ALGEBRAIC PROPERTIES OF Q-FUZZY GROUP, M-FUZZY GROUP, FUZZY S-SEMI GROUP, INTUITIONISTS FUZZY GROUP AND S-ANTI FUZZY LEFT H-DEALS OVER FUZZY GROUP, NEAR RING, AND HEMI RING

SYNOPSIS OF Ph. D. THESIS

Submitted to the

BHARATHIDASAN UNIVERSITY

for the award of the degree of

DOCTOR OF PHILOSOPHY IN MATHEMATICS

by

R. NAGARAJAN, M.Sc., B. Ed., M. Phil.,

Under the Guidance of

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JAMAL MOHAMED COLLEGE (AUTONOMOUS)

(Accredited at A Grade by NAAC - CGPA 3.6 out of 4.0)

TIRUCHIRAPPALLI - 620 020, TAMILNADU, INDIA.

JUNE – 2010

REPORT OF DOCTORAL COMMITTEE CERTIFYING PRESENTATION OF SYNOPSIS FOR Ph. D. IN MATHEMATICS

Name of the Candidate: R. NAGARAJAN

Title of Synopsis : Algebraic properties of Q-Fuzzy group, M-Fuzzy group, Fuzzy S-Semi group, Intuitionists Fuzzy group, and S-Anti Fuzzy left h-ideal over Fuzzy group, Near ring and Hemi ring

Venue : Seminar Hall, PG & Research Department of Mathematics Jamal Mohamed College (Autonmous), Tiruchirappalli

Time & Date : 03.00 pm on 28-06-2010

The Doctoral Committee consisting of **Dr. A. SOLAIRAJU** (Research Guide and Convener), **Dr. K. Murugesan** as member examined the candidate Mr. R. Nagarajan while presenting the synopsis for Ph. D. research works. He has satisfied both the committee and other members present by suitably answering the questions raised. Therefore the Doctoral Committee recommends thesubmission of the synopsis to the University.

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CERTIFICATE

This is to certify that Mr. R. Nagarajan has been doing Ph.D. (Part-Time) Research work under my supervision. He has submitted progress report relating to his Ph.D. work in all the years, and has put in adequate number of contact days with the supervisor. Hence, he has satisfied all the guidelines, and he may be permitted to submit the synopsis of the Ph.D. Thesis.

Dr. A. SOLAIRAJU

Tiruchirappalli
28th June, 2010

From
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To
Controller of Examinations
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Sir,

Sub.: Ph. D. Programme in Mathematics – submission of synopsis–reg.
Ref.: No. 12717 / Ph.D./ PT / Maths. / Confirm, / Oct., 2005 dated 18.10.2006

Mr. R. Nagarajan, Part-Time Research Scholar, PG & Research Department of Mathematics, Jamal Mohamed College, Tiruchirappalli is doing Ph. D. Programme in Mathematics under my guidance.

I am herewith sending six copies of Synopsis that was presented in front of Doctoral Committee members, staff members and research scholars at Department of Mathematics, Jamal Mohamed College on 28th June, 2010.

I request you to take the necessary steps.

Thanking you,

Yours faithfully,

(Dr. A. SOLAIRAJU).

SYNOPSIS OF Ph. D. THESIS

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Subject : **MATHEMATICS**

Title : Algebraic properties of Q-Fuzzy group, M-Fuzzy group, Fuzzy S-semigroup, Intuitionsts fuzzy group, S-anti-fuzzy left h-ideal over Fuzzy group, Near ring and hemi ring

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SYNOPSIS OF THE Ph. D.THESIS ENTITLED

ALGEBRAIC PROPERTIES OF Q-FUZZY GROUP, M-FUZZY GROUP, FUZZY S-SEMI GROUP, INTUITIONISTS FUZZY GROUP AND S-ANTI FUZZY LEFT H-DEALS OVER FUZZY GROUP, NEAR RING, AND HEMI RING

Submitted by

R. NAGARAJAN

Assistant Professor of Mathematics, JJ College of Engineering & Technology, Tiruchirappalli, Tamil Nadu, India.

