bodily as a single unit (Cormstock, 1988). Third mode of collapse is
sensitive to spacing of bulkheads or deepframes and the scantlings of supporting ring
frames. The general instability is very much sensitive to initial imperfections.

The simultaneous occurrence of all modes of failures described earlier has
been argued by theoreticians as being the only criterion to be considered for the
optimum design.

1.4 STRUCTURAL ANALYSIS OF CYLINDRICAL SHELLS

Classical methods are available for deflections, stresses and buckling
pressures of ring stiffened cylindrical shells under hydrostatic pressure. But these
are not applicable to actual submarines with stiffeners of various shapes and
nonuniform spacing and shells with complex boundary conditions. Numerical
solution schemes like finite difference and finite element methods can effectively
be employed in these situations.

1.5 FINITE ELEMENT ANALYSIS OF STIFFENED CYLINDRICAL SHELLS

Finite element method is an efficient numerical technique for the study of
the behaviour of various structural forms. The finite element method requires the
actual submarine structure to be replaced by a finite element model, made up of
structural elements of known elastic and geometric properties. The objective therefore, is to develop a model, which simulates the elastic behaviour of continuous structure as closely as required. The finite element modeling of stiffened cylindrical shells can be done either using a smeared model or stiffener shell model. Various finite element models of stiffened cylindrical shells, viz., orthotropic shell model, discrete stiffener model and superelement model are generally used in the analysis.

The hydrostatic pressure can be idealized as uniformly distributed external load acting on the periphery of the shell, which can be converted into consistent load vector. Since hydrostatic pressure is a displacement dependant load, nonlinear analysis has become a necessity and hence finite element method is preferably adopted.

1.6 FINITE ELEMENT MODELING OF UNSTIFFENED CYLINDRICAL SHELLS

Unstiffened cylindrical shells subjected to external hydrostatic pressure can be modeled using axisymmetric elements, facet elements or general shell elements.

Singly curved shell finite elements were first developed in axisymmetric form for the analysis of shells of revolution. Since the hull of the submarine is stiffened cylindrical shell under axisymmetric loading, axisymmetric shell finite elements can be effectively used for analysis. Elements with axisymmetric geometry and asymmetric displacement functions (designated as rotational finite elements) can be effectively used for stability and geometric nonlinear analyses. In these types of elements shell nodes are nodal circles. The shape functions are obtained by combining polynomials along meridional direction and trigonometric functions in circumferential direction. Axisymmetric structures subjected to nonaxisymmetric loading can also be analysed using these elements.

Generally axisymmetric elements are efficient in achieving a state of constant strain and rigid body modes and in eliminating membrane locking and shear locking problems compared to general shell elements (Cook et al, 1989). The major
The drawback of these elements is that proper analysis is not possible in the presence of irregularities or discontinuities within the shell.

In facet element modeling, the assembly of elements gives a geometry, which approximates the actual shell surface. The shell behaviour is achieved by the superposition of stretching behaviour (membrane element) and bending behaviour (plate bending element). The concept of the use of such elements in shell analysis was suggested by Greene et al (1961). The attractive features of this modeling are simplicity in formulation, easiness to mix with other types of elements and the capacity of modeling rigid body motion. Geometric nonlinear analysis based on corotational kinematics can be done effectively using these elements (Ramm, 1982). However, there are some drawbacks such as the lack of coupling between stretching and bending within the element and the discontinuity of slope between adjacent plate elements, which may produce bending moments in the regions where they do not exist. These are available in rectangular, quadrilateral and triangular shape together with coordinate transformations.

The curved elements have been developed with a view to overcome the limitations of facet elements and are generally used for general shells or shells with geometric discontinuity. Based on basic assumptions and theories, two types of curved elements have been formulated, viz., elements based on classical shell theories and degenerated shell elements.

1.7 FINITE ELEMENT MODELING OF STIFFENED CYLINDRICAL SHELLS

Various finite element models of stiffened cylindrical shells are orthotropic shell model; discrete stiffener model and superelement model and are described subsequently.

