Chapter 7

Total Graph of Non-commutative Ring

The total graph of a commutative ring with unity was introduced by Anderson and Badawi in [9]. They considered the total graph $T(\Gamma(R))$ of a commutative ring $R$ as an undirected graph with vertex set $R$ and any two vertices of $T(\Gamma(R))$ are adjacent if and only if their ring sum is a zero-divisor of $R$. In [2], Akbari et al. continued this concept of total graph. We have developed the concept of total graph in non-commutative ring. For a non-commutative ring $R$, the left total graph $T_l(\Gamma(R))$ and the right total graph $T_r(\Gamma(R))$ have been introduced. Basic results of two induced subgraphs $Z_l(\Gamma(R))$ and $Reg_l(\Gamma(R))$ of $T_l(\Gamma(R))$, with vertex sets $Z_l(R)$ and $Reg_l(R)$ respectively, have been discussed. We have investigated some interesting properties, when $Z_l(R)$ is a left ideal and not a left ideal of $R$ respectively. The coloring idea of total graph has been observed with the help of clique concept. The dual concepts i.e. for the right total graph $T_r(\Gamma(R))$, have also been taken under consideration. The chapter is the outcome of our paper ”On total graphs of non-commutative rings”.

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7.1 Definitions and examples

Throughout this chapter $R$ denotes a non-commutative ring and $G$ is a directed graph unless otherwise stated.

The left total graph $T_l(\Gamma(R))$ of $R$ is a directed graph with $R$ as vertex. Any two vertices $x$ and $y$ of $T_l(\Gamma(R))$, $x$ is adjacent to $y$ ($x \ adj y$) if and only if there exists a non-zero element $r$, which is distinct from $x$ and $y$, in $(Z_l(R))$ with $rx + yr \in Z_l(R)$. Similarly, for the right total graph $T_r(\Gamma(R))$, any two vertices $x$ and $y$, $x$ is adjacent to $y$ ($x \ adj y$) if and only if there exists a non-zero element $r$, which is distinct from $x$ and $y$, in $Z_r(R)$ with $rx + yr \in Z_r(R)$. $Z_l(\Gamma(R))$ and $Reg_l(\Gamma(R))$ are two induced subgraphs of $T_l(\Gamma(R))$ with vertex sets $Z_l(R)$ and $Reg_l(R)$ respectively. Similarly, $Z_r(\Gamma(R))$ and $Reg_r(\Gamma(R))$ are two induced subgraphs of $T_r(\Gamma(R))$ with vertex sets $Z_r(R)$ and $Reg_r(R)$ respectively.

For any subset $A$ of $R$, $A^*$ contains all non-zero elements of $R$.

Now we produce two examples of total graph.

**Example. 7.1.1.** Consider the non-commutative ring $R = \{(a_{ij})_{2 \times 2} : a_{11}, a_{12} \in \mathbb{Z}_2, a_{21} = 0 = a_{22}\} = \{A_1, A_2, A_3, A_4\}$, where

\[
A_1 = \begin{bmatrix}
0 & 0 \\
0 & 0 
\end{bmatrix}, \quad A_2 = \begin{bmatrix}
0 & 1 \\
0 & 0 
\end{bmatrix}, \quad A_3 = \begin{bmatrix}
1 & 0 \\
0 & 0 
\end{bmatrix}, \quad A_4 = \begin{bmatrix}
1 & 0 \\
0 & 0 
\end{bmatrix}.
\]

Here $T_l(\Gamma(R))$ is a symmetric digraph.

**Example. 7.1.2.** Consider the ring $R = \{0, a, b, c\}$ with addition and multiplication operations defined as follows:

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and

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Here $T_r(\Gamma(R))$ is a symmetric diagraph.

7.2 Total graph of $R$

In this section, we discuss some interesting properties of total graph and its induced subgraphs. The first result gives the idea of connectedness in total graph. Consequently, we obtain some immediate results of ring homomorphism. Interesting results for sources and sinks are also established. We investigate various characteristics of the total graph using our adjacency relations, and obtain corresponding results related with the ring theoretic concepts.