Introduction: Demirci [2001] showed that the concept of smooth groups and smooth homomorphism are introduced and their basic properties are investigated. Dib [1994] explained that The concept of fuzzy space is introduced. This concept corresponds to the concept of the universal set in the ordinary case. The algebra of fuzzy spaces and fuzzy subspaces are studied. Using the concept of fuzzy space and fuzzy binary operation, a new approach can be considered as a generalization and reformulation of the Rosenfeld theory of fuzzy groups. Therefore it is an active tool to develop the theory of fuzzy groups.

Dobrista and yahhyaeva [2002] showed that the Classification is suggested for the early notion of homomorphism's of fuzzy groups conditions are considered for fulfillment of various properties of homomorphism's of ordinary groups as well as properties specific to systems with a fuzzy operation correctness is discussed of the introduction notion of preservation of fuzzy operation. The theory of fuzzy sets has developed in many directions and is finding applications in a wide variety of fields. Rosenfeld in 1971 used this concept to develop the theory of fuzzy groups in this paper we have given independent proof of several theorems on fuzzy groups. We discuss about Fuzzy groups and investigate some of their structures on the concept of Fuzzy Group family. Ray [1999] discussed that in the short communication, some properties of the product of two fuzzy subsets and fuzzy subgroups.

Rovento [2001] proved that the crisp environment the notions of normal subgroup and group operating on a set are well known due to many applications. In this paper, we study extensions of these classical notions to the larger universe of fuzzy sets. We obtain a characterization of operations of fuzzy group on a fuzzy set in terms of homomorphism's of crisp groups. Ray [1999] studied some results of the product of fuzzy sets and fuzzy subgroups. Ray's results will be generalized. p-level subset and p-level subgroups are introduced in the thesis, and then we study of their algebraic properties.

Section 1: Fuzzy group, Fuzzy normal subgroup, and Fuzzy subgrouupoid

Rosenfeld [1971] defined fuzzy subgroupoids and proved that a homomorphic image of a fuzzy subgroupoid with the sup property was a fuzzy groupoid and hence that a homomorphic image of a fuzzy subgroup with sup property was a fuzzy subgroup. This theorem needs the sup property, but we can show the theorem without sup property. Moreover Mukherjee and Battacharya [1991] showed that if [Ậ] is a fuzzy subgroup of a finite group G such that all the level subgroups of G are normal subgroups then [Ậ] is a fuzzy normal subgroup. They can also obtained the theorem without finites using the transfer principle which is a fundamental tool developed here.

There is a result that if A is fuzzy subgroup of G, then gAg^{-1} is also fuzzy subgroup of G for all g in G and$\cap$ gAg^{-1} is a normal subgroup of G (under a t- norm as A). If A and B be two fuzzy subgroups of G under the t- norms T_1 and T_2 respectively, then A$\cap$ B is a fuzzy subgroups under any t- norm T such that $T_1,T_2 \geq T$. The intersection of any two normal fuzzy subgroups of G is also a normal fuzzy subgroup of G under any t- norm weaker than the t- norms of the two fuzzy subgroups. Mukherjee and Battacharya [1991] also explained let $f : G \rightarrow H$ be a group homomorphism if A is a normal fuzzy subgroups of H, then $f^{-1}(A)$ is a normal fuzzy subgroup of G and if f is an epimorphiism then f(A) is a normal fuzzy subgroup of H. Also author derived that $G^t = \{ x \, \varepsilon \, G \, / \, A(x) \geq t \}$ is a subgroup of G. The normaliser of a fuzzy subgroup of G is a subgroup of G.