In the orthotropic approach, the ring stiffeners are blended with the shell such that the ring-stiffened shell is represented as an unstiffened but orthotropic cylindrical shell having different constitutive relationships in longitudinal and circumferential directions.
In the orthotropic shell modeling, stiffeners are assumed to interact to such a
degree that these can be smeared into the shell. The compatibility of the plate and the
stiffener gives rise to internal stresses, which results in change in constitutive
relations in two mutually perpendicular directions. These constitutive relations can
be effectively derived from the compatibility of the shell and the stiffener. The
orthotropic approximation is applicable to geometries where there are a large number
of closely and equally spaced rings and/or stringers, in which the stiffened hull is
modeled using orthotropic shell elements.

In discrete stiffener model the stiffener is modeled as rings or an assembly
of curved beam finite elements defined by cross sectional area and eccentricity of the
cross section from the shell middle surface. In this model the stiffeners are assumed
to be concentrated along the nodes of the shell elements. This model introduces
certain inconsistencies such as the lumped stiffeners, indicating a coupling only
along the nodes to which it is connected. Secondly the stiffeners inside the shell
element are shifted to a new position in the lumped model.

The superelement modeling generally consists of merging a group of
subelements into an assembly followed by the reduction of internal degrees of
freedom that are local to a given superelement. The remaining degrees of freedom
are termed as retained or super degrees of freedom. It is the process of substructuring
technique followed by static condensation. The degrees of freedom normally retained
are those, which are required to connect the superelement. The superelements may
in turn be used as subelements for new assemblies on higher level. In this way a
multilevel hierarchy of superelement may be established. The highest level in such a
hierarchy will represent the complete structure. Hybrid beam elements (in which
axial and bending stiffnesses are based on different cross sections) or eccentric beam
elements (in which element nodes are not located along the stiffener centroidal axis)
can be effectively used as the special elements or superelements (Hughes, 1986).

1.8 TYPES OF ANALYSES PERFORMED

Finite element analyses performed for the stiffened cylindrical shells of
submarine are linear static analysis, linear buckling analysis and geometric nonlinear
analysis.
1.8.1 Linear Static Analysis

Linear static analysis is the strength analysis in which the principle of superposition is valid. It is based on the small deflection theory where stress strain relations and strain displacement relations are linear. In this method of analysis the change in geometry of the structure is not taken into account while deriving the equilibrium equations. The linear static analysis of the stiffened cylindrical shell can be performed by solving the general finite element equilibrium equations, consisting of linear elastic stiffness matrix and load vector. Deformation pattern and stress resultants can be calculated.

1.8.2 Linear Buckling Analysis

Buckling phenomenon is the major failure mode associated with thin walled cylindrical structures subjected to external pressure. The structure can suffer instability at a pressure, which may be only a small fraction to cause material failure. The buckling phenomenon associated with thin walled circular cylindrical shell subjected to uniform external pressure can be explained using the load deflection curve shown in fig. 1.1 (Rajagopalan, 1993).

![Fig. 1.1 Bifurcation buckling](image)

The first regime OR, called the prebuckling state, determines the axisymmetric state of stress due to axisymmetric pressure load on the perfect
cylinder. The prebuckling path is linear. The second regime RS, called the buckling stage and the load deflection curve for a perfectly circular cylinder subjected to uniform external pressure splits into two at the point R. At this point the load deflection curve can be either RS or RS* and the pressure \( P_b \) is called bifurcation-buckling pressure.

In the linear prebuckling analysis, change in geometry prior to buckling is neglected. The prebuckling deformations are neglected and hence stiffness matrices are evaluated at the original undeformed configuration.

Bifurcation buckling pressure is determined from linear buckling analysis. Linear buckling analysis is performed by constructing linear elastic stiffness matrix signifying the internal strain energy and geometric stiffness matrix representing the work done by the prebuckling stresses on the buckling displacement of the complete structure. The elastic stiffness matrix, \([K_o]\) and the geometric stiffness matrix \([K_g]\) are evaluated at the original undeformed configuration. The geometric stiffness matrix at any load level \([K_G]\) is linearly related to the initial geometric stiffness matrix \([K_g]\) by a parameter \( \lambda \), which is a nondimensional function of load applied (Felippa, 1999).

\[
[K_G] = \lambda [K_g]
\] 

During buckling the total stiffness matrix becomes singular or the determinant of the total stiffness matrix vanishes. The eigen value problem of instability is therefore formulated as

\[
([K_o] + [K_G]) \{\delta\} = 0
\]

\[
([K_o] + \lambda_b [K_g]) \{\delta\} = 0
\]

The buckling pressure is evaluated for the condition

\[
\left| [K_o] + \lambda_b [K_g] \right| = 0
\]

where \( \lambda_b \) is the nondimensional buckling pressure.

