In this chapter an attempt is made to derive the demand function for a two commodity model. Generally, a rational consumer wants to maximise his utility. Let us assume that there are two commodities and let their quantities be \( q_1 \) and \( q_2 \) respectively. Hence, the budget constraint will be

\[
y^* = P_1 q_1 + P_2 q_2 \quad \ldots (1)
\]

where \( P_1 \) and \( P_2 \) are the prices respectively.

\( Y^* \) = amount of fixed income.

In such a situation utility function is:

\[
u = f(q_1, q_2) \quad \ldots (2)\]

subject to budget constraint. So the lagrangian function will be:

\[
L = f(q_1, q_2) + \lambda (Y^* - P_1 q_1 - P_2 q_2) \quad \ldots (3)
\]

Condition:

(1) According to first order condition, all the first order partial derivatives are zero. Now differentiating equation (3) with respect to \( q_1, q_2 \) and \( \lambda \) we get:

\[
(3) \quad \frac{\partial L}{\partial q_1} = \frac{\partial L}{\partial q_2} - \lambda P_1 = 0 \quad \text{or} \quad f_1 = \lambda P_1 \quad \ldots (4)
\]
Solution of these equations will give us the point at which the utility would either be maximum or minimum.

\[
\frac{\Delta f_1}{\Delta P_1} = \lambda \quad f_1 = \text{marginal utility of } q_1
\]

\[
\frac{\Delta U_1}{\Delta P_1} = \frac{\Delta U_2}{\Delta P_2} = \lambda
\]

Thus, the consumer, in order to maximise the utility, must allocate his income so as to equalise the ratio of marginal utility to the price of each commodity. These ratios must also be equal to the M.U. of money which is represented by \(\lambda\), while \(\frac{f_1}{P_1}\) represents the slope of indifference curve and \(\frac{P_1}{P_2}\) represents the slope of the budget line. Another proposition which comes out from this analysis is that equilibrium will be at the point where slope of indifference curve is equal to the slope of price line. Fig. 1 below makes it clear (Figure on next page, please)
II. **Second Order Condition:**

Second order condition for maximising the utility is that the relevant bordered Hessian determinant should be positive.

\[
\begin{vmatrix}
0 & -P_1 & -P_2 \\
-P_1 & f_{11} & f_{12} \\
-P_2 & f_{21} & f_{22}
\end{vmatrix} > 0
\]

\[-P_1^2 f_{22} + 2P_1P_2f_{12} + P_2^2 f_{11} > 0\]

This matrix is bordered by the prices of two goods. From (4), (5) and (6), we will get

\[
\frac{\partial^2 L}{\partial q_2^2} = f_{11} \quad \ldots (8)
\]
Thus, two properties of the demand equation are clear: the demand function is single valued function of income and prices. Second, the demand functions are homogeneous of degree zero which means that when income and prices change in the same proportion and in the same direction, then demand for the commodity remains constant.

**Price elasticity:**

Price elasticity is nothing but a proportionate change in demand/proportionate change in price. The elasticity is negative if the corresponding demand curve is downward sloping.

\[
\varepsilon_{11} = \frac{\text{Change in demand}}{\text{Total demand}} \div \frac{\text{Change in price}}{\text{Total price}}
\]

\[
\varepsilon_{11} = \text{Price elasticity.}
\]

**Cross elasticity:**

It is the ratio of proportionate change in quantity of one commodity to the proportionate change in the price of the other commodity:

\[
\frac{\partial^2 L}{\partial q_1 \partial q_2} = \varepsilon_{22} \quad \ldots (9)
\]

\[
\frac{\partial \Delta L}{\partial q_1 \partial q_1} = \varepsilon_{12} \quad \ldots (10)
\]

\[
\frac{\partial \Delta L}{\partial q_1 \partial q_2} = \varepsilon_{21} \quad \ldots (11)
\]
\[ \varepsilon_{ij} = \frac{p_i}{q_i} \left( \frac{dq_j}{dp_i} \right) \]

\( \varepsilon_{ij} \) = Cross price elasticity. Whether these elasticities are positive or negative depends upon the type of the commodity consumed.