We conclude that the concept of a normal fuzzy subgroup and proved some properties of this new concept. The theory of fuzzy sets applications in a many directions and is finding applications in a wide variety of fields. Rosenfeld in 1971 used this concept to develop the theory of fuzzy groups We have given Independent proof of Several theorems on pseudo fuzzy cosets of fuzzy normal subgroups

Organization of the thesis

In Chapter I, basic concepts, previous works, needed definitions, organization of the thesis, all new results in various chapters, and contributions to the field of Q-Fuzzy group, S-semigroup, S-product of antifuzzy right R-subgroup of near rings, β-fuzzy congruence relations on a Q-Fuzzy groups, fuzzy M-group from group theory, some new theorems from fuzzy groups, Max^i interval valued anti fuzzy left h-ideals in hemi rings, fuzzy α-cut and L-fuzzy number in fuzzy group, algebraic properties of intuitionistic fuzzy groups, both homomorphic image and preimage of Pseudo fuzzy cosets in fuzzy groups, and their applications in decision making approach to optimize the product mix in assignment level under fuzzy group parameters. All the contributions mentioned above are newly introduced in each chapter.

In Chapter II, we fuzzify the new class of algebraic structures introduced by K.H.Kim [2006]. In this fuzzification, we introduce the notion of Q- fuzzy groups (QFG) and investigate some of their related properties. Some properties on group theory in Q- fuzzy groups are obtained. This fuzzification leads to development of new notions over fuzzy groups. Characterizations of Q- fuzzy groups (QFCG) and normal Q- fuzzy groups (QFNG) are given.

In Lattice Valued Q- fuzzy left R- sub modules of near rings with ewspect to T-norms, a technique of generating of Q- fuzzy R- sub module by a given arbitrary Q- fuzzy set was provided. It is shown that (i) The sum of two Q- fuzzy R- sub module of a module M is the Q- fuzzy R- sub module generated by their union and (ii) The set of all Q-fuzzy sub module of a given module forms a complete lattice. Consequently the collection of all Q-fuzzy R- sub module, having the same values at zero of M of the lattice of Q- fuzzy R- sub module of M. Interrelationship of these finite range sub lattices was established. Finally it was shown that the lattice of all Q- fuzzy R- sub module of M can be embedded into a lattice of Q- fuzzy R- sub module of M where M denote as Q-fuzzy R- sub module and R is the commutative near ring with unity. Characterization of Q-fuzzy left R- sub modules with respect to t- norm was also given.

In chapter III, the purpose is to define the β_q- fuzzy congruence by using special fuzzy equivalence relation of Q- fuzzy subgroups which is defined in this study and we define suitable Q-fuzzy subgroupoids and Q- fuzzy quotient subgroup of finite group G/H differently then we investigate some basic properties.

In chapter IV, we fuzzify the new class of algebraic structure introduced by Kim [2006]. In this fuzzification (called fuzzy S-semi groups), introduced the notions of fuzzy sub S-semi groups and investigate some of their related properties. The purpose of leads to development of new notions over fuzzy S-semi groups. Introduced the notions of fuzzy sub algebra, intuitionistic fuzzy sub algebra in d-algebras and investigate some of their results.

In Chapter V, the theory of fuzzy sets has developed in many directions and is finding application in a wide variety of fields.Rosenfeld [1971] used this concept to develop the theory of fuzzy groups, We have given independent proof of several theorems on M- fuzzy groups. We discuss about M- fuzzy groups and investigate some of their structures on the concept of M-fuzzy group family.

In chapter VI, the concept of $\max^i$- interval valued anti fuzzy left h- ideals in a hemi rings and extension principle of interval valued fuzzy set are introduced. Some of their properties and structural characteristics, some theorems for homomorphic image are investigated and its inverse image on $\max^i$ – interval valued anti fuzzy left h- ideals of a hemi rings is verified. Relationship between anti fuzzy left h- ideals in a hemi ring and fuzzy left h- ideals is also given. Using lower level set, a characterization of interval value anti fuzzy left h ideals is given.

