4. **METHOD OF SOLUTION**

In Chapter 3, the characteristics and compatibility equations corresponding to the unsteady one-dimensional flow equations are presented. In the present chapter, the numerical procedure for solving these equations is described. The characteristic equations, Eqns. (3.1.8 & 3.1.10) and the compatibility equations, Eqns. (3.1.9 & 3.1.11) are non-linear total differential equations and their solution is obtained numerically by applying finite difference technique. Zucrow-Hoffman algorithm (1977) is employed for the solution and the unit processes for determining the flow properties at the interior points and at the boundary points are developed.

4.1 **Numerical Solution Procedure**

Eqn. (3.1.8) defines the pathline $C_o$ while the compatibility equation, Eqn. (3.1.9) provides a differential relationship between pressure and density along the pathline. Eqn. (3.1.10) defines two characteristics (right running $C_r$ and left running $C_l$) and Eqn. (3.1.11) specifies one relationship between velocity and pressure on each of the two characteristics. To obtain three independent relationships between velocity $u$, pressure $p$ and density $\rho$ at a point in the flow field, a network shown in Fig. (4.1) is devised wherein the two characteristics and the pathline intersect at a common point. At the point of intersection the three compatibility equations valid along the two characteristics and along the pathline are solved simultaneously for the three unknowns $p$, $u$ and $\rho$ for an interior point. For flow properties at a boundary point, one or more of the three compatibility equations, depending upon the boundary conditions, are replaced by the same number of boundary equations.

In the present study, modified Euler predictor-corrector method using the inverse marching procedure is employed for the integration of the characteristic and compatibility equations following Zucrow and Hoffman (1977). Fig. (4.2) illustrates schematically the finite difference grid in the $x$-$t$ (position-time) plane for determining the flow properties at an interior point based on the inverse marching method. Points 5, 6 and 7 denoted by (●) are the solution points at positions $x-\Delta x$, $x$ and $x+\Delta x$. 
respectively along the pipe at time $t = t_1$. Point 4 denoted by (*) is the current solution point at position $x$ at time $t = t_1 + \Delta t$. Points 1, 2 and 3 denoted by $\Theta$ are located at the intersection of the characteristics $C_+$, $C_-$ and pathline $C_o$, respectively with the previous solution line at time $t = t_1$. $\Delta x$ and $\Delta t$ are the spatial and time steps. The location of the current solution point 4 is specified a priori where the two characteristics and the pathline have to pass. The flow properties at points 1, 2 and 3 are obtained by interpolation from the previous solution points such as points 5, 6 and 7. The Courant-Friedrichs Lewy stability criterion (Courant et al, 1928) must be satisfied to ensure that the solution is stable. The criterion requires that the initial data points (1,2 and 3) fall between the previous solution points (5,6 and 7) which are employed in the interpolation for determining the flow properties at points 1, 2 and 3.

4.1.1 Finite Difference Equations

To obtain the finite difference equations corresponding to the characteristics and compatibility equations, Eqns. (3.1.8 - 3.1.11), a finite difference grid is constructed with the characteristics curves, wherein the portion of a characteristic curve connecting two points of the grid is replaced by a straight line as shown in Fig. (4.2). The slope of this line is determined using average values between the end points of the pertinent segment of the characteristic (e.g., points 1 and 4). According to the Euler method, the finite difference equations corresponding to the differential equations, Eqns. (3.1.8 - 3.1.11) are obtained by replacing the differentials $dt$, $dx$, $du$, $dp$ and $d\rho$ by the differences $\Delta t$, $\Delta x$, $\Delta u$, $\Delta p$, and $\Delta \rho$, respectively. The finite difference equations corresponding to Eqns. (3.1.8 - 3.1.11) are given in Table (4.1).
Table (4.1) Finite Difference Equations for Unsteady One-dimensional Flow

<table>
<thead>
<tr>
<th>Differential Equations</th>
<th>Finite Difference Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Characteristic Equations</strong></td>
<td></td>
</tr>
<tr>
<td>Pathline Eqn. (3.1.8)</td>
<td>( \Delta t_0 = \lambda_0 \Delta x_0 ) (4.1.1)</td>
</tr>
<tr>
<td>Right running characteristic Eqn. (3.1.10a)</td>
<td>( \Delta t_+ = \lambda_+ \Delta x_+ ) (4.1.2a)</td>
</tr>
<tr>
<td>Left running characteristic Eqn. (3.1.10b)</td>
<td>( \Delta t_- = \lambda_- \Delta x_- ) (4.1.2b)</td>
</tr>
</tbody>
</table>

| **Compatibility Equations** | | |
| Along path line, Eqn. (3.1.9) | \( \Delta \rho - A_0 \Delta \rho_0 = B_0 \Delta x_0 \) (4.1.3) |
| Along right running characteristic, Eqn. (3.1.11a) | \( \Delta \rho_+ + Q_+ \Delta u_+ = S_+ \Delta t_+ \) (4.1.4a) |
| Along left running characteristic, Eqn. (3.1.11b) | \( \Delta \rho_- - Q_- \Delta u_- = S_- \Delta t_- \) (4.1.4b) |

Subscripts + and – denote right and left running characteristics, respectively and the subscript o denotes the pathline. \( \lambda_0, \lambda_+, A_0, B_0, Q_+, S_+ \) and \( S_- \) are the coefficients of the difference equations and are defined later in this section.

The finite difference equations, Eqns. (4.1.1 – 4.1.4) are expressed in terms of points 3 and 4 along the pathline \( C_0 \) (see Fig. 4.2), and in terms of points 1 and 4 along the right running characteristic \( C_+ \) and points 2 and 4 along the left running characteristic \( C_- \). The resulting computational equations based on the average property method for determining the coefficients of finite difference equations are presented below in Table (4.2).
Table (4.2) Computational Equations for Unsteady One Dimensional Flow Using Zucrow-Hoffman Algorithm

<table>
<thead>
<tr>
<th>Finite Difference Equations</th>
<th>Computational Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pathline Eqn. (4.1.1)</td>
<td>$\Delta t_0 = \lambda_0 (x_4 - x_3)$ (4.1.5)</td>
</tr>
<tr>
<td>Compatibility Eqn. (4.1.3)</td>
<td>$p_4 - A_0 \rho_4 = T_0$ (4.1.6)</td>
</tr>
<tr>
<td>along the path line</td>
<td>$T_0 = B_0 (x_4 - x_3) + p_3 - A_0 \rho_3$ (4.1.7)</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>$= 1/u_0$ (4.1.8)</td>
</tr>
<tr>
<td>$A_0$</td>
<td>$= a_o^2$ (4.1.9)</td>
</tr>
<tr>
<td>$B_0$</td>
<td>$= \psi_0/u_0$ (4.1.10)</td>
</tr>
<tr>
<td>$\psi_0$</td>
<td>$= (\gamma_0 - 1) (\rho_o u_0 \delta Q_{xo} - u_0 \beta_o)$ (4.1.11)</td>
</tr>
<tr>
<td>$\delta Q_{xo}$</td>
<td>$= \frac{f}{2} \left( \frac{\gamma_0 R_o}{\gamma_0 - 1} \right) u_0 \left( \frac{T_{wp} - T_g}{x_6 - x_5} \right)$ (4.1.12)</td>
</tr>
<tr>
<td>$\beta_o$</td>
<td>$= -\left( \rho_0 u_0^2 / 2 \right) (4f/D_o)$ (4.1.13)</td>
</tr>
<tr>
<td>Right running characteristic</td>
<td>$\Delta t_+ = \lambda_+ (x_4 - x_1)$ (4.1.14a)</td>
</tr>
<tr>
<td>Eqn. (4.1.2a)</td>
<td></td>
</tr>
<tr>
<td>Left running characteristic</td>
<td>$\Delta t_- = \lambda_- (x_4 - x_2)$ (4.1.14b)</td>
</tr>
<tr>
<td>Eqn. (4.1.2b)</td>
<td></td>
</tr>
<tr>
<td>Compatibility equation along right running characteristic</td>
<td>$p_4 + Q_4 u_4 = T_+$ (4.1.15a)</td>
</tr>
<tr>
<td>Eqn. (4.1.4a)</td>
<td></td>
</tr>
<tr>
<td>Compatibility equation along left running characteristic</td>
<td>$p_4 - Q_4 u_4 = T_-$ (4.1.15b)</td>
</tr>
<tr>
<td>Eqn. (4.1.4b)</td>
<td></td>
</tr>
<tr>
<td>$T_+ = S_+ \Delta t + p_1 + Q_+ u_1$ (4.1.16a)</td>
<td></td>
</tr>
<tr>
<td>$T_- = S_- \Delta t + p_2 - Q_- u_2$ (4.1.16b)</td>
<td></td>
</tr>
</tbody>
</table>
\[ \lambda_+ = \frac{1}{u_+ + a_+} \]  
\[ \lambda_- = \frac{1}{u_- - a_-} \]  
\[ Q_+ = \rho_+ a_+ \]  
\[ Q_- = \rho_- a_- \]  
\[ S_+ = -\delta_s \rho_+ u_+ a_+^2/x_+ + a_+ \beta_+ + \psi_+ \]  
\[ S_- = -\delta_s \rho_- u_- a_-^2/x_- + a_- \beta_- + \psi_- \]  
\[ \delta_+ = \frac{2}{D_+} (dD/dx) \]  
\[ \delta_- = \frac{2}{D_-} (dD/dx) \]  
\[ \beta_+ = - (\rho_+ u_+^2/2) (4f/D_+) \]  
\[ \beta_- = - (\rho_- u_-^2/2) (4f/D_-) \]  
\[ \psi_+ = (\gamma_+ - 1)(\rho_+ u_+ \delta Q_{x+} - u_+ \beta_+) \]  
\[ \psi_- = (\gamma_- - 1)(\rho_- u_- \delta Q_{x-} - u_- \beta_-) \]  
\[ \delta Q_{x+} = \frac{f}{2} \left( \frac{\gamma_+ R_+}{\gamma_+ - 1} \right) u_+ \left( \frac{T_{wp} - T_s}{x_6 - x_5} \right) \]  
\[ \delta Q_{x-} = \frac{f}{2} \left( \frac{\gamma_- R_-}{\gamma_- - 1} \right) u_- \left( \frac{T_{wp} - T_s}{x_7 - x_6} \right) \]
4.1.2 Integration Method

The integration of the characteristic and compatibility Eqns. (3.1.8-3.1.11) is performed in the following steps:

1. Determination of the locations of the initial data points 1, 2 and 3 (Fig. 4.2) on the previous t-line.

2. Determination of the coefficients $\lambda_0, \lambda_x, A_0, B_0, S_x, A, B, S$ and $Q_x$ of the finite difference equations (characteristic and compatibility). These coefficients are obtained in the following two steps according to the modified Euler predictor-corrector method.

   (i) In the predictor step, all the coefficients are calculated at the known initial data points; 1, 2 and 3.

   (ii) In the corrector step, the coefficients are calculated based on the average values of the flow properties of the initial data points 1, 2 and 3 and the solution point 4.

The detailed integration procedure for various finite difference equations is described in the following sections.