**Theorem 7.2.1.** If $|Z_l(R)| \geq 3$, then $Z_l(\Gamma(R))$ is strongly connected.

**Proof.** Suppose $x$ and $y$ are two non-zero elements of $Z_l(R)$. Then $yx \in Z_l(R)$ gives $x \to 0 \to y$ is a path in $Z_l(\Gamma(R))$. Similarly $xy \in Z_l(R)$, so $y \to 0 \to x$ is a path in $Z_l(\Gamma(R))$. Hence $Z_l(\Gamma(R))$ is strongly connected.

The above result asserts that $diam(Z_l(\Gamma(R))) = 2$. In the same way, if $|Z_r(R)| \geq 3$, then $Z_r(\Gamma(R))$ is strongly connected. Also $diam(Z_r(\Gamma(R))) = 2$.

**Lemma 7.2.1.** Let $f : R_1 \to R_2$ be a ring monomorphism, where $R_1$ and $R_2$ are two non-commutative rings. If $x$ adj $y$ then $f(x)$ adj $f(y)$, for $x, y \in R_1$. 

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**Proof.** Let \( x \text{ adj } y \). Then, there exists a non-zero \( r \) in \( Z_l(R_1) \) such that \( rx + yr \in Z_l(R_1) \). So \((rx + yr)z = 0\), for some \( z \neq 0 \). This gives \( f(rx + yr)f(z) = 0 \) with \( f(z) \neq 0 \) i.e. \( f(r)f(x) + f(y)f(r) \in Z_l(R_2) \). Hence \( f(x) \text{ adj } f(y) \). 

**Theorem 7.2.2.** Let \( f : R_1 \rightarrow R_2 \) be a ring monomorphism, where \( R_1 \) and \( R_2 \) are two non-commutative rings. If \( T_l(\Gamma(R_1)) \) is a tournament then \( T_l(\Gamma(f(R_1))) \) is also a tournament.

**Proof.** Suppose that \( T_l(\Gamma(R_1)) \) is a tournament. To show \( T_l(\Gamma(f(R_1))) \) is also a tournament. For this, we assume \( y_1, y_2 \in f(R_1) \). So, \( y_1 = f(x_1) \) and \( y_2 = f(x_2) \) for the elements \( x_1 \) and \( x_2 \) in \( R_1 \) respectively. As \( T_l(\Gamma(R_1)) \) is a tournament, therefore either \( x_1 \text{adj} x_2 \) or \( x_2 \text{adj} x_1 \). Then, from the above lemma we get, either \( y_1 \text{adj} y_2 \) or \( y_2 \text{adj} y_1 \). Thus \( T_l(\Gamma(f(R_1))) \) is also a tournament.

**Lemma 7.2.2.** Let \( f : R_1 \rightarrow R_2 \) be a ring isomorphism, where \( R_1 \) and \( R_2 \) are two non-commutative rings. Then \( f \) is also an isomorphism from \( T_l(\Gamma(R_1)) \) onto \( T_l(\Gamma(R_2)) \).

**Proof.** We need only to show that adjacency relation is preserved. For this, we assume that \( x \text{ adj } y \), for \( x, y \in R_1 \). Then, there exists a non-zero \( r \) in \( Z_l(R_1) \) with \( rx + yr \in Z_l(R_1) \). So

\[
(rx + yr)z = 0 \text{ for some } z(\neq 0) \in R_1
\]

\[
\Rightarrow f((rx + yr)z) = f(0)
\]

\[
\Rightarrow (f(r)f(x) + f(y)f(r))f(z) = 0.
\]

Since \( f \) is an isomorphism, so \( f(z) \neq 0 \). This gives \( f(r)f(x) + f(y)f(r) \in Z_l(R_2) \). Again \( f(r) \neq 0 \). Thus \( f(x) \text{ adj } f(y) \). Hence the result.

