CHAPTER-3

MMIB BASED LINK ADAPTATION

3.1 INTRODUCTION

Link performance prediction based on EESM has been discussed in the previous chapter. In this method adjusting parameter has to be calculated for every modulation order and stored in the base station to perform link adaptation. Another well known approach which predicts the link accurately without any adjusting parameter is MMIB based approach [48]. Bayes theorem states that if the logarithmic likelihood value at any instant is maximum for one particular signal level, then the error can be minimized. The error reduces to maximum when difference in Euclidean distance is maximum [102]. Keeping this in mind the SINR of the subcarrier is linked to Mutual information (MI) between the coded bit and its Logarithmic likelihood ratio (LLR) that yields Mutual Information per coded bit (MIB). If the likelihood value is higher, then the MI increases [63] and subsequently the error reduces. From this mean MIB for all the subcarriers is calibrated to obtain Mean Mutual Information per Coded Bit (MMIB) and that is attributed as the Link Quality metric for the entire codeword which is used to predict the Block Error Rate for that channel. In this method the MMIB estimate is the equivalent estimate of the selective channel in AWGN channel and the mapping is performed so as to obtain the desired BER.

3.2 PRINCIPLE

For OFDM system which employs multiple channel state, the BLER depends upon the multiple SINR vectors received over multiple subcarriers. Link Error prediction is performed by 2 step mapping process, first mapping converts the
multiple state SINRs to one single effective SINR vector and second mapping function maps this vector to BLER. Mathematically the first mapping function MMIB is defined as

$$ M = I_m(SNR) = \frac{1}{m} \sum_{i=1}^{m} I_{m,i}(SNR) $$  \hspace{1cm} (3.1)$$

Where $ I_{m,i}(SNR) $ is the mutual information that depends on the modulation type where $ m = \{2, 4, 6\} $ corresponding to QPSK, 16-QAM, 64-QAM and $ i $ represents the bit in the word respectively. $ I_{m,i}(SNR) $ maps the sub-carrier SNR to the mean mutual information between the log-likelihood ratio and the binary codeword bits [105] comprising the QAM symbol. In a symbol each bit experiences a different equivalent channel, hence for an $ m $-tuple input word there exists $ m $ mutual information functions that represents the mean $ SNR $ obtained over different channel states. The above quantity refers to MIB and MMIB refers to mean obtained over different channel states. The second mapping function to estimate BLER is derived from the specific coding type and decoder performance under AWGN conditions. A distinct function is required for each possible MCS type supported in future.

### 3.2.1 Bit LLR Channel

Since the metric MMIB is defined between the encoded bit and LLR, the channel is defined at the coder–decoder level. Hence mutual information of the equivalent channel is expressed as

$$ I(b, LLR) = \frac{1}{m} \sum_{i=1}^{m} I(b_i, LLR(b_i)) $$ \hspace{1cm} (3.2)$$

Where $ I(b_i, LLR(b_i)) $ is the mutual information between the input bit to the mapper and the output LLR at the demapper. The Mean Mutual Information $ M $ at all N sub-carriers over the codeword may be computed as [48]
The relationship is required for each modulation type to construct the reference curve in future.

### 3.2.2 Mutual Information Computation

Generally, if $H(X)$ is the entropy of $X$, then

$$I(b, LLR) = H(b) - H(b/LLR)$$

That is, the mutual information between the coded bit value $b$ and the LLR is equal to the difference between the uncertainty concerning $b$ (which is assumed to be unity) and the uncertainty concerning $b$ given that LLR is available.

The received signal can be expressed as

$$y = x + n$$

where $E[x^2] = 1$, -1 for bit 0 and 1 transmitted and $\sigma_n^2 = N_0/2$ being the noise variance in the channel.

LLR simplifies to

$$LLR = \frac{2}{\sigma_n^2}(x + n)$$

Shows that the LLR is a scaled value of the received signal strength and is Gaussian conditioned on $x$. The metric MIB can be defined after simplification as [48]
\[ I(b, LLR) = 1 - \int_{-\alpha}^{\alpha} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(z-\sigma^2/2)^2}{2\sigma^2}} \log_2(1+e^{-z})dz \]

\[ = J(\sigma) = J(\sqrt{\frac{8E_s}{N_0}}) = J(\sqrt{8SNR}) \]  

(3.7)

shows that the MIB is function of SNR.

### 3.2.3 Implementation Algorithm for MMIBM

1. The post processing SNRs are calculated for all subcarriers.
2. The mean MI per coded bit is calibrated for current MCS and MI per symbol is computed by numerical integration for all the bits.
3. MI per symbol is mapped to BLER by look up function stored.
4. MCS search is performed until the current MCS achieves BLER less than the target.

