ACCELERATION AND GRAVITATIONAL THERMODYNAMICS IN BRANE-GRAVITY BASED PHANTOM COSMOLOGY

ABSTRACT

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IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN MATHEMATICS

To
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SHILLONG - 793022
NOVEMBER, 2010
Abstract

General Relativity, in spite of being the most successful gravitational theory has left some problems without answer. After the advent of General Relativity, a gravitational theory in a 4-dimensional space-time, Kaluza-Klein theory emerged as a higher dimensional theory. This is a 5-dimensional theory with metric tensor components depending on electromagnetic fields. It emerged from the desire to unify gravity and electromagnetism. In this theory, the extra dimensional space is very small and compact. Moreover, this theory has a problem that standard model particles are not observed in Kaluza-Klein framework. In Ref. [1], it is proposed that the reason for not observing the standard model particles in Kaluza Klein setup is confinement of these particles in 4-dimensional space-time, called 3-brane. This idea was the key ingredient of brane-gravity. Thus according to the theory of brane gravity, major portion of gravity lies in higher-dimensional "bulk" space-time as well as very small portion of gravity and standard model particles are trapped to the 3-brane [2]. The 3-brane is identified with our observable universe. This kind of cosmic picture is inspired by the string-theory/M-theory.

One of the first examples of this type was the Hořava and Witten setup [3] of 11-dimensional M-Theory compactified on $S^1/Z_2$ orbifold. In this theory,
two (1+9)-dimensional branes are located at two ends of the orbifold. These two branes are endowed with the product manifold of (1+3)-dimensional non compact and 6-dimensional compact manifold. When six extra dimensions are compactified on a very small scale close to the fundamental scale, their effect is realised on (1+3)-dimensional brane. Thus, Hořava-Witten set up provided an effective 5-dimensional model where the extra dimension can be large relative to the fundamental scale in contrast to Kaluza-Klein theory, where extra dimension is very small [4, 5, 6]. This solution was used by L. Randall and R. Sundrum in their seminal paper to solve the “hierarchy problem” by a warped or curved space-time showing that fundamental scale could be brought down from the Planck scale to 100 GeV by this approach. Thus, Randall-Sundrum approach brought the theory to scales below 100 GeV being the electroweak scale (so far results could be verified experimentally up to this scale only). In this model, extra-dimension is large having (1 + 3)-branes at its ends. These branes are \( \mathbb{Z}_2 \)-symmetric (mirror symmetry) and have tension to counter the negative cosmological constant in the “bulk”, which is AdS\(_5\). The model, having two (1 + 3)-branes at the ends of the orbifold \( S^1/\mathbb{Z}_2 \), is known as RS-I model [7]. In RS-I model, one brane is hidden and other is visible.

In another paper, published in the same year, these authors proposed another brane-model as an alternative to compactification. In this model, there is only one (1 + 3)-brane at one end of the extra-dimension and the other end tends to infinity. This model is known as RS-II model [8]. In this thesis, we consider RS-II model which is very popular among cosmologist due to its simplicity and predictive power. Singularity problem of phantom
cosmology is resolved easily using RS-II model of brane-gravity.

Another approach for brane-gravity was proposed by Dvali, Gabadaze and Porrati in the year 2000 [9, 10, 11, 12]. This is an induced gravity model. The main idea of the DGP model is the inclusion of a four dimensional Ricci scalar into the action. The model is then characterized by a cross over length scale

$$r_c = \frac{M_P^2}{2M_5^2},$$

such that gravity is 4-dimensional theory at scales $a \ll r_c$ where matter behaves as pressure less dust but gravity *leaks out* into the bulk at scales $a \gg r_c$ and matter approaches the behaviour of a cosmological constant.

Braneworld cosmology for these models is a strong candidate from the beginning of this century after *Wilkinson Microwave Anisotropy Probe* (WMAP) observations having conclusive evidence for cosmic acceleration beginning in the recent past. Being String theory inspired, braneworld models provide corrections to the General Relativity which is considered low energy limit of String theory. Here, we find that novel cosmologies are obtained which potentially answers to some longstanding problems of modern cosmology, such as origin and nature of *Dark Energy* (DE). At the same time success of standard 4-dimensional cosmology is preserved and in some cases the treatment in the framework of brane cosmology is even more satisfactory. A review on brane-gravity and its various applications with special attention to cosmology is available in [2, 13, 14, 15]. The phantom dark energy, being a non-luminous cosmic fluid and having gravitational effect, is found more suitable source to cause late acceleration observed by scientist working in *WMAP* project.
In General Relativity based models, lot of works in phantom cosmology are available in the literature. But phantom cosmology, in the framework of both brane-gravity theories, needs proper attention as brane-corrections in Friedmann equations bring drastic changes in results obtained from General Relativity [16, 17, 18]. A part of this thesis is addressed to this area.