**Theorem 7.2.3.** Let \( x, y \) be two left invertible elements of \( R \). If \( x \text{ adj } y \), then \( x_i^{-1} \text{adj } y_i^{-1} \), where \( x_i^{-1} \) and \( y_i^{-1} \) are left inverses of \( x \) and \( y \) respectively.
Proof. Let $x \text{ adj } y$. Then, there exists a non-zero $r$ in $Z_l(R)$ with $rx + yr \in Z_l(R)$. So
\[
(rx + yr)z = 0 \text{ for some } z \neq 0
\]
\[
\Rightarrow (y^{-1}_l rx + yr)x^{-1}_lz = 0
\]
\[
\Rightarrow (rx^{-1}_l + y^{-1}_lr)xz = 0.
\]
Hence $x^{-1}_l \text{ adj } y^{-1}_l$, as $xz \neq 0$.

Theorem 7.2.4. Let $x, y$ be two right invertible elements of $R$. If $x \text{ adj } y$, then $x^{-1}_r \text{ adj } y^{-1}_r$, where $x^{-1}_r$ and $y^{-1}_r$ are right inverses of $x$ and $y$ respectively.

Proof. Let $x \text{ adj } y$. Then, there exists a non-zero $r$ in $Z_r(R)$ with $rx + yr \in Z_r(R)$. So
\[
z(rx + yr) = 0 \text{ for some } z \neq 0
\]
\[
\Rightarrow zyy^{-1}_r (r + yr^{-1}_r) = 0
\]
\[
\Rightarrow zy(rx^{-1}_r + y^{-1}_r) = 0.
\]
Hence $x^{-1}_r \text{ adj } y^{-1}_r$, as $zy \neq 0$.

Theorem 7.2.5. Let $R$ and $S$ be two non-commutative rings. If $T_l(\Gamma(R))$ and $T_l(\Gamma(S))$ have no sources (respectively no sinks), then $T_l(\Gamma(R \times S))$ has no sources (respectively no sinks).

Proof. Let $(r, s) \in R \times S$ be arbitrary. Then $r \in R$, $s \in S$. So $r$ and $s$ are not sources of $T_l(\Gamma(R))$ and $T_l(\Gamma(S))$ respectively. Thus, there is an element $x \in R$ and an element $w \in S$ with $x \text{ adj } r$ and $w \text{ adj } s$ respectively. If $x \text{ adj } r$, then there exists an element $u \in Z^*_l(R)$ such that $ux + ru \in Z_l(R)$. Again, if $w \text{ adj } s$, then there exists an element $v \in Z^*_l(S)$ such that $vw + sv \in Z_l(S)$. From this, we get $(u, v) \in Z^*_l(R \times S)$ with $(u, v)(x, w) + (r, s)(u, v) \in Z_l(R \times S)$. Therefore, $(x, w) \text{ adj } (r, s)$. Hence $(r, s)$ is not a source of $T_l(\Gamma(R \times S))$. 

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Corollary 7.2.1. Let $R_1, R_2, \ldots, R_n$ be non-commutative rings for some positive integer $n$. If all $T_i(\Gamma(R_i))$ for $i = 1, 2, 3, \ldots, n$ have no sources (respectively no sinks,) then $T_i(\Gamma(R_1 \times R_2 \times \ldots \times R_n))$ has no sources (respectively no sinks).

Proof. Let $(r_1, r_2, \ldots, r_n) \in R_1 \times R_2 \times \ldots \times R_n$ be arbitrary. Then $r_i \in R_i, \forall i \in \{1, 2, \ldots, n\}$. So $r_i$ is not a source of $T_i(\Gamma(R_i)), \forall i \in \{1, 2, \ldots, n\}$. Thus, there is an element $x_i \in R_i$ with $x_i \text{ adj } r_i, \forall i \in \{1, 2, \ldots, n\}$. Then there exists an element $u_i \in Z_i^*(R_i)$ such that $u_i x_i + r_i u_i \in Z_i(R_i), \forall i \in \{1, 2, \ldots, n\}$. From this, we get $(u_1, u_2, \ldots, u_n) \in Z_i^*(R_1 \times R_2 \times \ldots \times R_n)$ with $(u_1, u_2, \ldots, u_n)(x_1, x_2, \ldots, x_n) + (r_1, r_2, \ldots, r_n)(u_1, u_2, \ldots, u_n) \in Z_i(R_1 \times R_2 \times \ldots \times R_n)$. Therefore, $(x_1, x_2, \ldots, x_n) \text{ adj } (r_1, r_2, \ldots, r_n)$. Hence $(r_1, r_2, \ldots, r_n)$ is not a source of $T_i(\Gamma(R_1 \times R_2 \times \ldots \times R_n))$.