Taking the total differential of the budget constraint (1) and putting \( dy^o = dP_2 = 0 \), we get

\[ dy^o = p_1 \, dq_1 + q_1 \, dp_1 + p_2 \, dq_2 = 0 \quad \ldots (12) \]

If we multiply the equation (12) by

\[ (p_1 \, q_1 \, q_2 / y^o \, q_1 \, q_2 \, dp_1) \]

then the equation will be

\[ = \left( p_1 dq_i \right) \left( \frac{p_1 q_1 q_2}{y^o q_1 q_2 dp_1} \right) + \left( q_1 dp_1 \right) \left( \frac{p_1 q_1 q_2}{y^o q_1 q_2 dq_1} \right) + \left( p_2 dq_2 \right) \left( \frac{p_1 q_1 q_2}{y^o q_1 q_2 dp_1} \right) \quad \ldots (13) \]

\[ = \left( p_1 dq_i \right) \left( \frac{p_1 q_1 q_2}{y^o q_2} \right) + \left( q_1 dp_1 \right) \left( \frac{p_1 q_1 q_2}{y^o q_2} \right) + \left( p_2 dq_2 \right) \left( \frac{p_1 q_1 q_2}{y^o q_2} \right) \quad \ldots (14) \]

\[ = \varepsilon_{i1} \alpha_1 + \varepsilon_{i2} \alpha_2 + \alpha_1 = 0 \]

\[ = \varepsilon_{i1} \alpha_1 + \varepsilon_{i2} \alpha_2 = - \alpha_1 \quad \ldots (15) \]

(a) \[ \frac{p_1 dq_i}{q_1 dp_1} \]

= Own price elasticity
The equation (15) tells us about the cross elasticity, when own price elasticity is given.

**Income Elasticity:**

Income elasticity is defined as the proportionate change in purchases of a commodity relative to the proportionate change in income with prices remaining constant.

\[ \eta_i = \frac{\Delta q_i}{\Delta y} \times \frac{\Delta y}{q_i} \]  

Income elasticity can be negative, positive or zero but normally it is assumed to be positive.

**Substitution and Income effects:**

Comparative static analysis examines the effect of disturbances in exogenous variables on the solution values for the endogenous variables in order to find out the magnitude of the effect of prices and income on the consumer purchases if we allow all the variables to vary simultaneously, then situation will be different.
When all the variables are changing simultaneously then total differentiation of these following equation will be

\[ U = f(q_1, q_2) \]

\[ L = f(q_1, q_2) + \lambda (y^0 - P_1 q_1 - P_2 q_2) \]

First order conditions are:

(a) \[ \frac{\delta L}{\delta q_1} = f_1 - \lambda P_1 = 0 \]

(b) \[ \frac{\delta L}{\delta q_2} = f_2 - \lambda P_2 = 0 \]

(c) \[ \frac{\delta L}{\delta \lambda} = y^0 - P_1 q_1 - P_2 q_2 = 0 \]

Second Order Condition:

Taking the total differential of above equations the new will be:

\[ d (dZ) = d(fx dx) + d(fy dy) \quad \ldots (17) \]

(a) \[ f_{11} dq_1 + f_{12} dq_2 - P_1 d\lambda - \lambda dP_1 = 0 \]

(b) \[ f_{21} dq_1 + f_{22} dq_2 - P_2 d\lambda - \lambda dP_2 = 0 \]

(c) \[ dy - P_1 dq_1 - P_2 dq_2 - q_1 dP_1 - q_2 dP_2 = 0 \]

or

\[ f_{11} dq_1 + f_{12} dq_2 - P_1 d\lambda = \lambda dP_1 \]
\[ f_{21} \, dq_1 + f_{22} \, dq_2 - P_2 \, d\lambda = \lambda d \, P_2 \]
\[ -P_1 \, dq_1 - P_2 \, dq_2 = -dy + q_1 \, dP_1 + q_2 \, dP_2 \]

In the matrix form

\[
\begin{pmatrix}
- \lambda d \, P_1 \\
\lambda d \, P_2 \\
\lambda d \, \lambda
\end{pmatrix}
= \begin{pmatrix}
f_{11} & f_{12} & -P_1 \\
f_{21} & f_{22} & -P_2 \\
-P_1 & -P_2 & 0
\end{pmatrix}
\begin{pmatrix}
dq_1 \\
dq_2 \\
d\lambda
\end{pmatrix}
- dy + q_1 \, dP_1 + q_2 \, dP_2
\]

But we want to find out the value of \( dq_1, dq_2 \) and \( d\lambda \). Rewriting the above matrices:

\[
\begin{pmatrix}
dq_1 \\
dq_2 \\
d\lambda
\end{pmatrix}
= \begin{pmatrix}
f_{11} & f_{12} & -P_1 \\
f_{21} & f_{22} & -P_2 \\
-P_1 & -P_2 & 0
\end{pmatrix}
- \begin{pmatrix}
dP_1 \\
dP_2 \\
-dy + q_1 \, dP_1 + q_2 \, dP_2
\end{pmatrix}
\]