4.1.2a Right Running Characteristic Equations

Fig. (4.3) shows a finite difference grid in the x-t plane for the right running characteristic $C_x$. The integration of the finite difference form of the characteristic and compatibility equations, Eqns. (4.1.2a) and (4.1.4a), respectively valid along the right running characteristic is carried out in the following steps:

1. The location of the solution point 4 denoted by $(x, t_1, +\Delta t)$ is prespecified as $x_4 = x_6 (x, t_1)$.

2. The initial location of point 1 is taken to be as the location of point $5 (x-\Delta x, t_1)$ and therefore, the properties at point 1 are assumed equal to those at point 5.
The Predictor Step

3. The initial estimation of the slope of the right running characteristic \( C^+ \) is determined by Eqn. (4.1.17a) given in Table (4.2).

\[
\lambda_+ = \frac{1}{u_+ + a_+}
\]

4. \( x_i \) is found from Eqn. (4.1.14a), \( \Delta t = \lambda_+ (x_4 - x_i) \). The time step \( \Delta t \) is taken the same for the pathline \( C_0 \) and for the two characteristics \( C^+ \) and \( C^- \) in the present case of inverse marching algorithm (Zucrow and Hoffman 1977), so that,

\[
\Delta t_0 = \Delta t_+ = \Delta t_-
\]

5. The properties at point 1 are determined by linear interpolation between points 5 and 6. The expressions for \( u_1 \) and \( a_1 \) are given as :

\[
u_1 = u_6 + \left( \frac{u_5 - u_6}{x_5 - x_6} \right) (x_1 - x_6)
\]

\[
a_1 = a_6 + \left( \frac{a_5 - a_6}{x_5 - x_6} \right) (x_1 - x_6)
\]

6. The modified value of \( \lambda_+ \) is recalculated from Eqn. (4.1.17a) using the properties at point 1, \( u_1 \) and \( a_1 \). Steps 4 and 5 above are repeated to obtain improved values of \( x_1 \) until specified convergence limit is achieved.

7. The coefficients of the compatibility equation, Eqn. (4.1.15a) valid along the right running characteristic \( Q^+ \), \( T^+ \) and \( S^+ \) are calculated for Euler predictor step using Eqns. (4.1.16a), (4.1.18a), (4.1.19a), (4.1.20a), (4.1.21a), (4.1.22a) and (4.1.23a).

For the Euler predictor step values of \( u_+, p_+, \rho_+, \gamma_+, a_+, R_+, D_+, \) \( x_+ \) are given by the following equations :

\[
u_+ = u_1 \quad , \quad p_+ = p_1
\]

\[
\rho_+ = \rho_1 \quad , \quad \gamma_+ = \gamma_1
\]

\[
a_+ = \left[ \frac{(\gamma_+ p_+)}{\rho_-} \right]^{1/2} \quad , \quad R_+ = R_1
\]

\[
D_+ = D_1 \quad , \quad x_+ = x_1
\]
8. Eqn. (4.1.15a) is solved simultaneously along with another equation either a boundary equation or a characteristic equation to obtain the predicted values of the flow properties at point 4 \((p_4, u_4)\).

**The Corrector Step**

9. To obtain corrected values of \(p_4\) and \(u_4\) obtained by the above Euler predictor method, steps 3-8 above are repeated for the Euler corrector step employing average values for the flow properties given by the following equations:

\[
\begin{align*}
  u_+ = \frac{(u_1 + u_4)}{2} , \\
  \rho_+ = \frac{(\rho_1 + \rho_4)}{2} , \\
  p_+ = \frac{(p_1 + p_4)}{2} , \\
  \gamma_+ = \frac{(\gamma_1 + \gamma_4)}{2} , \\
  a_+ = \left(\frac{(\gamma_+ p_+)}{\rho_+}\right)^{\frac{1}{2}} , \\
  R_+ = \frac{(R_1 + R_4)}{2} , \\
  D_+ = \frac{(D_1 + D_4)}{2} , \\
  x_+ = \frac{(x_1 + x_4)}{2}
\end{align*}
\]

During the iteration in steps 4, 5 and 6 above the values of \(p_4\) and \(u_4\) remain fixed at the values obtained at the predictor step. The coefficients of the compatibility equations (4.1.15a), \(Q_+\), \(T_+\) and \(S_+\) are recalculated at the corrector step using Eqns. (4.1.16a), (4.1.18a), (4.1.19a), (4.1.20a), (4.1.21a), (4.1.22a) and (4.1.23a).

10. Equation (4.1.15a) is solved simultaneously along with another equation, either a boundary equation or a characteristic equation to obtain the corrected values of the flow properties \(p_4\) and \(u_4\).

11. To improve the accuracy of the solution values, \(p_4\) and \(u_4\), the corrector steps 9 & 10 are repeated substituting the corrected values of \(p_4\) and \(u_4\) obtained in the first pass in the corrector step (step 10) instead of substituting the predicted values of \(p_4\) and \(u_4\) obtained by the predictor step (step 7).

The flow chart for computer programming of steps 1-11 above is presented in Fig. (4.4) and is called subroutine CHARR. In the computer programs the number of applications of the modified Euler corrector step is
governed by a control parameter called ICOR. Before starting the calculations, ICOR is assigned a value of 1, 2 or 3. Another control variable Iter is a counter representing the number of iterations after the first pass in the corrector step.

### 4.1.2b Left Running Characteristic Equations

Fig. (4.5) shows a finite difference grid in the x-t plane for the left running characteristic $C_-$. The integration of the finite difference form of the characteristic and compatibility equations, Eqns. (4.1.2b) and (4.1.4b), valid along the right running characteristic is carried out in the following steps.

1. The location of the solution point 4 $(x, t_1 + \Delta t)$ is prespecified as $x_4 = x_6 (x, t_1)$.

2. The initial location of point 2 is taken at point $7(x + \Delta x, t_1)$ and therefore the properties at point 2 are assumed equal to those at point 7.

**Predictor Step**

3. The initial estimation of the slope of the left running characteristic $C_-$ is determined by Eqn. (4.1.17b) from Table (4.2).

$$\lambda_- = \frac{1}{u_- - a_-}$$

4. $x_2$ is obtained from Eqn. (4.1.14b). $\Delta t = \lambda_+ (x_4 - x_2)$

5. Properties at point 2 are determined by linear interpolation between points 6 and 7. The expressions for $u_2$ and $a_2$ are given as:

$$u_2 = u_6 + \left( \frac{u_7 - u_6}{x_7 - x_6} \right) \left( x_2 - x_6 \right)$$

$$a_2 = a_6 + \left( \frac{a_7 - a_6}{x_7 - x_6} \right) \left( x_2 - x_6 \right)$$

6. Modified value of $\lambda_-$ is recalculated from Eqn. (4.1.17b) using properties at point 2, $u_2$ and $a_2$. Steps 4 and 5 above are repeated to obtain improved values of $x_2$ until specified convergence limit is achieved.

7. The coefficients of the compatibility Eqn. (4.1.15b) valid along the left
running characteristic, \( Q_\), \( T_\) and \( S_\) are calculated for Euler predictor step using Eqns. (4.1.16b), (4.1.18b), (4.1.19b), (4.1.20b), (4.1.21b), (4.1.22b) and (4.1.23b). For the Euler predictor step values of \( u_\), \( p_\), \( \rho_\), \( \gamma_\), \( a_\), \( R_\), \( D_\), \( x_\) are given by the following equations:

\[
\begin{align*}
  u_\ &= \ u_2  \\
  p_\ &= \ p_2  \\
  \rho_\ &= \ \rho_2  \\
  \gamma_\ &= \ \gamma_2  \\
  a_\ &= \ [(\gamma_\ p_\)/\rho_\]^{1/2}  \\
  R_\ &= \ R_2  \\
  D_\ &= \ D_2  \\
  x_\ &= \ x_2
\end{align*}
\]

8. Eqn. (4.1.15b) is solved simultaneously along with another equation, either a boundary equation or a characteristic equation, to obtain the predicted values of the flow properties at point 4, \( p_4 \) and \( u_4 \)

**Corrector Step**

9. To obtain corrected values of \( p_4 \) and \( u_4 \), steps 3-8 above are repeated for the Euler corrector employing the average values for the flow properties given by the following equations:

\[
\begin{align*}
  u_\ &= \ (u_2 + u_4)/2  \\
  \rho_\ &= \ (\rho_2 + \rho_4)/2  \\
  p_\ &= \ (p_2 + p_4)/2  \\
  \gamma_\ &= \ (\gamma_2 + \gamma_4)/2  \\
  a_\ &= \ [(\gamma_\ p_\)/\rho_\]^{1/2}  \\
  R_\ &= \ (R_2 + R_4)/2  \\
  D_\ &= \ (D_2 + D_4)/2  \\
  x_\ &= \ (x_2 + x_4)/2
\end{align*}
\]

During the iteration in steps 4, 5 and 6, the values of \( p_4 \) and \( u_4 \) remain fixed at the values determined by the predictor step. The coefficients of the compatibility equation, Eqn. (4.1.15b), \( Q_\), \( T_\) and \( S_\) are recalculated for the corrector step using Eqns. (4.1.16b), (4.1.18b), (4.1.19b), (4.1.20b), (4.1.21b), (4.1.22b) and (4.1.23b).

10. Eqn. (4.1.15b) is solved simultaneously along with another equation, either a boundary equation or a characteristic equation, to obtain the corrected values of the flow properties \( p_4 \) and \( u_4 \).

11. To improve accuracy of the solution values, \( p_4 \) and \( u_4 \), the corrector
steps 9 & 10 are repeated using the corrected values of \( p_4 \) and \( u_4 \) obtained in the first pass in the corrector (step 10) instead of using the predicted values of \( p_4 \) and \( u_4 \) obtained in the Euler predictor step (step 7).

The flow chart for computer programming of the above steps is shown in Fig. (4.6) and is called subroutine CHARL.

4.1.2c Pathline Equations

Fig. (4.7) shows a finite difference grid in the x-t plane for the path line \( C_o \). The integration of the finite difference form of the characteristic and compatibility equations, Eqn. (4.1.1) and Eqn. (4.1.3) in Table (4.1), valid along the pathline is carried out in the following steps.

1. The location of point 4 \( (x_4, t_1 + \Delta t) \) is prespecified as \( x_4 = x_6(x_1, t_1) \)

2. Initially point 3 is assumed to coincide with point 6 and therefore the properties at point 3 are those at point 6.

**Predictor Step**

3. The slope of the pathline is determined from Eqn. (4.1.8)

\[
\lambda_o = \frac{1}{u_0}
\]

4. \( x_3 \) is found from pathline Eqn. (4.1.5)

\[
\Delta t_o = \lambda_o (x_4 - x_3)
\]

5. Properties at point 3 are determined by linear interpolation. If the direction of flow is from left to right 'positive flow', values at point 3 are obtained by interpolation of values at point 5 and 6 as shown in Fig. (4.7a). If the direction of flow is from right to left 'reverse flow', values at point 3 are obtained using points 6 and 7 as shown in Fig. (4.7b).