### 3.2.4 Results and Discussion

The equation 3.7 represents the MIB for basic BPSK modulation and can also be extended for higher modulation orders. The numerically integrated value of MIB is shown in the Fig.3.1 which infers that the MIB value increases with the SNR, i.e. the mutual information of the bit increases with SNR of the bit. For QPSK, the LLR distribution is numerically calculated from the Probability density function (PDF) of the MIB response. The Fig.3.1 and Fig. 3.2 indicates that the LLR follows the Gaussian distribution for 16QAM and QPSK.
Fig. 3.1 (a, b) Mutual Information per bit for 16QAM and its LLR distribution
Fig. 3.2 (a, b) Mutual information per bit calibration for QPSK and its LLR distribution

The Fig. 3.1(a,b) and Fig. 3.2(a,b) implies that bit wise LLR distribution is Gaussian for any symbol mapping. Hence for higher order symbols the LLR PDF’s can be expressed as mixture of lower level Gaussians and the corresponding MIB’s
can be expressed as sum of $J(.)$ functions. The MIB’s are expressed numerically as sum of $J(.)$ functions in the table.

**Table 3.1 Representation of MIB as a function of mixture of Gaussian**

<table>
<thead>
<tr>
<th>MI Function</th>
<th>Numerical Approximation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1(SNR)BPSK$</td>
<td>$J(\sqrt{8SNR})$</td>
</tr>
<tr>
<td>$I_2(SNR)QPSK$</td>
<td>$J(2\sqrt{SNR})$</td>
</tr>
<tr>
<td>$I_4(SNR)16QAM$</td>
<td>$\frac{1}{2} J(0.8\sqrt{SNR}) + \frac{1}{4} J(2.17\sqrt{SNR}) + \frac{1}{4} J(0.965\sqrt{SNR})$</td>
</tr>
<tr>
<td>$I_6(SNR)64QAM$</td>
<td>$\frac{1}{3} J(1.47\sqrt{SNR}) + \frac{1}{3} J(0.529\sqrt{SNR}) + \frac{1}{3} J(0.366\sqrt{SNR})$</td>
</tr>
</tbody>
</table>

In a codeword whatever may be the channel state experienced by the bit, the LLR always takes a Gaussian distribution, hence the mapping of MMIB to BLER is now independent of the symbol level and depends on the code rate and code word length. It is worth to notice that the mapping function is reduced to one dimension compared to 2 dimensional mapping in the case of EESM. The base station can store the reference curves [48] for every channel in order to map the MMIB to BLER. The curves are parameterized as

$$y = \frac{a}{2} \left[1 - \text{erf}\left(\frac{x-b}{\sqrt{2c}}\right)\right], c \neq 0$$  \hspace{1cm} (3.8)

Where ‘a’ represents the transition height, ‘b’ is the centre and ‘c’ is the width of the error rate curve. In a linear curve where the error rate falls with the MMIB $a=1$ and the mapping requires only 2 parameters to model the curve. The b, c values are listed in the Table 3.2 as follows.
Table 3.2 Reference Parameters to model the channel for different constellation

<table>
<thead>
<tr>
<th>Modulation</th>
<th>Code rate</th>
<th>$b$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>QPSK</td>
<td>1/2</td>
<td>0.5512</td>
<td>0.0307</td>
</tr>
<tr>
<td>16QAM</td>
<td>3/4</td>
<td>0.7863</td>
<td>0.03375</td>
</tr>
<tr>
<td>64QAM</td>
<td>5/6</td>
<td>0.8565</td>
<td>0.02622</td>
</tr>
</tbody>
</table>

The Fig.3.3 depicts the BLER vs. MMIB performance for a Rayleigh channel with 5 taps. The OFDM system consists of 128 subcarriers using convolution linear codec system. The MMIB value varies between 0 to 1 for any channel AWGN or Rayleigh. In this case the metric represents the CQI for the entire codeword i.e. mean of MI of all subcarriers in the coded block Greater the value of MMIB depicts a good channel condition and lesser value represents a bad channel condition. The more it lies close to 1 more the information is certain so that receiver could detect the information with the target BLER. These reference curves are modeled and stored in the Base Station for mapping in future. To achieve a target bit error rate of $10^{-2}$ for the selected channel through QPSK (1/2) for the coded block an MMIB of 0.6 is needed. The transmitter will shift the code rate depending upon the received MMIB to achieve the target bit error rate.