In the solution, eigen values will be the buckling pressure and eigen vectors will be the buckling mode. Linear prebuckling analysis has the advantage of avoiding a full nonlinear analysis, which may be expensive and time consuming. This method
is effective in cases of cylinder subjected to hydrostatic loading, in which prebuckling deformations are small. Linear prebuckling analysis is effective in cylindrical shell structures made of steel in which buckling occurs in the elastic range. Cylindrical shell under hydrostatic pressure is not much sensitive to initial imperfections and hence linear buckling analysis can be adopted.

Linear buckling analysis predicts the collapse pressure at the bifurcation point and the postbuckling regime is left untouched. Geometric nonlinear analysis has been recommended to make the investigations of buckling behaviour complete.

1.8.3 Geometric Nonlinear Analysis

In structural mechanics a problem is nonlinear if the stiffness matrix or load vector depend on displacements. The cause of nonlinearity may be material or geometric. The material nonlinearity may be due to nonlinear stress-strain relations and geometric nonlinearity may be due to nonlinear kinematic relations i.e. nonlinear strain-displacement relations (large displacements) and large strains.

The prebuckling deformations of the cylindrical shell causes rotation of the structural elements and primary equilibrium path will be nonlinear from the outset. The ring stiffened shell with high degree of orthotropy may experience significant nonlinear prebuckling deformations. The critical load could not be determined with sufficient accuracy if prebuckling nonlinearity is neglected. Normally the loss of stability occurs at the limit point rather than at the bifurcation point. In such cases the critical load must be determined through the solutions of nonlinear system of equations.

The geometric nonlinearity in which the nonlinear effect arising from nonlinear strain displacement relations and nonlinearity due to follower force effect of hydrostatic pressure are to be taken into consideration for stiffened cylindrical shell subjected to hydrostatic pressure. These two are smooth nonlinearities and incremental iterative procedure can effectively be used as solution strategy.

The key component of the finite element nonlinear analysis is the solution of nonlinear algebraic equations that arise upon discretization. This difficulty is
overcome by the concept of continuation, which is also called incremental analysis (Crisfield, 1980). In this method the analysis is started from an easily computable solution (for e.g. the linear solution) and try to follow the behaviour of the system, as actions applied to it are changed by small steps called increments. In the incremental iterative methods one or more iteration steps are included to eliminate or reduce the drifting error, which are there in purely incremental methods (Felippa, 1999).

Out of three types of incremental iterative procedures, viz., load control, displacement control and arc length control, load control method is the basic one, and is generally adopted in the analyses mentioned earlier.

The essential feature of geometric nonlinear analysis is that the equilibrium equations must be written with respect to the deformed geometry, which is not known in advance (Bathe, 2001). Corotational kinematics is adopted for the generation of equilibrium equation at the deformed configuration i.e., for the generation of tangent stiffness matrix and the load vector at the deformed configuration. The reference configuration is split. Strains and stresses are measured from the corotated configuration where as the base configuration is maintained as a reference for measuring rigid body motion.

1.9 FOLLOWER FORCE EFFECT OF HYDROSTATIC PRESSURE

Conventional structural analysis involves loads that do not change their direction during deformation process and such loads are called conservative loads. The direction of the external loads such as water pressure or wind forces in the real situation may be changed during the deformation and the forces induced by such loads are called follower forces or polygenetic forces. These forces remain normal to the surface upon which they act throughout the load displacement history. Follower force effects are to be considered in the analysis of practical structures such as pressure vessels, cooling towers etc.