In chapter VII, the concept of $\max^i$- interval valued anti fuzzy left h- ideals in a hemi rings and extension principle of interval valued fuzzy set are introduced. Some of their properties and structural characteristics, some theorems for homomorphic image are investigated and its inverse image on $\max^i$ – interval valued anti fuzzy left h- ideals of a hemi rings is verified. Relationship between anti fuzzy left h- ideals in a hemi ring and fuzzy left h-ideals is also given. Using lower level set, a characterization of interval value anti fuzzy left h ideals is given.

In chapter VIII, the theory of fuzzy sets has developed in many directions and is finding applications in a wide variety of fields. Rosenfeld in 1971 used this concept to develop the theory of fuzzy groups. We have given independent proof of several theorems on fuzzy groups. We discuss about Fuzzy groups and investigate some of their structures on the concept of Fuzzy Group family. We have given Independent proof of Several theorems on pseudo fuzzy cosets of fuzzy normal subgroups

In chapter IX, the improved algorithm benefits from the advantage of reaching optimal solution. In the previous researchers all inputs were considered as crisp values. The assumption is not in real cases. This work considers product mix problem as a group decision making problem in which all inputs are fuzzy. A new algorithm for optimizing product mix under fuzzy parameters is developed. For this method, ordering methods are used in order to make decision in a fuzzy group decision making environment.

Section 2: A New structure and Construction of Q-Fuzzy groups

Introduction: In Q-fuzzy left R-subgroups of near rings with respect to T- norms [Y.U.Cho, Y.B.Jun, 2007], they showed that if A is an intuitionistic fuzzy right R-subgroup of a near ring R then the set $R_A = \{ x \varepsilon R / \mu_A(x) = \mu_A(0), \gamma_A(x) = \gamma_A(0)\}$ is a right R- subgroup of R.

K.H Kim and Y. B. Jun [2001] established the notion of a normal fuzz R-subgroup in a near ring and related properties discussed. They [2000] discussed about an S-norm on [0, 1]. If μ is sensible fuzzy R- subgroup of R with respect to S, then $\mu(0) \leq \mu(x)$ for all x ε R, and every sensible fuzzy R- subgroup of R with respect to S is an anti fuzzy R- subgroup of R.

Osmankazanci, sultan yamark and serife yilmaz [2007] identified that if $\{A_i\}_{i \varepsilon A}$ is a family of intuitionistic Q- fuzzy R- subgroups of R, then $\cap A_i$ is an intuitionistic Q- fuzzy R- subgroup of R. If $A = (\mu_A, \gamma_A)$ is an intuitionistic Q- fuzzy R- subgroup of R then if $\theta : R \rightarrow S$ be an epimorphism, $B = (\mu_B, \lambda_B)$ is an intuitionistic Q- fuzzy set in S, and $\theta^{-1}(B) = (\mu_{\theta\text{-}1}(B), \lambda_{\theta\text{-}1}(B))$ is an intuitionistic Q-fuzzy R-subgroup of R, then B is an intuitionistic Q- fuzzy R- subgroup of S.

The notion of Q-fuzzification of left R- subgroups is introduced in a near-ring and investigated some related properties. Characterization of Q-fuzzy left R-subgroups with respect to a t-norm are given.

In B. Davvaz, W. A. Dudek and Y. B. Yun, [2005] in the work titled "A new Structure and Construction of Q- fuzzy groups" they established the notion of intuitionistic fuzzy sets introduced by Atanassov as a generalization of the notion of fuzzy sets. They [2006] explained the concept of an intuitionistic fuzzy set to Hv- modules, They also introduced the notion of intuitionistic fuzzy Hv-sub modules of an Hv-submodules and discussed with some properties.

F.H.Rho, K.H.Kim, J.G.Lu [2006] showed that if A in X is an intuitionistic fuzzy Q- sub algebra of X, then $\mu_A(0,q) \geq \mu_A(x,q)$ and $\lambda_A(0,q) \leq \lambda_A(x,q)$, and they concluded that the intuitionistic Q-fuzzification of the concept of sub algebra in BCK/BCI algebra.