Apart from many developments in cosmology, thermodynamics in expanding universe is a very important issue which requires serious attention. In the past it has been the subject of many papers based on General Relativity [19, 21, 22, 23, 24, 25]. Recently the connection between gravity and thermodynamics has been extended to braneworld scenario [27, 28, 29, 30].

In this thesis, we investigate certain aspects of braneworld modifications to cosmological dynamics. In particular, it is devoted to following two important aspects of Brane Cosmology.

- Acceleration and deceleration of the universe.
- Validity of Generalised second law of thermodynamics [21].

This thesis is a collection of six papers based on my research on the above mentioned two aspects of Brane Cosmology. Except last chapter, all the chapters are based on published papers in international journals.

This thesis contains seven chapters. First chapter gives a general introduction to the higher dimension, braneworld scenario, brane cosmology and accelerated phantom dominated universe. Chapter ends with a discussion on gravitational thermodynamics.

In 2nd chapter, we consider RS-II model of brane-gravity and analyze phantom universe using a non-linear equation of state. Phantom fluid is
known to violate the weak energy condition (WEC). It is found that this characteristic of phantom energy is affected drastically by the negative brane-tension $\lambda$ of the RS-II model. It has been found that upto a certain value of energy density $\rho$ satisfying $\rho/\lambda < 1$, WEC is violated and universe super-accelerates. Moreover, on sufficient increase in phantom energy density, even strong energy condition (SEC) is not violated due to effect of brane corrections. As a consequence, it is found that the present model of the universe accelerate up to a finite time, explaining present acceleration but decelerates later on. Also, expansion of the universe stops when $\rho = 2\lambda$. This is contrary to earlier results of phantom universe exhibiting acceleration only. Moreover, the model is free from big-rip problem. This chapter is based on [31].

In 3rd chapter, DGP model of brane-gravity is considered for the phantom universe using a nonlinear equation of state. Here, DGP model of brane-gravity is analyzed and compared with the standard general relativity (GR) and Randall-Sundrum cases. It is found that in DGP model, SEC is always violated and the universe accelerates only where as WEC is violated only for a special range of energy density. Chapter ends with an expression of the scale factor and analysis of its behaviour in the late universe. This chapter is based on [32].

In 4th chapter, cosmology of the late and future universe is obtained from $f(R)$– gravity with non-linear curvature terms $R^2$ and $R^3$ ($R$ being the Ricci scalar curvature). Here, curvature terms induce dark energy, dark matter and cosmological constant, which appear in the Friedmann equation for the late universe derived from $f(R)$– gravitational equations. It has been observed that curvature-induced dark energy, obtained here, mimics phan-
tom with the equation of state parameter \( \omega = -\frac{5}{4} \). Moreover, Friedmann equation contains phantom DE term as \( \rho_{\text{DE}}[1 - \rho_{\text{DE}}/2\lambda] \). The correction term \(-\rho_{\text{DE}}^2/2\lambda\), with \( \lambda \) being the cosmic tension, is analogous to such a term in RS-II model Friedmann equation. Different phases of this model, including acceleration and deceleration during phantom phase have been investigated. This chapter is based on [33].

5th chapter deals with gravitational thermodynamics in DGP braneworld. In particular, we study the validity of the generalised second law of gravitational thermodynamics (GSLT) of the universe bounded by the event horizon. Here, the radius of the event horizon is calculated by establishing a correspondence between holographic dark energy (HDE) and the effective energy density in the DGP braneworld. It is shown that in the absence of cold dark matter (CDM), GSLT is always respected. In the presence of CDM, we investigate validity of GSLT in three different models of DGP brane world. The result shows that, the relation between thermodynamics and gravity is not just accident in GR, but it has deep meaning which other theory of gravity also supports. This chapter is based on [34].

In 6th chapter, in contrast to 5th chapter, we study the validity of GSLT in both branches of DGP model, i.e. DGP(+) and DGP(-) model. Moreover, in this chapter matter in the universe is taken in the form of non-interacting two fluid system:- one component is the holographic dark energy and the other component is in the form of dust (CDM). Also, here we examine the validity of GSLT on both apparent horizon and event horizon. At the apparent horizon it is shown that GSLT is always respected regardless of specific form of DE. But in case of event horizon, GSLT may breakdown in the future.
universe. This chapter is based on [35].

Finally in chapter 7, we extend the investigation of the validity of the GSLT to interacting holographic dark energy model in the DGP braneworld. The thermodynamics of interacting HDE model in GR set up have been extensively studied in literature [36, 37]. In a recent paper [38], validity of GSLT has been studied on the event horizon for interacting DE. Assuming first law of thermodynamics on the event horizon, they have found conditions for validity of GSLT in both cases when FRW universe is filled with interacting two fluid system:- one in the form of cold dark matter and the other is either HDE or new agegraphic DE. Here, we use this method of extracting entropy for interacting HDE in the DGP model. This chapter is based on [39].