If we put the whole inverse matrix equal to \((D)^{-1}\)

\[
(D)^{-1} \begin{pmatrix}
\lambda d \, P_1 \\
\lambda d \, P_2 \\
\lambda d \, \lambda
\end{pmatrix}
= \begin{pmatrix}
dP_1 \\
dP_2 \\
-dy + q_1 \, dP_1 + q_2 \, dP_2
\end{pmatrix}
\]

\[\begin{pmatrix}
(D)^{-1} \\
\lambda d \, P_1 \\
\lambda d \, P_2
\end{pmatrix}
= \begin{pmatrix}
dP_1 \\
dP_2 \\
-dy + q_1 \, dP_1 + q_2 \, dP_2
\end{pmatrix}
\]

\[\begin{pmatrix}
\lambda d \, P_1 \\
1
\end{pmatrix}
= \frac{1}{|D|}
\]

(co-factors of the matrix)
whereas \( D_{11} = \begin{pmatrix} f_{22} & -p_2 \\ -p_2 & 0 \end{pmatrix} \)

So the values of \( dq_1, dq_2 \) and \( d\lambda \) will be

1. \( dq_1 = \frac{D_{11} dP_1 + D_{21} dP_2 + D_{31} (-dy+q_1 dP_1 + q_2 dP_2)}{D} \)

2. \( dq_2 = \frac{D_{12} dP_1 + D_{22} dP_2 + D_{32} (-dy+q_1 dP_1 + q_2 dP_2)}{D} \)

3. \( d\lambda = \frac{D_{31} dP_1 + D_{23} dP_2 + D_{33} (-dy+q_1 dP_1 + q_2 dP_2)}{D} \)

If we put \( dy^o = dP_2 = 0 \) and divide the both sides of the above equations by \( dP_1 \)

\[ \frac{dq_1}{dP_1} = \frac{\Delta_{11} dP_1 + \Delta_{21} dP_2 + \Delta_{31} (-dy+q_1 dP_1 + q_2 dP_2)}{(D) (d P_1)} \] \( \ldots \) (19)

\[ \frac{d\lambda}{dP_1} = \frac{\Delta_{31}}{\Delta} + q_1 \frac{D_{33}}{dP_1} \] \( \ldots \) (20)

where \( \frac{dq_1}{dP_1} \) is the rate of change of consumer's purchases of \( Q_1 \) with respect to change in \( P_1 \) while other things being equal. If we divide it by \( dy \), the rate of change with respect to income will be
Mathur-Ezekiel propounded the hypothesis that short-run supply of food-grains tends to be backward bending. With the help of a mathematical model, we try to present the main argument (26)[17][33]

If food-grain sector, \( Q_f \) denotes the total per capita output of the food-grains producers. This \( Q_f \) consists of different items, as

\[
Q_{ft} = O_{ft} + N_{ft} + (S_{ft} - S_{ft-1}) \quad \ldots (22)
\]

whereas \( Q_{ft} \) is total available quantity of the good in period \( t \); \( O_{ft} \) is total output in year \( t \); \( N_{ft} \) is purchased from outside and \( (S_{ft} - S_{ft-1}) \) is the changes in stocks of goods.

A producer, being a consumer also wants to maximise his utility while allocating this available output among different uses as human consumption, seeds, cattle feed and sales to the market. \( D_{ft} \) represents his total demand as follows:
\[ D_{ft} = C_{ft} + F_{ft} + Z_{ft} + \bar{M}_{ft} \]  \hspace{1cm} ... (23)

where \( C_{ft} \) is total human consumption, \( F_{ft} \) cattle feed, \( Z_{ft} \) = seed requirements, \( \bar{M}_{ft} \) = Marketed surplus. For a general equilibrium the equality between the two equations is essential. For equilibrium

\[ Q_{ft} = D_{ft} \]  \hspace{1cm} ... (24)

\[ Q_{ft} + N_{ft} + (S_{ft} - S_{ft-1}) = C_{ft} + F_{ft} + Z_{ft} + \bar{M}_{ft} \]  \hspace{1cm} ... (25)

so

\[ M_{ft} = \bar{M}_{ft} + (C_{ft} - S_{ft-1}) \]

\[ = Q_{ft} + N_{ft} - C_{ft} - F_{ft} - Z_{ft} \]  \hspace{1cm} ... (26)

So with a given output, \( Q_{f} \), the income of the producer will be

\[ Y = P_{f} Q_{f} \]  \hspace{1cm} ... (27)

and marketable surplus will be

\[ M = n ( Q_{1} - q_{1} ) \]  \hspace{1cm} ... (28)