In Lattice Valued Q- fuzzy left R- sub modules of near rings with respect to T-norms, a technique of generating of Q- fuzzy R- sub module by a given arbitrary Q-fuzzy set was provided. It is shown that (i) The sum of two Q- fuzzy R- sub module of a module M is the Q- fuzzy R- sub module generated by their union and (ii) The set of all Q-fuzzy sub module of a given module forms a complete lattice. Consequently the collection of all Q-fuzzy R-sub module, having the same values at zero of M of the lattice of Q- fuzzy R- sub module of M. Interrelationship of these finite range sub lattices was established. Finally it was shown that the lattice of all Q- fuzzy R- sub module of M can be embedded into a lattice of Q- fuzzy R- sub module of M where M denote as Q-fuzzy R- sub module and R is the commutative near ring with unity. Characterization of Q-fuzzy left R- sub modules with respect to t- norm was also given.

We fuzzify the new class of algebraic structures introduced by [K.H.Kim, 2006]. In this fuzzification, we introduce the notion of Q- fuzzy groups (QFG) and investigate some of their related properties. Some properties on group theory in Q- fuzzy groups are obtained. This fuzzification leads to development of new notions over fuzzy groups. Characterizations of Q- fuzzy groups (QFCG) and normal Q- fuzzy groups (QFNG) are given.

The following results on the properties of Q-fuzzy group are obtained.

1. Let 'A' be a Q- fuzzy group of G. Then $A(x,q) \leq A(e,q)$ for all x εG and q ε Q. (ii) The subset G_A = { xε G / A(x,q) = A(e,q)} is a Q- fuzzy group of G. Let A and B be two Q- fuzzy groups of a group G. Then (A ∩ B) is Q-fuzzy group of G. If 'A' is a Q- fuzzy group of G, then A^C is also Q-fuzzy group of G.

2. If 'A' is Q- fuzzy group of G, then the set U(A; t) is also Q-fuzzy group for all q ε Q, t ε Im(A).

3. Let G and G^1 be two groups and θ: G→G^1 a homomorphisms. If 'B' is Q – fuzzy group of G^1 then the pre image $\theta^{-1}(B)$ is Q- fuzzy group of G.

4. Let θ: G → G^1 be an epimorphisms and B is Q – fuzzy set in G^1. If $\theta^{-1}(B)$ is Q- fuzzy group of G, then B is Q- fuzzy group of G^1.

5. If $\{A_i\}_{i\varepsilon A}$ is a family of Q- fuzzy groups of G, then $\cap A_i$ is Q-fuzzy group of G where $\cap A_i = \{((x,q) \ \Lambda \ \mu_{Ai}(x,q)) / x \ \varepsilon \ G \ , q \ \varepsilon \ Q\}$, where $i \in A$.

6. If A is Q- fuzzy set in G such that all non- empty level subset U(A; t) is Q- fuzzy group of G, then A is Q- fuzzy group of G.

7. A set of necessary and sufficient conditions for a Q- fuzzy set of a group G to be a Q- fuzzy group of G is that $A(xy^{-1}, q) \geq \min (A(x, q), A(y,q)\}$ for all x,y in G and q in Q.

8. If 'A' is Q- fuzzy group of G and θ is a homomorphism of G, then the Q-fuzzy set A^θ of G given by A^θ = {< (x,q), $\mu_A{}^\theta$ (x,q)>; xεG, qεQ}is Q-fuzzy group of G.

9. Let A be Q–fuzzy group of G. Let A^+ be a Q- fuzzy set in G defined by $A^+(x, q) = A(x, q) + 1 - A(e,q)$ for all x ε G. Then A^+ is normal Q- fuzzy group of G which contains A.