The Fig.3.4 depicts the mapping performed at the transmitter after the CQI report for 40 time different time instants using matlab as software tool. The system adapts between four different constellations to achieve the desired target error rate($10^{-2}$).It is inferred that the code rate switches for 1/2 for bad channel condition and 5/6 for good channel condition provided the target is achieved. The throughput achieved using fixed coded system and dynamically adapted system is shown in Fig.3.5 It is inferred that there is throughput limitation in fixed coded system even if the channel condition changes whereas the adaptive system dynamically switches the MCS to achieve higher throughput.
Fig. 3.3 BLER vs. MMIB mapping for Rayleigh channel for different modulation and coding orders.

Fig. 3.4 MCS selection for different time instants.
3.3 MMIB FOR UNEQUAL GROUPING

The MMIB based LA algorithm has been modified for multimedia data transmission that consists of sensitive low frequency which are necessary for the decoder to decode the data and the high frequency data in which tolerable loss can be accepted by the receiver to decode. In that case the transceiver can be modeled to process the two data separately at different code rates [53] so as to achieve a better throughput. For LA, the transmitter needs 2 separate CQI in the control channel to process the data. Modified MMIB algorithm has been proposed considering 2 different code rates within a codeword.
3.3.1 Transceiver Structure

Fig. 3.6 Transceiver structure for unequal grouping of bits using MMIBM

The MMIBM has been implemented for Multimedia transmission in which the low frequency detail coefficients are transmitted at lower coding rates as they are sensitive to channel impairments. Similarly, the approximate coefficients are transmitted in higher coding rates as their loss can be tolerable. In the above Fig. 3.6 the bits are separated LFSB and HFSB. They are separately interleaved, coded and transmitted in the channel. At the receiver, channel estimation is performed separately so that the feedback consists of 2 MMIB metric according to which the transmitter modifies the loading techniques.

3.3.2 Results and Discussion

The following Fig. 3.7 demonstrates the selection of MCS corresponding to group1 and group2 depending upon the 2 received CQI. The subcarriers carrying a single codeword is divided into 2 groups and the code rate shifts between the 2 groups based on the MMIB metric received. The code rate shifts between
QPSK(1/2), 16QAM(3/4) and 64QAM(5/6). The group with highest MMIB is allotted to low frequency information and low MMIB with high frequency data. The Fig.3.8 shows the throughput performance of MMIB with and without grouping of bits in a codeword.

**Fig.3.7** MCS selection for 2 groups at any instant

**Fig.3.8** Throughput performance for MMIBM with equal and unequal constellation
3.4 MATHEMATICAL MODEL FOR UNEQUAL GROUPING

3.4.1 Introduction

If x, y are two Random variables defined on same probability space and if the outcome depends on both x, y it is referred as Bivariate. Let x and y be the two mutual information defined on the same space (codeword), then the Joint Mutual information within a codeword can be defined in terms of Bivariate Gaussian distribution provided the MI follows the Gaussian. It is shown in the previous chapter that the mutual information between the encoded bit and LLR follows Gaussian distribution [101]. Under the same property for codeword with 2 different MCS a mathematical model has been derived based on Bivariate Gaussian distribution [63]. The probability density function (pdf) and cumulative distribution function (cdf) for the proposed distribution has been stated and based on that Moment Generating Function (MGF) has been derived. Considering transmission of bivariate source on a Nakagami-m fading channel the Probability of error (Pe) has been derived [61].

3.4.2 Model Description for OFDM Systems

An OFDM signal is a sum of subcarriers that are modulated using phase shift keying or QAM. The symbol can be written as

\[ s(t) = \sum_{k=0}^{N-1} D_k e^{j2\pi f_o t} e^{j2\pi kf_o t}, 0 \leq t \leq T \]  

(3.9)

Where N is the number of subcarriers, T is the symbol duration, \( D_k \) represents the data sequences in a symbol, \( f_o = 1/T \) represents the subcarrier spacing and k represents the power factor. After IFFT the time domain OFDM signal is represented as
\[
s(n) = \frac{1}{N} \sum_{k=0}^{N-1} D_k e^{j\frac{2\pi k f_c n}{N}} \tag{3.10}
\]

After IFFT the signal is up converted to carrier frequency \(f_c\). The signal ready for transmission is given by:

\[
x(t) = \text{Re}\left\{ \sum_{k=0}^{N-1} D_k e^{j2\pi k(f_o+f_c)t} \right\}, 0 \leq t \leq T \tag{3.11}
\]

\(X(t)\) is the signal in which the subcarriers shall undergo frequency selective fading.