For \( x_3 < x_6 \)

\[
u_3 = u_6 + \left( \frac{u_5 - u_6}{x_5 - x_6} \right) (x_3 - x_6)
\]
For $x_3 > x_6$

$$u_3 = u_6 + \left( \frac{u_7 - u_6}{x_7 - x_6} \right) \left( x_3 - x_6 \right)$$

6. The modified values of $\lambda_o$ is recalculated from Eqn. (4.1.8) using the properties at point 3 estimated in step 5. Steps 4 and 5 are repeated to obtain improved values of $x_3$ until specified convergence tolerance is achieved.

7. The coefficients for the compatibility equation (4.1.6) valid along pathline, $T_o, B_o$ and $A_o$ are calculated for Euler predictor step using Eqns. (4.1.7), (4.1.9), (4.1.10), (4.1.11), (4.1.12), (4.1.13). For the Euler predictor $u_o, p_o, \rho_o, \gamma_o, a_o, R_o, D_o,$ are given by the following expressions.

$$u_o = u_3, \quad p_o = p_3$$
$$\rho_o = \rho_3, \quad \gamma_o = \gamma_3$$
$$a_o = \left[ (\gamma_o p_o) / \rho_o \right]^{1/2}, \quad R_o = R_3$$
$$D_o = D_3$$

8. Eqn. (4.1.6) is solved simultaneously along with either a boundary equation or another characteristic equation to obtain predicted values of the flow properties at point 4, $p^4$ and $\rho^4$.

The Corrector Step

9. To obtain corrected values of $p^4$ and $\rho^4$, steps 3-8 above are repeated employing Euler corrector step using the values of $u_o, p_o, \rho_o, \gamma_o, a_o, R_o, D_o,$ as given by the following expressions.

$$u_o = (u_3 + u_4)/2, \quad \rho_o = (\rho_3 + \rho_4)/2$$
$$p_o = (p_3 + p_4)/2, \quad \gamma_o = (\gamma_3 + \gamma_4)/2$$
$$a_o = \left[ (\gamma_o p_o) / \rho_o \right]^{1/2}, \quad R_o = (R_3 + R_4)/2$$
$$D_o = (D_3 + D_4)/2$$

During the iteration in steps 4, 5 and 6, $p^4$ and $\rho^4$ remain fixed at values determined by the predictor procedure.
10. To improve the accuracy of the solution, the corrector step 9 is repeated substituting the corrected values of $p_4$ and $p_4$, obtained in the first pass in the corrector (step 9) instead of using values of $p_4$ and $p_4$ obtained by the predictor (step 7).

The flow chart for computer programming of the above algorithm is shown in Fig. (4.8a) for positive flow (subroutine PATHR) and in Fig. (4.8b) for reverse flow (subroutine PATHL).

4.2 Unit Processes

Three equations are required in order to determine the flow properties $p$, $u$ and $\rho$ at any point in the flow field. These equations are either the three compatibility equations in case of internal points of the flow field or combinations of compatibility and boundary equations in case of points on boundaries. The valid equations at any point are solved simultaneously to form computational equations for determining the three unknowns $p$, $u$ and $\rho$. The complete numerical procedure to obtain the flow properties at any point is called a unit process. In the present work, various unit processes have been developed for flow analysis and are given in the following sections.

4.2.1 Unit Process for an Interior Point

Fig. (4.2), described earlier, also illustrates the finite difference grid for the unit process of an interior point (point 4). At point 4, three characteristics are originating from inside the flow field and the compatibility equations valid along these characteristics are:

*Along right running characteristic*

$$p_4 + Q_+ u_4 = T_+$$ (4.2.1)

*Along left running characteristic*

$$p_4 - Q_- u_4 = T_-$$ (4.2.2)
Along path line

\[ p_4 - A_0 \rho_4 = T_0 \]  \hspace{1cm} (4.2.3)

The above equations are solved simultaneously for three flow properties \( p_4, u_4 \) and \( \rho_4 \) so that

\[ u_4 = \frac{(T_+ - T_-)}{(Q_+ + Q_-)} \]  \hspace{1cm} (4.2.4)

\[ p_4 = (T_+ - Q_+ u_4) \]  \hspace{1cm} (4.2.5)

\[ \rho_4 = \frac{(p_4 - T_o)}{A_o} \]  \hspace{1cm} (4.2.6)

and \[ a_4 = \left[ \frac{\gamma_4 p_4}{\rho_4} \right]^{\frac{1}{2}} \]  \hspace{1cm} (4.2.7)

The computational procedure of the unit process for an interior point consists of the following steps:

1. Coefficients \( Q_+ \) and \( T_+ \) of the compatibility equation valid along the right running characteristic Eqn. (4.2.1) are determined for the predictor step as described in Section (4.1.2a) and the coefficients \( Q_- \) and \( T_- \) of Eqn. (4.2.2) valid along the left running characteristic are determined as described in Section (4.1.2b). The coefficients \( A_o \) and \( T_o \) of the path line Eqn. (4.1.3) are determined for the predictor step as described in Section (4.1.2c).

2. Predicted values of \( u_4, p_4 \) and \( \rho_4 \) are obtained from Eqns. (4.2.4), (4.2.5) and (4.2.6), respectively and \( a_4 \) is obtained from Eqn. (4.2.7).

3. Coefficients \( Q_+, T_+, Q_-, T_-, A_o \) and \( T_o \) are determined again for the corrector step as described in Section (4.1.2) and corrected values of \( u_4, p_4 \) and \( \rho_4 \) are obtained from Eqns. (4.2.4), (4.2.5) and (4.2.6) respectively and corrected value of \( a_4 \) is obtained from Eqn. (4.2.7).

The flow chart describing the calculation procedure of the unit process for an interior point is presented in Fig. (4.9) and forms subroutine INTER of the computer program.
4.2.2 Unit Process for a Closed End

The finite difference grid for the unit process of a closed end at the left hand side of the flow field is illustrated in Fig. (4.10). Two characteristics pass at the solution point 4, the left running characteristic $C_-$ and the pathline $C_0$ which is located along the closed boundary as shown in the figure. The two compatibility equations valid are:

*Along left running characteristic*

\[ p_4 - Q_\cdot u_4 = T_- \quad (4.2.8) \]

*Along the path line*

\[ p_4 - A_0 \rho_4 = T_0 \quad (4.2.9) \]

The third equation required to solve for the three unknowns $p_4$, $u_4$ and $\rho_4$ is supplied by the boundary condition for a closed end, see Section (3.2.1), and is given below:

\[ u_4 = 0 \quad (4.2.10) \]

Eqns. (4.2.8), (4.2.9) and (4.2.10) are solved simultaneously for $p_4$ and $\rho_4$ to give

\[ p_4 = T_- \quad (4.2.11) \]

\[ \rho_4 = (p_4 - T_0)/A_0 \quad (4.2.12) \]

\[ a_4 = [\gamma_4 p_4/\rho_4]^{\frac{1}{2}} \quad (4.2.13) \]

For the predictor step, the coefficients $Q_-$ and $T_-$ of the compatibility Eqn. (4.2.8) of the left running characteristic are determined as described in Section (4.1.2b) and the coefficients $A_0$ and $T_0$ of the pathline compatibility equation are obtained as described in Section (4.1.2c),

The coefficients $T_-$, $A_0$ and $T_0$ are determined again for the corrector step as described in Section (4.1.2) and corrected values of $p_4$ and $\rho_4$ are obtained. $a_4$ is obtained from Eqn. (4.2.13).

The flow chart describing the programming procedure for the unit process of a closed end located on the left hand side of the pipe is shown in Fig. (4.11).
and the related computer program is called subroutine CLOSL. If the closed end is at the right hand side of the pipe, Eqn. (4.2.8) is replaced by the compatibility equation of the right running characteristics therefore:

\[ p_4 = T_+ \text{ for } u_4 = 0 \]

And the program describing this unit process is called subroutine CLOSR. This boundary also represents cylinder boundary conditions when the ports are closed.

### 4.2.3 Unit Process for an Open End

As described in Section (3.2.2), there are two cases to be considered, namely subsonic outflow and subsonic inflow.

**i) Subsonic Outflow**

Fig. (4.12a) illustrates the finite difference grid for the unit process of an open end with subsonic outflow at the right hand side of the internal flow field. Two characteristic curves originate from inside the flow passage, the pathline \( C_o \) and the right running characteristic \( C_+ \). These characteristics provide two compatibility equations, and the third equation is supplied by the boundary equation for subsonic outflow in an open end, given by Eqn. (3.2.2)

\[ p_4 = p_o \]  \hspace{1cm} (4.2.14)

The two known compatibility equations are,

*Along right running characteristic*

\[ p_4 + Q_+ u_4 = T_+ \]  \hspace{1cm} (4.2.15)

*Along Pathline*

\[ p_4 - A_o \rho_4 = T_o \]  \hspace{1cm} (4.2.16)

Eqns. (4.2.14), (4.2.15) and (4.2.16) are solved simultaneously for \( u_4 \) and \( \rho_4 \) to give:

\[ u_4 = (T_+ - p_4)/Q_+ \]  \hspace{1cm} (4.2.17)

\[ \rho_4 = (p_4 - T_o)/A_o \]  \hspace{1cm} (4.2.18)
The speed of sound $a_4$ is calculated as

$$a_4 = \sqrt{\frac{\gamma_4 p_4}{\rho_4}} \quad (4.2.19)$$

The coefficients $Q_+$ and $T_+$ of the right running characteristics and $A_0$, $T_0$ of the pathline are determined for both predictor and corrector steps as described in Section (4.1.2) and the set of equations, Eqns. (4.2.17), (4.2.18) and (4.2.19) are solved once to obtain predicted values of $p_4$, $u_4$ and $\rho_4$ and once to obtain corrected values of $p_4$, $u_4$ and $\rho_4$ as described in Section (4.1.2).

The flow chart for the subsonic outflow in an open end is described in Fig. (4.13) and the related program is called subroutine OPENR. If the open end is at the left side of the pipe, the compatibility equation of the right running characteristic, Eqn. (4.2.15) is replaced by that of the left running characteristic:

$$p_4 - Q_+ u_4 = T_+$$

The program for this case is called subroutine OPENL.

(ii) Subsonic Inflow

Fig. (4.12b) illustrates the finite difference grid for the unit process of subsonic inflow at an open end. In this case, single characteristic $C_+$ reaches point 4 from inside the flow passage and therefore two boundary conditions must be specified from the external flow field. Eqn. (3.2.3) is the boundary equation which relates the two flow properties $p_4$ and $u_4$ at the solution point 4 to the stagnation external conditions and is rewritten as:

$$\left(\frac{p_4}{p_0}\right)^{(\gamma - 1)/\gamma} + \frac{\gamma - 1}{2} \left(\frac{u_4}{a_0}\right)^2 = 1 \quad (4.2.20)$$

where, $p_0$ and $a_0$ are known stagnation external conditions.