Theorem 7.2.6. Let $e_r, e_l$ be right and left identity elements of $R$ respectively and $Z_i^*(R) \neq \phi$. Then $e_r \text{ adj } e_l$.

Proof. Let $r \in Z_i^*(R)$. Then $r e_r = r$ and $e_l r = r$, as $e_r$ and $e_l$ are right and left identity elements of $R$ respectively. Therefore, $r e_r + e_l r = 2r \in Z_i(R)$. This gives $e_r \text{ adj } e_l$.

In the same way, if $Z_i^*(R) \neq \phi$, then $e_r \text{ adj } e_l$.

For the following results, we consider $Z_i(R)$ and $Z_r(R)$ are left and right ideal of $R$ respectively whenever necessary.

Theorem 7.2.7. For any $x, y \in \text{Reg}_i(R)$, $x \text{ adj } y$ if and only if every element of $x + Z_i(R)$ is adjacent to every element of $y + Z_i(R)$.

Proof. Let $a = x + r_1 \in x + Z_i(R)$, $b = y + r_1 \in y + Z_i(R)$. If $x \text{ adj } y$, then there exists a non-zero $r$ in $Z_i(R)$ with $rx + yr \in Z_i(R)$. This gives $r(a - r_1) + (b - r_2)r \in Z_i(R)$ i.e. $(ra + br) - (rr_1 + r_2r) \in Z_i(R)$. As $Z_i(R)$ is a left ideal of $R$, so $ra + br \in Z_i(R)$. From this $a \text{ adj } b$. Conversely, if $a \text{ adj } b$, then there exists a non-zero $r$ in $Z_i(R)$ with
ra + br ∈ Z_l(R). From this r(x + r_1) + (y + r_2)r ∈ Z_l(R). Therefore, rx + yr ∈ Z_l(R). Hence x adj y.

In the same way, for any x, y ∈ Reg_r(R), xadjy if and only if every element of x + Z_r(R) is adjacent to every element of y + Z_r(R).

**Theorem 7.2.8.** Let e_l be a left identity element. Then e_l + Z_l(R) is a symmetric diagraph.

**Proof.** Let e_l + r_1, e_l + r_2 be any elements of e_l + Z_l(R). Then, at least one of r_1, r_2 is non-zero. From this, we get (e_l + r_1)adj(e_l + r_2), (e_l + r_2)adj(e_l + r_1). Hence, e_l + Z_l(R) is a symmetric diagraph.

Also, if e_r is a right identity element, then e_r + Z_r(R) is a symmetric diagraph.

**Lemma 7.2.3.** Let |Z_l(R)| ≥ 2 and e_r be a right identity element. Then, x + e_r is adjacent to every element of y + Reg_l(R).

**Proof.** If x ≠ 0, then x(x + e_r) + (y + r_l)x ∈ Z_l(R), and if y ≠ 0, then y(x + e_r) + (y + r_l)y ∈ Z_l(R), for any r_l ∈ Reg_l(R). Hence the result.

From the above lemma, it is obvious that every element of x + E_r(R) is adjacent to every element of y + Reg_l(R). With these immediate preceding discussions, we have, every element of x + E_l(R) is adjacent to every element of y + Reg_r(R).

**Theorem 7.2.9.** Let x ∈ Reg_l(R) and e_r ∈ E_r(R). Then every element of e_r + Z_l(R) is adjacent to every element of x + Z_l(R).