In the case of follower force the direction of the applied force is dependent on displacement, and to account for this additional stiffness terms, pressure stiffness
matrix must be added to the conventional stiffness matrix to take care of the pressure rotation effects.

Normally structures with follower force do not have proximate equilibrium position. These structural systems change to instability directly from the prebuckled equilibrium configuration and geometric nonlinear analysis becomes a necessity. The linear prebuckling analysis is restricted to static criterion, which is restricted to conservative loads. But for structures not having any loaded free edges or if a constant pressure is acting on a fully enclosed volume (like submarine pressure hull), polygenetic force effect will be weak and hence the structure is amenable to bifurcation buckling analysis. So the pressure rotation effects can also be handled within the realm of bifurcation buckling analysis.

Pressure rotation effects are important in cylindrical shells only when the shell buckles with a smaller number of waves in the circumferential direction, a phenomenon that occurs on long shells. Hence there is sufficient scope for including follower force effect originating from hydrostatic pressure in the collapse pressure prediction of submarine shells.

1.10 DESIGN ASPECTS OF SUBMARINE HULLS

A landmark paper on submarine design is presented by Arentzen and Mandel (1960). The design procedure forwarded by Kendrick (1970) has received acceptance in European codes (BS 5500 and DnV). According to Kendrick the advantage in submarine strength prediction is that the hydrostatic loading is well defined. Under static conditions the ring-framed cylinder may fail by general instability, inter frame buckling or yielding of the plate between frames. Overall collapse between bulkheads or general instability is a low order-buckling phenomenon due to insufficiently strong frames in relation to the compartment length. Reducing the effective compartment length and/or introducing stronger ring frames can markedly increase the buckling pressure. Kendrick has published about half a dozen design papers. His design method is based on the philosophy that it is more practical to arrange the prime mode of collapse that determine the main weight and cost of the vessel should have an adequate but not excessive strength margin. But
other forms of collapse mode that require considerable analytical effort for accurate collapse prediction but little material to avoid premature buckling can be avoided by using generous margins of elastic buckling pressure for the appropriate mode.

A rational submarine hull design proposes scantlings for an optimum structural form, which has adequate safety at the operational diving depth. The designer has to take into account many uncertainties and unavoidable situations like slight variation in material characteristics, deviations from circularity and other departures from ideal, which may occur in construction or service. Residual stresses particularly in frames, stress concentrations, inaccuracies in computing statically indeterminate systems and possibility of submarine exceeding its operational depth due to control malfunctions or as a deliberate manoeuvre to avoid attack as reported by Daniel (1983) etc., are also to be taken into account. There has to be reasonable stress analysis or strength estimation done before arriving at the final scantlings.

The stiffeners are the principal structural members that support the shell membrane and maintain its integrity. Actually externally welded frames are more stable than internal frames (Gorman & Louie, 1994). It also allows better utilization of internal spaces. However, these experience tensile stresses in a corrosive environment and are more likely to have separation from shell plating under dynamic loading and hence not adopted usually. From the hydrodynamic point of view internal frames are preferred.

1.11 ORGANISATION OF THE THESIS

This thesis is presented in six chapters. In the first chapter an introduction for submarines, structural action of underwater shells and method of structural analysis employed are given. Brief description of type of finite element analyses of stiffened cylindrical shells is presented.

In the second chapter a review of literature on finite element analysis of cylindrical shell is presented and the objectives of the present study are given here.

Third chapter describes the linear static analysis of stiffened cylindrical shells. The description of the all-cubic element and discrete stiffener element used in
the analysis are given. The validation of computer code developed and numerical investigations of stiffened cylindrical shell models of submarines are included.

Fourth chapter describes the linear buckling analysis of stiffened cylindrical shells, which predicts the collapse pressure of submarine hull. Validation and analytical investigation of submarine cylindrical shell models are included subsequently.

The description of the nonlinear analysis of stiffened cylindrical shell is given in the fifth chapter. Development of software and results of numerical investigations are described. Conclusions and scope for future work are given in chapter 6.

The details of elements of stiffness matrices are given in Appendix A and classical solutions and Rulebook provisions for the analysis of stiffened cylindrical shells are depicted in Appendix B.