The compatibility equation along the known characteristic $C_+$ is:

$$p_4 + Q_+ u_4 = T_+ \quad (4.2.21)$$

Using Eqns. (4.2.20) and (4.2.21) and defining $X_4 = p_4/p_0$, we have:
\[ F(X_4) = (X_4)^{(\gamma - 1)/\gamma} + (\gamma - 1)/2 \times (1/a_0^2) \times \{(T_+ - X_4p_0)/Q_+\}^2 - 1 = 0 \]  \hfill (4.2.22)

Eqn. (4.2.22) is solved by Newton-Raphson method for \( X_4 \) and the derivative \( dF(X_4)/dX_4 \) denoted by \( F'(X_4) \) is given by the following expression:

\[ F'(X_4) = (\gamma - 1)/\gamma \times 1/X_4^{(1/\gamma)} - (\gamma - 1) \times (p_o/a_0^2) \times (T_+ - X_4p_0)/Q_+^2 \]  \hfill (4.2.23)

Once \( X_4 \) is obtained, the flow properties are obtained as below:

\[ p_4 = X_4 \times p_o \]  \hfill (4.2.24)

\[ u_4 = (T_+ - p_4) / Q_+ \]  \hfill (4.2.25)

\[ \rho_4 = \left( \frac{\gamma p_4}{a_4} \right)^{\gamma/2} \]  \hfill (4.2.26)

and

\[ a_4 = a_o \left( \frac{p_4}{p_o} \right)^{(\gamma - 1)/2\gamma} \]  \hfill (4.2.27)

The coefficients of the compatibility equations \( Q_+, T_+ \) of the right running characteristics are determined, once for the predictor step to obtain predicted values of \( p_4, u_4, \rho_4 \) and once or more for the corrector step to obtain the corrected values as described in Section (4.1.2). The flow chart for the unit process of inflow in an open end is given in Fig. (4.14) and the respective computer program is called subroutine INFLR.

For subsonic inflow in an open end at the left side of a pipe. Eqn. (4.2.21) is replaced by the compatibility equation of the left running characteristic:

\[ p_4 - Q_- u_4 = T_- \]

### 4.2.4 Unit Process for Partially Open End (Nozzle)

There are three cases to be considered as described in Section (3.2.3).

(i) Subsonic Outflow

(ii) Sonic Outflow

(iii) Subsonic Inflow

The analysis is carried out here for a nozzle placed at the right end of the pipe. The unit process used for fully open end as illustrated in Fig. (4.12) is employed as well for a partially open end.
(i) **Subsonic Outflow**

Following the open end case as shown in Fig. (4.12a), the compatibility equations for the partially open end are also written as:

*Along the right running characteristic*

\[ p_4 + Q_4 u_4 = T_+ \]  \hspace{1cm} (4.2.28)

*Along the pathline*

\[ p_4 - A_o \rho_4 = T_0 \]  \hspace{1cm} (4.2.29)

The boundary equations for subsonic outflow in a nozzle, Eqn. (3.2.8) is rewritten here for conditions at solution point 4, as:

\[
\left( \frac{u_4}{a_o} \right)^2 = \frac{2}{\gamma_4 - 1} \left[ \left( \frac{p_4}{\rho_o} \right)^{(\gamma_4-1)/\gamma_4} - 1 \right] \left[ \left( \frac{p_4}{\rho_o} \right)^{2/\gamma_4} - \frac{1}{\phi^2} - 1 \right]
\]  \hspace{1cm} (4.2.30)

The isentropic relationship is:

\[
\left( \frac{a_4}{a_o} \right) = \left( \frac{p_4}{\rho_o} \right)^{(\gamma_4-1)/2\gamma_4}
\]  \hspace{1cm} (4.2.31)

The speed of sound is given by:

\[
a_4 = \left( \frac{\gamma_4 p_4}{\rho_4} \right)^{1/2}
\]  \hspace{1cm} (4.2.32)

Using Eqns. (4.2.29), (4.2.31) and (4.2.32), we get

\[
a_o = \left( \frac{\gamma_4 p_4 A_o}{p_4 - T_0} \right)^{1/2} \left( \frac{p_4}{\rho_o} \right)^{(1-\gamma_4)/2\gamma_4}
\]  \hspace{1cm} (4.2.33)

Using Eqn. (4.2.28), (4.2.33) and Eqn. (4.2.30) and rearranging gives:

\[
F(p_4) = \left[ \frac{T_+ - p_4}{Q_+} \right]^2 \left[ \left( \frac{p_4}{\rho_o} \right)^2 - \phi^2 \right] - \left( \frac{2}{\gamma_4 - 1} \right) \cdot \phi^2 \left[ \frac{\gamma_4 p_4 A_o}{p_4 - T_0} \right]
\]
Eqn. (4.2.34) is solved by Newton Raphson method for \( p_4 \). The derivative \( \frac{dF(p_4)}{dp_4} \), \( F'(p_4) \) is given below:

\[
F'(p_4) = \frac{2}{\gamma_4} \cdot \frac{1}{p_0} \left[ \frac{T_+-p_4}{Q_+} \right]^2 \left( \frac{p_4}{p_0} \right)^{2-\gamma_4/\gamma_4} - \frac{2}{Q_+} \left[ \frac{T_+-p_4}{Q_+} \right] \times \\
\left[ \left( \frac{p_4}{p_0} \right)^{\gamma_4/\gamma_4} - \phi^2 \right] + 2\phi^2 A_o T_o \left( \frac{1}{\gamma_4 - 1} \right) \left( \frac{1}{(p_4 - T_0)^2} \right) \left( \frac{p_4}{p_0} \right)^{(1-\gamma_4)/\gamma_4} \\
\times \left[ \left( \frac{p_4}{p_0} \right)^{(\gamma_4 - 1)/\gamma_4} - 1 \right] - 2\phi^2 A_o \left( \frac{1}{p_4 - T_0} \right) - 2\phi^2 A_o \left( \frac{1}{p_4 - T_0} \right) \\
\times \left( \frac{p_4}{p_0} \right)^{(1-\gamma_4)/\gamma_4} \left[ \left( \frac{p_4}{p_0} \right)^{(\gamma_4 - 1)/\gamma_4} - 1 \right] \quad (4.2.35)
\]

after obtaining \( p_4 \), \( u_4 \) and \( \rho_4 \) are calculated from Eqns. (4.2.28) and (4.2.29), respectively.

(ii) **Sonic Outflow**

When the flow in the throat is choked there is sonic flow and the Mach number \( M_{th} \) at the throat is equal to 1

\[
M_{th} = \left( \frac{u_{th}}{a_{th}} \right) = 1 \quad (4.2.36a)
\]

where

\[
u_{th} = \frac{\left( \frac{p_4}{p_{th}} \right)^{1/\gamma_4} \cdot u_4}{\phi} \quad (4.2.36b)
\]

and

\[
a_{th} = \left( \frac{\gamma_4 p_{th}}{\rho_{th}} \right)^{1/2} \quad (4.2.36c)
\]

for sonic conditions

\[
p_{th} = \frac{P_4}{\left( \frac{\gamma_4 + 1}{2} \right)^{\gamma_4/(\gamma_4 - 1)}}
\]
Referring back to Fig. (3.3), the solution point 4 represents full stream conditions. As for subsonic outflow, the right running characteristics and the path line pass at solution point 4 and their compatibility equations are given by Eqns. (4.2.28) and (4.2.29). The boundary equations for sonic flow given by Eqns. (3.2.10) and (3.2.13) are rewritten as

\[ \frac{u_4}{a_4} = \phi \left( \frac{p_{th}}{p_4} \right)^{\gamma_4+1/2\gamma_4} \quad (4.2.37) \]

or

\[ u_4 = \phi \ C \ a_4 \]

where \( C = \left( \frac{p_{th}}{p_4} \right)^{\gamma_4+1/2\gamma_4} \)

and

\[ \phi^2 = \left[ \frac{\gamma_4+1}{\gamma_4-1} - \frac{2}{\gamma_4-1} \left( \frac{p_4}{p_{th}} \right)^{\gamma_4-1/\gamma_4} \right] \left( \frac{p_4}{p_{th}} \right)^{2/\gamma_4} \quad (4.2.38) \]

using Eqn. (4.2.29) and Eqn. (4.2.32) gives :

\[ a_4 = \sqrt{\frac{\gamma_4 \cdot p_4 \cdot A_o}{(p_4 - T_o)}} \quad (4.2.39) \]

Combining Eqns. (4.2.37), (4.2.40) and Eqn. (4.2.28) gives

\[ F(p_4) = \left[ \frac{\gamma_4 \ p_4 \ A_o}{(p_4 - T_o)} \right]^{\frac{1}{2}} \phi \ C - \left[ \frac{T_o - p_4}{Q_o} \right] = 0 \quad (4.2.40) \]

Eqn. (4.2.41) is solved by Newton Raphson method for \( p_4 \) and its derivative with respect to \( p_4 \) \( F'(p_4) \) is given by

\[ F'(p_4) = -\frac{1}{2} \left( \gamma_4 \ T_o \ A_o \right) \left( \frac{\gamma_4 \ p_4 \ A_o}{(p_4 - T_o)^2} \right)^{-\frac{1}{2}} \cdot \left[ \frac{\gamma_4 \ p_4 \ A_o}{(p_4 - T_o)} \right]^{-\frac{1}{2}} \phi \ C + \frac{1}{Q_o} \quad (4.2.42) \]

where, \( C = \left( \frac{p_{th}}{p_4} \right)^{\gamma_4+1/2\gamma_4} \) is constant for a particular area ratio \( \phi \) and is found
from Eqn. (4.2.38) by Newton Raphson method. After $p_4$ is calculated $u_4$ and $\rho_4$, $a_4$ are determined from Eqns. (4.2.28), (4.2.29) and (4.2.39) respectively.

The coefficients $Q_+$, $T_+$ and $A_\omega$, $T_\omega$ are calculated for the predictor step as described in Section (4.2.1) and the predicted values of $p_4$, $u_4$ and $\rho_4$ are obtained. Again the coefficients are calculated from the corrector step and the corrected values of $p_4$, $u_4$ and $\rho_4$ are obtained.

The Flow chart for solving nozzle boundary is described in Fig. (4.15) and the corresponding computer program is called subroutine NOZZLE. The flow chart for the unit process of a partially open end is given in Fig. (4.16) and the related computer program is called subroutine NOZZR.

For a nozzle placed at the left hand end of the pipe, the same unit process is used with the left running characteristic originating from the internal flow instead of the right running characteristic, therefore, Eqn. (4.2.28) is replaced by

$$p_4 - Q_- u_4 = T_-$$

and the rest of the procedure is the same. The computer program for this case is called subroutine NOZZL (the computer flow chart is not given).

(iii) Subsonic Inflow

As discussed in Section (3.2.3), the boundary conditions for subsonic inflow in fully open end is same for subsonic inflow in a partially opened end. Therefore the unit process for subsonic inflow in an open end described in Section (4.2.3) is used here for subsonic inflow in a partially open end.