### 3.4.3 Channel Model

The envelope of the channel response can be modeled as nakagami-\(m\) model which is characterized by two parameters, fading figure and moments. The \(m\)-distributed pdf of the envelope is defined as [58]

\[
P(R) = \frac{2m^m R^{2m-1}}{\Gamma(m)\Omega^m} \exp\left(-\frac{mR^2}{\Omega}\right), R \geq 0
\]

where

\[
\Omega = E(R^2), m = \frac{\Omega^2}{E[(R^2-\Omega^2)^2]}, m \geq 1/2
\]

\(\Omega\) represents the average power and \(\Gamma(\cdot)\) represents the gamma function, \(\ \mbox{‘}m\mbox{’}\) is named fading figure, parameter related to fading range, \(r\) represents the nakagami distribution envelope. The fading model considered here is correlated identically distributed Nakagami-\(m\) fading channels. In this case if received codeword consists of two symbols, it can be shown that the PDF of the combined SNR per symbol, \(p_s(\gamma_1)\) is given by [61]

\[
p_s(\gamma_1) = \frac{\sqrt{\pi}}{\Gamma(m)} \left[ \frac{m^2}{\gamma_1^2(1-\rho)} \right]^{\frac{m}{2}} \left( \frac{\gamma_1}{2\beta} \right)^{(m-1)/2} I_{(m-1)/2}\left(\beta \gamma_1\right) e^{-\gamma_1}, \gamma_1 \geq 0 \tag{3.13}
\]
\( \bar{\gamma}_1, \bar{\gamma}_2 \) are the two SNR vectors representing the two symbol groups. Where the parameters \( \alpha' \) and \( \beta' \) are the parameters relating the correlation coefficient \( \rho \) to the SNR vectors and fading margin \( m \). Using the Laplace transform it is shown after some manipulations that the Moment generating function MGF of \( p_a(\gamma) \) of is given by [61]

\[
M_a(s) = \left[ 1 - \frac{(\bar{\gamma}_1 + \bar{\gamma}_2)}{m} s + \frac{(1 - \rho)\bar{\gamma}_1\bar{\gamma}_2 s^2}{m^2} \right]^{-m}, \ s \geq 0
\] (3.14)

In the case of fading channel the SNR per bit is a time invariant random variable with PDF \( P_\gamma(\gamma) \) defined by the type of fading. From nakagami-\( m \) pdf, the SNR per bit PDF [64] defined as

\[
P_\gamma(\gamma) = \frac{m^m \gamma^{m-1}}{\gamma^m \Gamma(m)} \exp\left(-\frac{m\gamma}{\gamma}\right)
\] (3.15)

Applying Laplace transform w.r.t \( \gamma \) to the above expression, the moment generating function can be evaluated as [61]

\[
M_\gamma(s) = \left(1 + \frac{s\gamma}{m}\right)^{-m}
\] (3.16)

\[
I = \frac{1}{\pi} \int_0^{\pi/2} \left(1 + \frac{a^2 \gamma}{2m \sin^2 \theta}\right)^{-m} \ d\theta
\]

From the MGF the average probability of error \( I \) is calibrated by the following formula [61]

\[
I = \frac{1}{\pi} \int_0^{\pi/2} \left(1 + \frac{a^2 \gamma}{2m \sin^2 \theta}\right)^{-m} \ d\theta
\] (3.17)
After simplification the equation finally reduces to

\[
I = \frac{\Gamma(2m+1/2)}{\varepsilon_{m+1} \Gamma(2m+1)} \cdot \frac{1}{(1-k^2)^m} \cdot \frac{1}{(\gamma/m) \sin^2 \left( \frac{\pi}{m} \right)^{2m}}
\]

(3.18)

Where the SNR vector \( \gamma = \gamma_1 + \gamma_2 \) is characterized by two fading envelopes. \( K \) is the power correlation coefficient of the two set of fading.

### 3.4.4 Results and Discussion

The two dimensional LLR PDF for QPSK and 16QAM on a single code word has been plotted and shown in Fig.3.9 and their respective CDF is represented in Fig.3.10. From the derived average probability of error results has been shown for different values fading margin \( m \). It is observed from Fig.3.11 that the increase in fading margin value decreases the requirement of SNR to achieve the target bit error rate.

![Fig. 3.9 Representation of CDF for Bivariate source using QPSK and 16 QAM](image-url)
Fig. 3.10  LLR pdf for Bivariate MIB using QPSK and 16 QAM

Fig. 3.11  Average BER performance for different fading margin values
3.5 SUMMARY

The Mean Mutual Information per Bit mapping (MMIBM) is modeled and the mapping results have been shown between the MMIB metric and BER for different code rates. Approximation function has been obtained and tabulated so that the transmitter can calibrate the BER for current channel conditions to decide the code rate for next instant. The MMIBM has been modeled for unequal grouping of bits in a single code word so that the BER performance can be improved further. The algorithm for unequal grouping has been simulated and mapping results has been shown for different code rates. Throughput performance has been improved for unequal compared to equal code rate assignment within a code word.