\( n \) = number of food-grain producers. With a given budget constraint, from the demand analysis, we get the following demand equation:

\[ dq_{1} = \lambda D_{11} dP_{1} + \lambda D_{21} dP_{2} + D_{31} (-dy + q_{1} dP_{1} + q_{2} dP_{2}) \]  \hspace{1cm} ... (29)
As \( dP_2 \) is zero and \( dy = dP_1 \), then demand equation will be

\[
dq_1 = (D_{11} + q_1 D_{31}) dp_1 - \frac{q_1 + nq_1}{n} D_{31} dp_1 \quad \ldots(30)
\]

This becomes

\[
\frac{p_1}{q_1} \cdot \frac{dp_1}{dp_f} = \frac{p_1}{q_1} \cdot \frac{dp_1}{dp} + \frac{y}{q_1} \cdot \frac{dp_1}{dy} \quad \ldots(31)
\]

Total short-run elasticity of \( q_1 \) with respect to \( P_1 = (\beta - \alpha) \). If we assume constant elasticity demand function, we can write

\[
q_f = A \frac{p_f - \alpha}{\gamma_f} \quad \ldots(32)
\]

where \( \alpha \) and \( \beta \) are price and income elasticities respectively. Substituting these values in the equation (28) of marketable surplus, we will get another equation:

\[
M = n \left( O_f - \frac{\beta}{\gamma_f} \right) \quad \ldots(33)
\]

or

\[
M = n \left( O_f - \frac{(\beta - \alpha)}{\gamma_f} \right) \quad \ldots(33a)
\]

Thus, the partial elasticity of marketable surplus with price is given by

\[
\frac{p_f}{M} \cdot \frac{dM}{dp_f} = \frac{p_f q_f}{M} \left( - (\beta - \alpha) \frac{(\beta - \alpha)}{\gamma_f} \right) \quad \ldots(33)
\]

\[
= - (\beta - \alpha) \frac{q_f}{Q_f} \quad \ldots(33)
\]
If $Q > q_f$, the sign of the elasticity of the marketable surplus will be that of $-(\bar{P} - \bar{x})$. For a superior good, income elasticity is always greater than the own price elasticity; i.e. $\beta > \alpha$, gives the sign of the elasticity of marketable surplus with respect to price. This implies that the increase in prices of food-grain should reduce the marketable surplus. Thus, the inverse relationship between price and marketable surplus postulated in Mathur-Ezekiel study can be derived independent of any assumption about the farmers monetary requirements. However, the extent will be affected.

**Marketed Surplus in the Short-run:**

Marketed surplus is equal to marketable surplus minus stocks. If we deduce stocks from marketable surplus, we will have the equation for marketed surplus.

$$\bar{M} = M - S = n (Q - q_f - S)$$

where $S$ is per capita stock of food-grains and $\bar{M}$ is per-capita marketed surplus.

$$\frac{P}{\bar{M}} \cdot \frac{d \bar{M}}{dP} = \frac{p_i}{\bar{M}} \cdot \frac{M}{M} \cdot \frac{dM}{dP_i} - \frac{p_i}{\bar{M}} \cdot \frac{S}{S} \cdot \frac{dS}{dP_i}$$

--- (34)

$$= \frac{M}{\bar{M}} \cdot \frac{p_i}{M} \cdot \frac{dM}{dP_i} - \frac{p_i}{\bar{M}} \cdot \frac{S}{S} \cdot \frac{dS}{dP_i}$$

--- (35)
Substituting the value of \( \left( \frac{p_i}{M} \cdot \frac{dM}{dp_i} \right) \) we will get
\[
\frac{M}{\bar{M}} \cdot \frac{q_f}{q_i} \left( \beta - \alpha \right) \frac{q_f}{q_i} - \frac{5}{\bar{M}} \cdot \frac{n}{n} \cdot \frac{d}{ds} \frac{ds}{dp_i} \quad \cdots (36)
\]
\[
= -\frac{M}{\bar{M}} \left( \beta - \alpha \right) \frac{q_f}{q_i} - \frac{5}{\bar{M}} \cdot \frac{n}{n} \cdot \frac{d}{ds} \frac{ds}{dp_i} \quad \cdots (37)
\]

Elasticity of marketed surplus is equal to \( \frac{M}{\bar{M}} \) times elasticity of marketable surplus minus \( \frac{5}{\bar{M}} \) times elasticity of stocks with respect to price. As \( M > \bar{M} \) the first term itself makes elasticity of marketed surplus usually more negative than that of marketable surplus.

In the subsequent chapters, we have made an attempt to estimate demand function, and marketable surplus for food-grains for Haryana farmers!