4.2.5 Unit Process for Cylinder Boundary

As described in Section (3.2.4), The important cases to be considered are,

(i) Sonic outflow through port throat
(ii) Subsonic outflow through port throat
(iii) Inflow from pipe into the cylinder

The unit process for sonic and subsonic outflow in the port throat is illustrated in Fig. (4.17) where the cylinder boundary is situated at the left hand end of the pipe. Only one characteristic originates form inside the flow, the left running characteristic,
and its compatibility equation is given by

\[ p_4 - Q_- u_4 = T_- \]  \hspace{1cm} (4.2.43)

(i) Sonic Outflow

In this case, the other two conditions are obtained from the boundary Eqn. (3.2.23) for sonic outflow from cylinder into pipe. This equation expressed in terms of the solution point 4 is rewritten as:

\[
\frac{p_4}{p_o} = \psi \left( \frac{2}{\gamma_4+1} \right)^{(\gamma_4+1)/(2(\gamma_4-1))} \left[ 1 - \left( \frac{\gamma_4-1}{2} \right) \left( \frac{u_4}{a_o} \right)^2 \right] \]  \hspace{1cm} (4.2.44)

Eqn. (4.2.44) relates \( p_4 \) to cylinder stagnation conditions \( p_o \) and \( a_o \).

Using Eqn. (4.2.43) and Eqn. (4.2.44), we get,

\[
F(u_4) = \psi \left( \frac{2}{\gamma_4+1} \right)^{(\gamma_4+1)/(2(\gamma_4-1))} \left[ 1 - \left( \frac{\gamma_4-1}{2} \right) \left( \frac{u_4}{a_o} \right)^2 \right]
- \left( \frac{u_4}{a_o} \right) \left( \frac{T_- + Q_- u_4}{p_o} \right) \]  \hspace{1cm} (4.2.45)

Eqn. (4.2.45) can be solved by Newton Raphson method to obtain \( u_4 \). The derivative \( dF(u_4) / du_4 \), \( F'(u_4) \) is given by,

\[
F'(u_4) = -2\psi \left( \frac{2}{\gamma_4+1} \right)^{(\gamma_4+1)/(2(\gamma_4-1))} \left( \frac{\gamma_4-1}{2} \right) \left( \frac{u_4}{a_o^2} \right) - \left( \frac{T_- + Q_- u_4}{p_o a_o} \right) - \left( \frac{Q_- u_4}{p_o a_o} \right) = 0.0 \]  \hspace{1cm} (4.2.46)

(ii) Subsonic Outflow

The effective area for sonic threshold is given by Eqn. (3.2.22) as
\[
\psi_{cr} = \left( \frac{2}{\gamma_4+1} \right)^{\frac{1}{2}} \left[ \frac{\left( \frac{u_4}{a_0} \right)}{1 - \left( \frac{\gamma_4-1}{2} \right) \left( \frac{u_4}{a_0} \right)} \right] \tag{4.2.47}
\]

When the effective area at the given crank angle is greater than that calculated by Eqn. (4.2.47), the flow is subsonic in the port throat and Eqn. (3.2.19) for subsonic outflow is solved. Eqn. (3.2.19) is rewritten in terms of conditions at the solution point 4 as:

\[
\psi \left( \frac{2}{\gamma_4-1} \right)^{\frac{1}{2}} \left[ 1 - \frac{\gamma_4-1}{2} \frac{u_4^2}{a_0^2} \right] \left[ 1 - \left( \frac{p_4}{p_0} \right)^{(\gamma_4-1)/\gamma_4} \right] \left( \frac{u_4}{a_0} \right) = \left( \frac{p_o}{p_0} \right)^{(\gamma_4-1)/\gamma_4} \left( \frac{u_4}{a_0} \right) \tag{4.2.48}
\]

substituting from Eqn. (4.2.43) for \( p_4 \) into Eqn. (4.2.48), we get,

\[
F(u_4) = \psi \left( \frac{2}{\gamma_4-1} \right)^{\frac{1}{2}} \sqrt{1 - \left[ \frac{T_+ + Q_- u_4}{p_0} \right]^{(\gamma_4-1)/\gamma_4}} \left\{ 1 - \left( \frac{\gamma_4-1}{2} \right) \left( \frac{u_4}{a_0} \right)^2 \right\}
\]

\[
- \left( \frac{u_4}{a_0} \right) \left[ \frac{T_- + Q_- u_4}{p_0} \right]^{(\gamma_4-1)/\gamma_4} = 0 \tag{4.2.49}
\]

which is solved by Newton Raphson method for \( u_4 \). The derivative of equation \(4.2.49\) with respect to \( u_4 \), \( F'(u_4) \) is given by

\[
F'(u_4) = \psi \left( \frac{2}{\gamma_4-1} \right)^{\frac{1}{2}} \sqrt{1 - \left[ \frac{T_+ + Q_- u_4}{p_0} \right]^{\gamma_4}} \left\{ -2 \left( \frac{\gamma_4-1}{2} \right) \left( \frac{u_4}{a_0} \right)^2 \right\} + \psi \left( \frac{2}{\gamma_4-1} \right)^{\frac{1}{2}} \left\{ 1 - \left( \frac{\gamma_4-1}{2} \right) \left( \frac{u_4}{a_0} \right)^2 \right\} \times 
\]

\[
\left\{ \frac{- \frac{1}{2} \left( \frac{\gamma_4-1}{\gamma_4} \right) \left( \frac{Q_-}{p_0} \right) \left( \frac{T_- + Q_- u_4}{p_0} \right) - \frac{1}{\gamma_4}} \sqrt{1 - \left( \frac{\gamma_4-1}{2} \right) \left( \frac{u_4}{a_0} \right)^2} \right\}
\]

\[
\sqrt{1 - \left( \frac{T_- + Q_- u_4}{p_0} \right)^{(\gamma_4-1)/\gamma_4}}
\]
\[ P_4 = \frac{T_+ + Q_+ u_4}{p_o} \left( \frac{1}{a_o} \right)^{\gamma_4 - 1} - \frac{u_4}{a_o} \left( \frac{\gamma_4 - 1}{\gamma_4} \right) \left( \frac{Q_+}{p_o} \right) \left( \frac{T_+ + Q_+ u_4}{p_o} \right)^{\gamma_4 - 1} \]  

(4.2.50)

\[ p_4 \text{ is obtained from Eqn. (4.2.43) and } a_4 \text{ is obtained from the energy equation as} \]

\[ a_4 = \sqrt{a_o^2 - \left( \frac{\gamma_4 - 1}{2} \right) u_4^2} \]  

(4.2.51)

\[ \rho_4 = \frac{\gamma_4 P_4}{a_4^2} \]  

(4.2.52)

The flow chart describing the solution of cylinder boundary conditions represented by Eqns. (4.2.43 - 4.2.52) is given in Fig. (4.18) and the related computer program is called subroutine CYLINDER.

The calculation procedure for the unit process of cylinder boundary consists of the following steps:

1. Coefficients \( Q_-, T_- \) of the compatibility Eqn. (4.2.43) are determined for the Euler predictor as described in Section (4.1.2b) for the left running characteristics. Subroutine CYLINDER is called to determine the predicted values of \( p_4, u_4 \) and \( \rho_4 \).

2. The coefficient \( Q_-, T_- \) are calculated for the Euler corrector as described in Section (4.1.2c) and subroutine CYLINDER is called to determine the corrected values of \( p_4, u_4, \rho_4 \), and \( a_4 \).

The flow chart for the computational steps of the unit process of cylinder boundary situated on the left side of the pipe is presented in Fig. (4.19) and the corresponding computer program is called CBOUNDL.

For a cylinder boundary situated at the right hand side of the pipe, the same unit process for outflow is used but with the compatibility Eqn. (4.2.43) for left running characteristic is replaced by the compatibility equation for the right running characteristic, \( p_4 + Q_+ u_4 = T_+ \) and the corresponding computer program for this case is called CBOUND and the flow chart is not given here.
(iii) Subsonic Inflow

For the case of inflow from pipe into cylinder, the unit process used for outflow in a partially open end (nozzle) described in Section (4.2.4) is used here also for a cylinder boundary situated at the left hand side of the pipe.

4.2.6 Unit Process for a Joint of Two Pipes

At the junction of two pipes, the known characteristics depend on the direction of flow. Fig. (4.20) illustrates the finite difference grid arrangement for flow from pipe 1 to pipe 2. At the junction of pipe 1 and pipe 2, represented by thick line, we have to determine $u_4$, $p_4$ and $\rho_4$ for pipe 1 and $u_4'$, $p_4'$ and $\rho_4'$ for pipe 2. Six equations are therefore needed. For flow from pipe 1 to pipe 2, the pathline and the right running characteristic originate in pipe 1. The two compatibility equations valid along these characteristics are:

**Along right running characteristic**

$$p_4 + Q_4 u_4 = T_4$$  \hspace{1cm} (4.2.53)

**Along pathline**

$$p_4 - A_4 \rho_4 = T_0$$  \hspace{1cm} (4.2.54)

For pipe 2, only the left running characteristic originates from inside the flow, its compatibility equation is given by:

$$p_4' - Q_- u_4' = T_-$$  \hspace{1cm} (4.2.55)

From boundary conditions at the joint of two pipes as described in Section (3.2.5)

$$p_4' = p_4$$  \hspace{1cm} (4.2.56)
$$u_4' = u_4$$  \hspace{1cm} (4.2.57)
$$\rho_4' = \rho_4$$  \hspace{1cm} (4.2.58)

Using Eqns. (4.2.56), (4.2.57) and (4.2.55) we have

$$p_4 - Q_- u_4 = T_-$$  \hspace{1cm} (4.2.59)

Eqns. (4.2.53), (4.2.54) and (4.2.59) are solved simultaneously for $u_4$, $p_4$ and $\rho_4'$, so that
\[ u_4 = \frac{(T_+ - T_-)}{(Q_+ + Q_-)} \quad (4.2.60) \]
\[ p_4 = \frac{T_+ - Q_+ u_4}{A_o} \quad (4.2.61) \]
\[ \rho_4 = \frac{p_4 - T_0}{A_o} \quad (4.2.62) \]
\[ a_4 = \left( \frac{\gamma_4 p_4}{\rho_4} \right)^{\frac{1}{2}} \quad (4.2.63) \]

The computational procedure of the unit process for a joint of two pipes is based on the following steps.

1. Determination of the coefficients of the compatibility equations \( Q_+ \), \( T_+ \), \( Q_- \), \( T_- \), \( A_o \) and \( T_0 \) as given in Section (4.1.2) and calculation of \( u_4 \), \( p_4 \) and \( \rho_4 \) from Eqns. (4.2.60) to (4.2.62) using Euler predictor procedure. Predicted value of \( a_4 \) is calculated from Eqn. (4.2.63).

2. The coefficients of the compatibility equations are determined again according to the Euler corrector and corrected values of \( u_4 \), \( p_4 \), \( \rho_4 \) and \( a_4 \) are obtained.

The flow chart for the unit process for flow in a joint boundary is given in Fig. (4.21) and the related computer program is called subroutine JOIN. For the case of reverse flow (flow from pipe 2 to pipe 1) subroutine PATHR is replaced by subroutine PATHL when determining the coefficients of the compatibility equation of the pathline.