**Proof.** Let Z_l^*(R) ≠ φ, x ∈ Reg_l(R) and e_r ∈ E_r(R). Then e_r adj x, when Z_l(R) is a left ideal of R.

**Theorem 7.2.10.** Let Z_l^*(R) ≠ φ, x ∈ Reg_l(R) and e_r ∈ E_r(R). Then e_r adj x.

**Proof.** Let z ∈ Z_l^*(R). Then ze_r + xz ∈ Z_l(R). Hence e_r adj x.
Above two results conclude that if $Z_l^*(R) \neq \phi$, then every right identity element is adjacent to every left regular element of $R$. In the same way, if $Z_r^*(R) \neq \phi$, then every left identity element is adjacent to every right regular element of $R$. Again, we observe that the graphs with $E_r(R) + Z_l(R)$ and $E_l(R) + Z_r(R)$ as vertex sets respectively, are two symmetric diagraphs. Every element of $E_r(R) + Z_l(R)$ is adjacent to every element of $Reg_l(R) + Z_l(R)$. Similarly, every element of $E_l(R) + Z_r(R)$ is adjacent to every element of $Reg_r(R) + Z_r(R)$.

**Theorem 7.2.11.** If $|Z_l(R)| \geq 2$, then $Z_l(R)$ is not disjoint from $Reg_l(R)$.

**Proof.** Let $z \in Z_l^*(R)$. Then, for $z' \neq z \in Z_l(R)$ and $r \in Reg_l(R)$, we get $zz' + rz \in Z_l(R)$. Hence $Z_l(R)$ is not disjoint from $Reg_l(R)$.

**Corollary 7.2.2.** If $|Z_l(R)| \geq 2$, then $Z_l(R)$ is not disjoint from $Reg_l(R) + Z_l(R)$.

The dual concept also holds for the immediate preceding theorem and corollary respectively. From these ideas, we have $Reg_l(R)$ and $Reg_r(R)$ are not totally disconnected if and only if $Reg_l(R) + Z_l(R)$ and $Reg_r(R) + Z_r(R)$ are not disconnected respectively.

### 7.3 Coloring of total graph

In this section, we discuss some results on coloring of total graph. We develop this coloring idea of total graph using the concept of clique. We consider only left total graph. Exact results can be obtained for coloring of right total graph.

**Theorem 7.3.1.** Let $R$ be a reduced ring. Then $\chi(T_l(\Gamma(R))) = 1$ if and only if $R$ has no non-zero left zero-divisors.

**Proof.** Suppose $x$ is a non-zero left zero divisor of $R$. Then there exists a non-zero $y$ in $R$ with $xy = 0$ Now $x$ is distinct from $y$ and $x + y$, so $xy + (x + y)x \in Z_l(R)$. Thus
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\[ y \text{ adj } (x+y). \] But it is a contradiction to \( \chi(T_l(\Gamma(R))) = 1. \) Hence \( R \) has no non-zero left zero-divisors.

**Theorem 7.3.2.** Suppose \( R \) has no left identity and \( Z_l(R) \) is not a left ideal of \( R. \) If there exists a non-zero element \( x \) in \( R \) with \( x^2 = 0 \) and \( |Rx| = n, \) then \( \chi(T_l(\Gamma(R/Z_l(R)))) \geq n. \)

**Proof.** Let \( r_1x, r_2x \in Rx. \) As \( x(r_1x) + (r_2x)x = (xr_1)x \in Z_l(R). \) So \( r_1x \text{ adj } r_2x. \) From this, we get \( Rx \) is a clique. Now \( Rx \sim R/Z_l(x). \) Therefore, \( T_l(\Gamma(Rx)) \sim T_l(\Gamma(R/Z_l(x))), \) by Lemma 7.2.2. This gives \( R/Z_l(x) \) is also a clique, since \( Rx \) is a clique. Thus \( \chi(T_l(\Gamma(R/Z_l(R)))) \geq n. \) Hence the result.

**Theorem 7.3.3.** If \( y_jy_i = y_ky_i \) for \( y_i, y_j, y_k \in R \) and \( k > j > i, \) then \( R \) contains an infinite clique.