### 4.2.7 Unit Process for Carburettor Boundary

Fig. (4.22a) illustrates the finite difference grid for forward flow in carburettor, and Fig. (4.22b) illustrates the finite difference grid for reverse flow. For the case of forward flow, it can be seen that, upstream the carburettor, the right running characteristic and pathline originate from inside the flow and their compatibility equations are:

**Along right running characteristic.**
\[
 p_4 + Q_+ u_4 = T_+ \quad (4.2.64)
\]

**Along pathline**
\[
 p_4 - A_o \rho_4 = T_0 \quad (4.2.65)
\]
In the downstream end of the carburettor, a left running characteristic originates from inside the flow and its compatibility equation is given by

\[ p_4' - Q_\nu u_4' = T_\nu \]  \hspace{1cm} (4.2.66)

The boundary conditions for a carburettor are given by Eqns. (3.2.63) and (3.2.64) and are rewritten here for the boundary points 4 and 4'

\[ \frac{p_4'}{p_4} = 1 - k M_4^2 \]  \hspace{1cm} (4.2.67)

and

\[ \frac{2}{\gamma_4 - 1} + M_4^2 = \left( \frac{F_4}{F_4'} \right)^2 \frac{M_4^2}{M_4'^2 (1 - k M_4^2)^2} \]  \hspace{1cm} (4.2.68)

where,

\[ M_4 = \frac{u_4}{a_4} \] \hspace{1cm} and \hspace{1cm} \[ M_4' = \frac{u_4'}{a_4'} \]

Combining the isentropic relation \[ a_4^2 = \frac{\gamma_4 p_4}{\rho_4} \] with Eqn. (4.2.65) we get \[ a_4 \] as

\[ a_4 = \sqrt{\frac{\gamma_4 p_4 A_0}{(p_4 - T_0)}} \]  \hspace{1cm} (4.2.69)

It is required to express \( M_4' \) in terms of \( M_4 \) in Eqn. (4.2.68).

Combining Eqns. (4.2.66) and (4.2.67), we get

\[ u_4' = \frac{p_4 (1 - k M_4^2) - T_\nu}{Q_\nu} \]  \hspace{1cm} (4.2.70)

From isentropic relationship

\[ \frac{a_4}{a_4'} = \left( \frac{p_4}{p_4'} \right)^{\gamma_4 - 1/2 \gamma_4} \]  \hspace{1cm} (4.2.71)
Using Eqn. (4.2.67) and (4.2.71), and (4.2.69), we have

\[ a_4' = a_4 \left(1 - k M_4^2\right)^{(\gamma_4 - 1)/2\gamma_4} \]  

(4.2.72)

and

\[ a_4' = \sqrt{\frac{\gamma_4 P_4 A_0}{(p_4 - T_o)}} \left(1 - k M_4^2\right)^{(\gamma_4 - 1)/2\gamma_4} \]  

(4.2.73)

Therefore,

\[ M_4' = \left(\frac{u_4'}{a_4'}\right) = \frac{\left[p_4 \left(1 - k M_4^2\right) - T_o\right]}{Q_+ \sqrt{\frac{\gamma_4 P_4 A_0}{(p_4 - T_o)}} \left(1 - k M_4^2\right)^{(\gamma_4 - 1)/2\gamma_4}} \]  

(4.2.74)

In Eqn. (4.2.74), the downstream Mach number \( M_4' \) is expressed in terms of the upstream Mach number \( M_4 \) where,

\[ M_4 = \frac{T_+ - p_4}{Q_+ \sqrt{\frac{\gamma_4 P_4 A_0}{(p_4 - T_o)}}} \]  

(4.2.75)

substituting for \( M_4' \) and \( M_4 \) from Eqns. (4.2.74) and (4.2.75), respectively into the carburettor boundary Eqns. (4.2.68) we get an equation which is function only of \( M_4 \) as follows.

\[ F(M_4) = \frac{2}{\gamma_4 - 1} + \left[p_4\left(1-kM_4^2\right)-T_o\right]^2 \times a_4^{-2} \left(1-kM_4^2\right)^{(1-\gamma_4)/\gamma_4} \times \]

\[ \left[p_4\left(1-kM_4^2\right)-T_o\right]^2 \times a_4^{-2} \left(1-kM_4^2\right)^{(1+\gamma_4)/\gamma_4} - M_4^2 \times \frac{F_{F_4}}{F_{F_4^0}} \left(\frac{2}{\gamma_4 - 1} + M_4^2\right) \]  

(4.2.76)

The above equation is expressed in terms of \( p_4 \) after substituting for \( M_4 \) from Eqn. (4.2.75) to get \( F(p_4) \) and then is solved by Newton Raphson technique for \( p_4 \).

Once \( p_4 \) is known, the other upstream flow conditions at point 4, \( u_4, \rho_4 \) and \( a_4 \) are obtained using Eqns. (4.2.64), (4.2.65) and (4.2.69), respectively. The downstream
flow conditions at point 4', \( p_4' \), \( u_4' \) and \( a_4' \) are given by Eqns. (4.2.67), (4.2.70) and (4.2.72), respectively and \( \rho_4' \) is given by:

\[
\rho_4' = \frac{\gamma_4 p_4'}{a_4'^2}
\]  \hspace{1cm} (4.2.77)

Eqns. (4.2.64) to (4.2.77) represent the set of equations for the flow properties upstream and downstream of the carburettor. The flow chart describing the unit process used for solving carburettor boundary is shown in Fig. (4.23) and the related computer program is called subroutine CARB.

The computational procedure of the unit process of carburettor boundary consists of the following steps:

1. Coefficients \( Q_+ \) and \( T_+ \) in Eqn. (4.2.64) and \( A_0 , T_0 \) in Eqn. (4.2.65) and \( Q_- , T_- \) in Eqn. (4.2.66) are determined for the predictor step as described in Section (4.1.2) and subroutine CARB is called to determine the predicted values of the flow properties \( p_4 , u_4 , \rho_4 , p_4' , u_4' , \rho_4' \).

2. The coefficients in step 1 are recalculated for the corrector step as described in Section (4.1.2) and the corrected values of \( p_4 , u_4 , \rho_4 , p_4' , u_4' , \rho_4' \) are determined.

For reverse flow in carburettor, similar procedure is followed with exchanging the notation \((p_4, u_4, \rho_4)\) and \((p_4', u_4', \rho_4')\).
Fig. (4.1)  Grid Structure in Time-distance Field for Inverse Marching Method
- Specified solution point
(x,t₁+Δt)

- Left-running characteristic C⁻

- Right-running characteristic C⁺

Pathline C₀

- 5, 6, 7 - solution points at previous time t₁
- ♦ 1, 2, 3 - Interpolated points on C⁺, C⁻ and C₀ at time t₁
- ✗ 4 - current solution point at time t₁ + Δt

Characteristics

--- straight line segments

Distance along the pipe

Fig. (4.2) Finite Difference Grid for the Unit Process of an interior point based on inverse marching method
Fig. (4.3) Finite Difference Grid for the Unit Process of Right Running Characteristic

- 5, 6 - Previous solution points at time $t_i$
- $\odot$ 1 - Interpolated point along $C_+$ characteristic at time $t_i$
- $\blacksquare$ 4 - Current solution point at time $t_i + \Delta t$
Referring to Fig. (4.3)

Input $X_r$, $D_r$, $V_r$, $p_r$, $\gamma_r$, $R_r$, $X_u$, $D_u$, $V_u$, $p_u$, $\gamma_u$, $R_u$, $\Delta t$, Iter, $dD/dx$

Set $X_i = X_r$, $D_i = D_r$, $X_u = X_u$, $V_i = V_u$, $p_i = p_u$, $\gamma_i = \gamma_u$, $R_i = R_u$, $D_i = D_u$

1. $\text{Iter} = 0$
   - No
   - Set $p_i = p_u$, $V_i = V_u$, $\gamma_i = \gamma_u$, $R_i = R_u$

2. Set $V_i = (V_r + V_u)/2$, $p_i = (p_r + p_u)/2$, $\gamma_i = (\gamma_r + \gamma_u)/2$, $R_i = (R_r + R_u)/2$

3. Calculate:
   - $F_i = \pi/4 D_i^2$, $\lambda_i = 1/(\lambda_i + a_i)$
   - $u_i = V_i/F$, $X_i = X_u - \Delta t/\lambda_i$
   - $a_i = (\gamma_i p_i / \rho_i)^{1/2}$

4. $|X_i - X_u| > 0.0001$
   - $X_i = X_u$
   - Calculate:
     - $V_i = V_i + (V_i - V_u)(X_i - X_u)/(X_u - X_u)$
     - $p_i = p_i + (p_i - p_u)(X_i - X_u)/(X_u - X_u)$
     - $\gamma_i = \gamma_i + (\gamma_i - \gamma_u)(X_i - X_u)/(X_u - X_u)$
     - $R_i = R_i + (R_i - R_u)(X_i - X_u)/(X_u - X_u)$
     - $a_i = a_i + (a_i - a_u)(X_i - X_u)/(X_u - X_u)$
     - $D_i = D_i + dD/dX(X_i - X_u)$

5. Exit

\[ F = \pi/4 D_i^2, \quad u_i = V_i / F \]

\[ \beta_i \rightarrow \text{Eqn (4.121a)} \]

\[ 6Q_i \rightarrow \text{Eqn (4.123a)} \]

\[ \psi_i \rightarrow \text{Eqn (4.122a)} \]

\[ S_i \rightarrow \text{Eqn (4.119a)} \]

\[ Q_i \rightarrow \text{Eqn (4.118a)} \]

\[ T_i \rightarrow \text{Eqn (4.116a)} \]
Fig. (4.5) **Finite Difference Grid for the Unit Process of Left Running Characteristic**
Fig. (4.6) Flow Chart for the Unit Process of Left Running Characteristic, Subroutine CHARL

Refering to Fig. (4.5)

**Input**

\[
X, D, V, p, r, R, X, D, V, p, r, R, X, D, V, p, r, R
\]

**Set**

\[
X_1 = X, \\
X_2 = X, \\
p = p, \\
r = r, \\
D = D, \\
V = V
\]

Iter = 0

**No**

Set

\[
p_1 = p, \\
p_2 = p, \\
V_1 = V, \\
r_1 = r, \\
R_1 = R
\]

**Yes**

Set

\[
V_2 = (V_1 + V_2) / 2, \\
p_2 = (p_1 + p_2) / 2, \\
r_2 = (r_1 + r_2) / 2, \\
R_2 = (R_1 + R_2) / 2
\]

**Calculate**

\[
F = \pi D^2, \\
n = \alpha / (\Delta V), \\
\lambda = 1 / (u - a), \\
X_2 = X_1 - \Delta V, \\
a_2 = \sqrt{(a, p_2 / p_2)}
\]

\[|X_2 - X_1| < 0.0001 \Rightarrow X_2 = X_1\]

\[|X_2 - X_1| > 0.0001 \Rightarrow X_2 = X_1\]

**Calculate**

\[
V = V_1 + (V_2 - V_1) (X_2 - X_1) / (X_2 - X_1), \\
p = p_1 + (p_2 - p_1) (X_2 - X_1) / (X_2 - X_1), \\
r = r_1 + (r_2 - r_1) (X_2 - X_1) / (X_2 - X_1), \\
R = R_1 + (R_2 - R_1) (X_2 - X_1) / (X_2 - X_1), \\
a_2 = a_1 + (a_2 - a_1) (X_2 - X_1) / (X_2 - X_1), \\
D = D + dD / dX (X_2 - X_1)
\]

**Calculate**

\[
F = \pi D^2, \\
n = V / F, \\
\beta \rightarrow \text{Eqn (4.1.21b)}, \\
\delta Q \rightarrow \text{Eqn (4.1.23b)}, \\
\psi \rightarrow \text{Eqn (4.1.22b)}, \\
S \rightarrow \text{Eqn (4.1.19b)}, \\
Q \rightarrow \text{Eqn (4.1.18b)}, \\
T \rightarrow \text{Eqn (4.1.16b)}
\]

**EXIT**
Fig. (4.7) Finite Difference Grid for the Unit Process of Pathline
Fig. (4.8a) Flow Chart for the Unit Process of Pathline (Positive Flow), Subroutine PATHR.

Referring to Fig. (4.7a)
Input \( X_1, D_1, V_1, p_1, \rho_1, \gamma_1, R_1, X_0, D_0, V_0, p_0, \rho_0, \gamma_0, R_0, \)
\( \Delta t, \int dD/dx \)

Set \( X_n = X_n \quad D_n = D_n \)
\( D_n = D_n \quad V_n = V_n \)
\( X_n = X_n \quad \rho_n = \rho_n \)
\( p_n = p_n \quad \gamma_n = \gamma_n \)

Iter = 0

No

Set \( p_n = p_n \quad V_n = V_n \)
\( \rho_n = \rho_n \quad \gamma_n = \gamma_n \)
\( R_n = R_n \)

Set \( V_n = (V_n + V_n)/2 \quad p_n = (p_n + p_n)/2 \)
\( \rho_n = (\rho_n + \rho_n)/2 \quad \gamma_n = (\gamma_n + \gamma_n)/2 \)
\( R_n = (R_n + R_n)/2 \)

Calculate:
\( F = \pi/4 D_0^2 \quad \lambda_n = 1/u_n \)
\( u_n = V_n/F \quad X_n = X_n - \Delta t \lambda_n \)
\( a_n = (Y_n p_n p_n)^{1/2} \)

\(|X_n - X_n| < 0.0001\)

Calculate:
\( V_n = V_n + (V_n - V_n) (X_n - X_n) / (X_n - X_n) \)
\( p_n = p_n + (p_n - p_n) (X_n - X_n) / (X_n - X_n) \)
\( \rho_n = \rho_n + (\rho_n - \rho_n) (X_n - X_n) / (X_n - X_n) \)
\( \gamma_n = \gamma_n + (\gamma_n - \gamma_n) (X_n - X_n) / (X_n - X_n) \)
\( R_n = R_n + (R_n - R_n) (X_n - X_n) / (X_n - X_n) \)
\( D_n = D_n + dD/dx (X_n - X_n) \)

|X_n - X_n| > 0.0001

Calculate:
\( F = \pi/4 D_0^2 \quad u_n = V_n / F \quad \gamma_n = \gamma_n \quad R_n = R_n \)
\( A_n \rightarrow \text{Eqn (4.19)} \)
\( \beta_n \rightarrow \text{Eqn (4.113)} \)
\( \psi_n \rightarrow \text{Eqn (4.111)} \)
\( \delta_j_n \rightarrow \text{Eqn (4.112)} \)
\( \theta_n \rightarrow \text{Eqn (4.110)} \)
\( T_n \rightarrow \text{Eqn (4.17)} \)

EXIT
Fig. (4.8b) Flow Chart for the Unit Process of Pathline (Reverse Flow), Subroutine PATHL.

Referring to Fig. (4.7b)

Input: X, D, V, p, p, y, R, X, D, V, p, p, y, R.

2. Iter = 0

Yes: $V = (V + V) / 2$, $p = (p + p) / 2$, $y = (y + y) / 2$, $R = (R + R) / 2$.

Calculate:
- $F = \pi / 4 D^2 / 2$, $u = V / F$, $y = y$, $R = R$.

No:
- Referring to Eqn (4.19): $V = V + (V - V) (X - X) / (X - X)$.
- Referring to Eqn (4.13): $p = p + (p - p) (X - X) / (X - X)$.
- Referring to Eqn (4.11): $a = a + (a - a) (X - X) / (X - X)$.
- Referring to Eqn (4.10): $y = y + (y - y) (X - X) / (X - X)$.
- Referring to Eqn (4.7): $R = R + (R - R) (X - X) / (X - X)$.

Set $X = X$.

EXIT
Referring to Fig. (4.2)

Input $X_1$, $D_1$, $V_1$, $p_1$, $r_1$, $y_1$, $R_1$, $X_2$, $D_2$, $V_2$, $p_2$, $r_2$, $y_2$, $R_2$, $X_3$, $D_3$, $V_3$, $p_3$, $r_3$, $y_3$, $R_3$, $\Delta t$, $dD/dx$

---

Set

- $\text{Iter} = 0$
- $X_0 = X_1$
- $D_0 = D_1$
- $X_{2n} = X_1$
- $V_0 = V_1$
- $V_n = V_{2n}$
- $p_0 = p_1$
- $p_n = p_{2n}$
- $r_0 = r_1$
- $r_n = r_{2n}$
- $y_0 = y_1$
- $y_n = y_{2n}$

---

Set

- $V_s = V_0$
- $p_s = p_0$
- $r_s = r_0$
- $y_s = y_0$

---

Set

- $V_s = (V_s + V_n)/2$
- $p_s = (p_s + p_n)/2$
- $r_s = (r_s + r_n)/2$
- $y_s = (y_s + y_n)/2$

---

Calculate

- $F = \pi/4 D_s$
- $\lambda = 1/(u_s + \alpha_s)$
- $u_s = V_s/F$
- $X_s = X_0 - \Delta t/\lambda$

---

No

Set

- $X_{s+1} = X_s$

---

No

Set

- $V_{s+1} = V_{s+1} + (V_{s+1} - V_s) (X_s - X_{s+1}) / (X_s - X_s)$
- $p_{s+1} = p_{s+1} + (p_{s+1} - p_s) (X_s - X_{s+1}) / (X_s - X_s)$
- $r_{s+1} = r_{s+1} + (r_{s+1} - r_s) (X_s - X_{s+1}) / (X_s - X_s)$
- $\gamma_{s+1} = \gamma_{s+1} + (\gamma_{s+1} - \gamma_s) (X_s - X_{s+1}) / (X_s - X_s)$
- $R_{s+1} = R_{s+1} + (R_{s+1} - R_s) (X_s - X_{s+1}) / (X_s - X_s)$
- $D_s = D_s + dD/dX (X_s - X_s)$

---

Yes

Set

- $X_{s+1} = X_{s+1}$

---

Yes

Set

- $V_{s+1} = V_{s+1} + (V_{s+1} - V_s) (X_s - X_{s+1}) / (X_s - X_s)$
- $p_{s+1} = p_{s+1} + (p_{s+1} - p_s) (X_s - X_{s+1}) / (X_s - X_s)$
- $r_{s+1} = r_{s+1} + (r_{s+1} - r_s) (X_s - X_{s+1}) / (X_s - X_s)$
- $\gamma_{s+1} = \gamma_{s+1} + (\gamma_{s+1} - \gamma_s) (X_s - X_{s+1}) / (X_s - X_s)$
- $R_{s+1} = R_{s+1} + (R_{s+1} - R_s) (X_s - X_{s+1}) / (X_s - X_s)$
- $D_s = D_s + dD/dX (X_s - X_s)$

---

$|X_s - X_{s+1}| < 0.0001$

---

$|X_s - X_{s+1}| > 0.0001$

---
Calculate:

\[ F = \pi/4 \cdot D^2 \cdot u_c = V_c / F, \]
\[ \beta = \text{Eqn (4.1.21a)} \]
\[ S_Q = \text{Eqn (4.1.23a)} \]
\[ \psi_c = \text{Eqn (4.1.22a)} \]
\[ S_c = \text{Eqn (4.1.19a)} \]
\[ Q_c = \text{Eqn (4.1.18a)} \]
\[ T_c = \text{Eqn (4.1.16a)} \]

Locate point 2 and repeat the same above procedure to determine \( Q_c \) and \( T_c \).

Locate point 3 and repeat the same above procedure to determine \( A_c \) and \( T_c \).

\[ u_c = (T_c - T_1) / (Q_c + Q_2) \quad \text{Eqn (4.2.4)} \]
\[ p_c = (T_c - Q_c) \quad \text{Eqn (4.2.5)} \]
\[ \rho_c = (p_c - T_c) / A_c \quad \text{Eqn (4.2.6)} \]
\[ \gamma_c = \gamma_1 \cdot R_c = R_i \]
\[ a_c = (\gamma_c p_c / \rho_c)^{1/2} \]

\[ \text{Iter} = \text{Iter} + 1 \]

No

\[ \text{Iter} > \text{ICOR} \]

Yes

EXIT
Fig. (4.10) Finite Difference Grid for the Unit Process of a Closed End
Fig. (4.11) Flow Chart for the Unit Process of a Closed End, Subroutine CLOSL

1. \textbf{ENTRY}

2. \textbf{Input}
   - Referring to Fig. (4.10)
   - \( X_c, D_c, V_c, p_c, \gamma_c, R_c \)
   - \( X_r, D_r, V_r, p_r, \gamma_r, R_r \)
   - \( p_v, V_v, u_v, p_t \)
   - \( \Delta \text{ICOR}, \frac{dD}{dx} \)

3. \textbf{Iter} = 0

4. Call subroutine CHARL to determine \( Q \) and \( T \).

5. Call subroutine PATHL to determine \( A, T, \gamma, \) and \( R \).

6. \textbf{Calculate}:
   - \( u_s = 0 \), Eqn (4.2.10)
   - \( p_s = T \), Eqn (4.2.11)
   - \( \rho_s = \left( \frac{p_s - T_s}{A_s} \right)/A_s \), Eqn (4.2.12)
   - \( a_s = \sqrt{\gamma_s p_s \rho_s} \), Eqn (4.2.13)

7. \textbf{Iter} = \textbf{Iter} + 1

8. \textbf{Iter} \leq \text{ICOR}
   - \textbf{No}

9. \textbf{EXIT}
Fig. (4.12)  Finite Difference Grid for the Unit Process of an Open End or Nozzle Boundary.
Fig. (4.13) Flow Chart for the Unit Process of Outflow in an Open End, Subroutine OPENR

ENTRY

Referring to Fig. (4.12a)
Input \( X, D, V, P, \rho, \gamma, R, \)
\( \Delta, ICOR, dD/dx \)

Iter = 0

Call subroutine CHARR to determine \( Q \) and \( T \).

Call subroutine PATHR to determine \( A, T, \gamma, \) and \( R \).