**Proof.** Let \( y_jy_i = y_ky_i \) for \( y_i, y_j, y_k \in R \) and \( k > j > i. \) If \( z_{i,j} = y_i - y_j, j > i, \) then

\[
    z_{k,r}z_{i,j} = (y_k - y_r)(y_i - y_j)
    = y_ky_i - y_ky_j - y_ry_i + y_ry_j
    = 0.
\]

Thus \( z_{k,r} \in Z_l^*(R) \) and so \( \{z_{3,4}, z_{3,5}, \ldots\} \subseteq Z_l^*(R). \) Now \( z_{3,4}z_{1,2} + z_{3,5}z_{3,4} \in Z_l(R). \) This gives \( \{z_{1,2}, z_{3,5}\} \) is a clique. If \( z_{6,7} \notin \{z_{1,2}, z_{3,5}\}, \) then \( z_{3,4}z_{1,2} + z_{6,7}z_{3,4} \in Z_l(R) \) and \( z_{8,9}z_{3,5} + z_{6,7}z_{8,9} \in Z_l(R). \) From this, \( \{z_{1,2}, z_{3,5}, z_{6,7}\} \) is a clique. Proceeding in this way, we get an infinite clique. Hence the result.

**Theorem 7.3.4.** If \( I \) is a finite ideal in \( R, \) then \( R \) contains an infinite clique if and only if \( R/I \) has an infinite clique.

**Proof.** If \( R \) has an infinite clique \( C, \) the homomorphic image \( \overline{C} \) of \( C \) is a clique in \( \overline{R} = R/I, \) and since \( I \) is finite, \( \overline{C} \) is still infinite. We assume \( C \) is an infinite clique. If \( x \text{ adj } y, \) for
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\( x, y \in C \), then there exists an non-zero \( r \in Z_l(R) \) distinct from \( x \) and \( y \) with \( rx + yr \in Z_l(R) \). This gives \( (r + I)(x + I) + (y + I)(r + I) \in Z_l(R/I) \). Thus \( (x+I) \ adj (y+I) \). Therefore \( C \) is a clique. Conversely, let \( \{x_i\} \) be a clique in \( C \). Then it is easy to verify that \( \{x_i\} \) is a clique in \( C \). Hence the theorem.

**Theorem 7.3.5.** Let \( x \) be a nilpotent element of degree \( n \geq 3 \) of \( R \). If every right ideal is a left ideal and \( x^{n-1} \notin x^2R \), then there is an infinite clique in \( R \).

**Proof.** Let \( x^n = n, n \geq 3 \). If we put \( y = x^2 \), then \( y^{n-1} = (x^2)^{n-1} = 0 \). If \( yR \) is infinite, then \( R \) has an infinite clique. Let \( yR \) be finite. This gives \( xR/yR \) is infinite. Now \( xR/yR \) is an infinite clique in \( \overline{R} = R/yR \). As

\[
\overline{r} = x^{n-1} + yR
\]

\[
\neq yR.
\]

and

\[
\overline{r}^2 = (x^{n-1} + yR)(x^{n-1} + yR)
\]

\[
= x^{2n}x^{-2} + yR
\]

\[
= yR.
\]

Let \( \overline{r_1}, \overline{r_2} \in xR/yR \). Then \( \overline{r_1} = r_1x + yR, \overline{r_2} = r_2x + yR \). Since

\[
\overline{r} \overline{r_1} + \overline{r_2} \overline{r} = (x^{n-1} + yR)(r_1x + yR) + (r_2x + yR)(x^{n-1} + yR)
\]

\[
= (x^{n-1} + yR)(r_1x + yR) + r_2x^{n} + yR
\]

\[
= (x^{n-1} + yR)(r_1x + yR)
\]

\[
\in Z_r(R/yR).
\]

Therefore, \( \overline{xR} = xR/yR \) is an infinite clique in \( \overline{R} = R/yR \). As \( yR \) is finite, so \( R \) has an infinite clique. Hence the result.
**Conclusion** : In this chapter, we have defined total graph of non-commutative ring and have discussed some basic results. We have also investigated some properties of coloring of total graph. This chapter is just an opening for making another bridge between graph theory and ring theory. The study of connectedness for total graph will be an interesting part of research. A different field can be made for the study of total graph of matrix ring. The approach, for the development of concept of total graph in any direction, will be an exciting field of research.