Calculate:
\[
\begin{align*}
    p_* &= p_p \quad \text{Eqn (4.2.14)} \\
    u_* &= (T_* - p_*)/Q_* \quad \text{Eqn (4.2.17)} \\
    \rho_* &= (p_* - T_*)/A_* \quad \text{Eqn (4.2.18)} \\
    a_* &= \sqrt{(T_* p_*)/\rho_*} \quad \text{Eqn (4.2.19)}
\end{align*}
\]

Iter = Iter + 1

Iter > ICOR

No

Yes

EXIT
Fig. (4.14) Flow Chart for the Unit Process of Subsonic Inflow in an open end, Subroutine INFLR

```
Referring to Fig. (4.12b)

Iter = 0

T4 = T,
R, = R.

Call subroutine CHARR to determine Q, and T,

Set
p, = p
X, = p/p.

Calculate:
F(X,), Eqn. (4.2.22)
F(X,), Eqn. (4.2.23)

(X,₁₁) = (X,₁) - (F(X,₁)/F'(X,₁))₁
ΔX, = (X,₁₁)₁ - (X,₁)

| ΔX, |
> 10⁻⁶

(X,₁₁)₁ = (X,₁₁)

< 10⁻⁶

X,₁₁ = (X,₁₁)

p,₁₁ = X,₁₁ p

Calculate:
u, Eqn. (4.2.25)
p, Eqn. (4.2.26)
a, Eqn. (4.2.27)

Iter = Iter + 1

No

Iter > ICOR

Yes

EXIT
```
Fig. (4.15) Flow Chart for Nozzle Boundary Conditions, Subroutine NOZZLE

ENTRY

Input
\( \phi, p_0, u_0, y_0, p_0, T_0, T_e, Q_0, A_0 \)

set
\( p_e = p_0 \)

Calculate, \( F(p_e) \), Eqn. (4.2.34)
Calculate, \( F(p_e) \), Eqn. (4.2.35)

\[ (p_{e\lambda})_{e+1} = (p_{e\lambda})_e - \{F(p_e)F'(p_e)\} \]

\[ \Delta p = (p_{e\lambda})_{e+1} - (p_{e\lambda})_e \]

\[ \frac{\Delta p}{p_e} > 10^{-6} \]

\[ \frac{\Delta p}{p_e} \leq 10^{-6} \]

\[ p_e = (p_{e\lambda})_{e+1} \]

Subsonic Flow

Calculate \( M_{e\lambda} \), Eqns. (4.2.36a-c)

\[ 1.0 > M_{e\lambda} \geq 1.0 \]

Calculate \( (p/p_e)_\lambda \), Eqn. (4.2.38) by Newton Raphson method

Subsonic Flow

Calculate, \( F(p_e) \), Eqn (4.2.40)
Calculate, \( F(p_e) \), Eqn (4.2.41)

\[ (p_{e\lambda})_{s+1} = (p_{e\lambda})_s - \{F(p_e)F'(p_e)\} \]

\[ \Delta p = (p_{e\lambda})_{s+1} - (p_{e\lambda})_s \]

\[ \frac{\Delta p}{p_e} > 10^{-6} \]

\[ \frac{\Delta p}{p_e} \leq 10^{-6} \]

\[ p_e = (p_{e\lambda})_{s+1} \]

Sonic Flow

Calculate: \( u_e, Eqn (4.2.28) \)
\( \rho_e, Eqn (4.2.29) \)
\( a_e, Eqn (4.2.39) \)

EXIT
Fig. (4.16) Flow Chart for the Unit Process of a Partially Open End, Subroutine NOZZR

Referring to Fig. (4.12)

Input: \( \Delta t, ICOR, \phi, p_1, a_1, dD/dx \)
\( X_x, D_x, V_x, p_x, \gamma_x, R_x \)
\( X_y, D_y, V_y, p_y, \gamma_y, R_y \)
\( p_4, u_4, p_4, a_4 \)

Iter = 0

Call subroutine CHARR to determine \( Q_0 \) and \( T_o \)

Call subroutine PATHR to determine \( A_0, T_o, \gamma_x, R_x \)

Call subroutine NOZZLE to determine the flow properties at point 4, \( p_4, u_4, p_4, a_4 \)

Iter = Iter + 1

Iter > ICOR

Yes

EXIT

No
Cylinder stagnation conditions \( p_0, a_0 \)

(a) Outflow from Cylinder to Pipe

Distance along the pipe

(b) Inflow from Cylinder to Pipe

Fig. (4.17) Finite Difference Grid for the Unit Process of Cylinder Boundary
Fig. (4.18) Flow Chart for Cylinder Boundary Conditions, Subroutine CYLINDER

ENTRY

Input \( u, u_a, p, T, Q, \gamma, R \)

\[ u_a = \left( p_a - T \right) / Q \]

Calculate, \( F(u_a) \), Eqn. (4.2.45)
Calculate, \( F(u) \), Eqn. (4.2.46)

\[ (u_{a+1}) = (u_a) - \left( F(u_a) / F(u) \right) \]

\[ \Delta u = (u_{a+1}) - (u_a) \]

\(|\Delta u| \]

\[ \Delta u < 10^{-10} \]

\[ u_a = (u_{a+1}) \]

Calculate, \( \psi_{ao} \), Eqn. (4.2.47)

\[ 0 > \psi - \psi_o \]

\[ \psi > 0 \]

Calculate, \( F(u_a) \), Eqn. (4.2.49)
Calculate, \( F(u) \), Eqn. (4.2.50)

\[ (u_{a+1}) = (u_a) - \left( F(u_a) / F(u) \right) \]

\[ \Delta u = (u_{a+1}) - (u_a) \]

\(|\Delta u| \]

\[ \Delta u < 10^{-10} \]

\[ u_a = (u_{a+1}) \]

Calculate:
- \( \rho \), Eqn. (4.2.43)
- \( s \), Eqn. (4.2.51)
- \( \rho_s \), Eqn. (4.2.52)

EXIT
Fig. (4.19) Flow Chart for the Unit Process of Cylinder Boundary, Subroutine CBOUNDL

Referring to Fig. (4.17)

Input
- $\Delta t$, ICOR, $\psi$, $p_1$, $a_1$, $(dD/dx)$
- $X_0$, $D_0$, $u_0$, $c_0$, $p_0$, $\gamma_0$, $R_0$
- $X_1$, $D_1$, $u_1$, $c_1$, $p_1$, $p_1$, $\gamma_1$, $R_1$
- $V_i$, $p_i$, $u_i$, $p_i$

Call subroutine CHARL to determine the coefficients $T_1$ and $Q_1$

0 = $u_i$< 0

No Flow

Inflow

> 0

Outflow

Call subroutine PATHR to determine $A_1$, $T_1$, $\gamma_1$, $R_1$

Call subroutine cylinder to determine $p_1$, $u_1$, $p_1$, $a_1$

Iter = Iter + 1

No

Iter > ICOR

Yes

EXIT

Call subroutine PATHL to determine $A_i$, $T_i$, $\gamma_i$, $R_i$

Set $p_1 = p_1$

$a_1 = a_1$

Call subroutine "NOZZLE" to determine $p_1$, $u_1$, $p_1$, $a_1$

Iter = Iter + 1

No

Iter > ICOR

Yes

EXIT
Fig. (4.20) Finite Difference Grid for the Unit Process of a Joint of Two Pipes
Fig. (4.21) Flow Chart for the Unit Process of a joint of Two Pipes, Subroutine JOIN

Referring to Fig. (4.20)

Input: ICOR, (dD/dx),, (dD/dx),

\( X, D, V, p, \gamma, R \)

\( X, D, V, p, \gamma, R \)

\( X, D, V, p, \gamma, R \)

\( X, D, V, p, \gamma, R \)

\( p, V, u, p \)

Iter = 0

Call subroutine CHARR to determine \( T \) and \( Q \).

Call subroutine CHARL to determine \( T \) and \( Q \).

Calculate:

\[ u = \frac{(T - T) + (Q + Q)}{A \gamma} \text{ Eqn. (4.2.60)} \]

\[ p = T + Q \gamma u \text{ Eqn. (4.2.61)} \]

0 =

No Flow

\( u = 0 \)

\( p = T \)

\( \gamma = \gamma \)

\( R = R \)

Direct Flow

\( u > 0 \)

\( u < 0 \)

reverse Flow

Call subroutine PATHL to determine \( A \) , \( T \) , \( \gamma \) , \( R \)

Call subroutine PATHR to determine \( A \) , \( T \) , \( \gamma \) , \( R \)

\[ p = (T - T) / A \text{ Eqn. (4.2.62)} \]

\[ u = \sqrt{(T - p) / \rho} \text{ Eqn. (4.2.63)} \]

Iter = Iter + 1

Iter > ICOR

Yes

EXIT

No
Distance along the pipe

(a) Forward Flow

Distance along the pipe

(b) Reverse Flow

Fig. (4.22) Finite Difference Grid for the Unit Process of a Carburettor Boundary
Fig. (4.23) Flow Chart for the Unit Process of a Carburettor Boundary, Subroutine CARB

ENTRY

Referring to Fig. (4.22)

Input

Yes

No

Set

Iter = 0

\( u_i = 0 \)

\( a_i = \sqrt{\gamma_i \rho_i} / \rho_i \)

\( \gamma_i = \gamma_i \)

\( R_i = R_i \)

Call subroutine CHARR to determine the coefficients \( T^h \) and \( Q \).

Call subroutine CHARL to determine the coefficients \( T \) and \( Q \).

Reverse Flow

Forward Flow

0 >

\( u_i \)

> 0

Set initial estimate of pressure at points 4 and 4' as

\( P_4 = P_i \)

\( P_4' = P_i' \)

Set initial estimate of pressure at points 4 and 4' as

\( P_4 = P_i \)

\( P_4' = P_i' \)

Call subroutine PATHL to determine \( A \), \( T \), \( Y \), \( R \).

Call subroutine PATHR to determine \( A \), \( T \), \( Y \), \( R \).

\( F_i = \frac{3}{4} D_i^2 \)

\( F_i = \frac{3}{4} D_i^2 \)

\( K = K_i \)

\( K = K_i \)

Solve Eqn (4.2.761) by Newton Raphson Method for \( p \).
Calculate:
\[ u, \quad \text{Eqn (4.2.64)} \]
\[ a, \quad \text{Eqn (4.2.69)} \]
\[ \rho, \quad \text{Eqn (4.2.65)} \]
\[ M, \quad \text{Eqn (4.2.75)} \]
\[ p, \quad \text{Eqn (4.2.67)} \]
\[ u, \quad \text{Eqn (4.2.70)} \]
\[ a, \quad \text{Eqn (4.2.77)} \]
\[ a, \quad \text{Eqn (4.2.72)} \]

\[ \frac{u_s}{u} < 0 \]

Set
\[ a = a_s \]

Calculate:
\[ u, \quad \text{Eqn (4.2.64)} \]
\[ a, \quad \text{Eqn (4.2.69)} \]
\[ \rho, \quad \text{Eqn (4.2.65)} \]
\[ M, \quad \text{Eqn (4.2.75)} \]
\[ p, \quad \text{Eqn (4.2.67)} \]
\[ u, \quad \text{Eqn (4.2.70)} \]
\[ a, \quad \text{Eqn (4.2.77)} \]

\[ \text{Iter} = \text{Iter} + 1 \]

\[ \text{Iter} > \text{ICOR} \]

Yes

EXIT

No