10. Let A be a QFNG of a group G. Then for all x,y ε G, A([x,y], q) = A(e,q).

11. If 'A' is QFCG of a group G, then A is QFNG of G.

12. Let T be a t- norm. Then every imaginable Q- fuzzy left R- subgroup μ of a near ring ' S' is a fuzzy left R-subgroup of S.

13 If μ is a Q-fuzzy left R- subgroups of a near ring S and Θ is an endomorphism of S, then μ[Θ] is a Q- fuzzy left R- subgroup of S.

14. An onto homomorphisms of a Q- fuzzy left R-subgroup of near ring S is Q- fuzzy left R-subgroup.

15. An onto homomorphic image of a fuzzy left R- subgroup with the sup property is a fuzzy left R- subgroup.

16. Let T be a continous t-norm and f be a homomorphism on a near ring S. If μ is Q-fuzzy left R- subgroup of S, then μ^f is a Q- fuzzy left R- subgroup of f(S).

17. Let μ be a Q fuzzy R- sub module of M. Then the Q- fuzzy subset<μ> is a Q- fuzzy R- sub module of M generated by. More over <μ> is the smallest Q- fuzzy R- sub module containing μ

18. Let μ and θ be a Q- fuzzy R- sub modules of M such that μ (0,q) = θ (0,q). Then $\mu \subseteq \mu + \theta$, $\theta \subseteq \mu + \theta$.

19. Let μ and θ be a Q- fuzzy R- sub module of M such that Let 'T' be a t- norm. Then every imaginable Q- fuzzy left R- sub module 'μ' of a near ring ' R' is a fuzzy left R-sub module of R. μ (0,q) = θ (0,q) implies that μ + θ = < μ + θ >.

20. Let 'μ' be a Q- fuzzy R- sub module of a near ring and let μ* be a Q- fuzzy set in R defined by μ*(x,q) = μ(x,q) +1- μ(0,q) for all x ε R. Then μ* is a normal Q- fuzzy R- sub module of R containing μ.

21. If 'μ' is a Q- fuzzy left R- sub module of a near ring 'R' and 'Ɵ' is an endomorphism of R, then $\mu_{[\Theta]}$ is a Q- fuzzy left R- sub module of 'R'.

22. An onto homomorphism of a Q- fuzzy left R- sub module of near ring R is Q- fuzzy left R-sub module.

23. Let T be a continuous t-norm and let 'f' be a homomorphism on a near ring R. If μ is Q- fuzzy left R- sub module of R, then μ^f is a Q- fuzzy left R-sub module of f(R).

25. L is a lattice under the usual ordering of Q- fuzzy set inclusion. More over L is a complete lattice, and L_t is a complete lattice of L.

25. L_t is a sub lattice of L. Let L_t denote the set of all Q- fuzzy R- sub modules θ of M such that θ (0,q) = t and $I_m\theta$ is finite

27. L(M), the lattice of all sub modules of M can be embedded in L_2

Section 3: Q- Fuzzy subgroups of β-Fuzzy Congruence relation on a group

Introduction The concept of fuzzy sets was first introduced by Zadeh in [1965] and since then there has been a tremendous interest in the subject due to diverse applications ranging from engineering and computer science to social behavior studies. The concept of fuzzy relation on a set was defined by Zadeh [1965] and other authors like Rosenfeld [1971] , The notion of fuzzy congruence on a group was introduced by Kuroki [1992] and that the universal algebra was studied.The study of fuzzy algebraic structures was started with the introduction of the concept of fuzzy subgroups by Rosenfeld in [1971]. Several mathematicians have followed the Rosenfeld approach in investigating the fuzzy subgroup theory. Fuzzy normal subgroups were studied by Wu [1981] and Dib [1998] , Kumar et.al. [1995] and Mukherjee [1985] . The concept of fuzzy quotient group was studied.

We introduced the notion of Q- fuzzy subgroups. In this study, we define some new special fuzzy equivalence relations and derive some simple consequences. Then using those relations we define suitable Q-fuzzy subgroupoids and Q- fuzzy quotient subgroup of G/H differently.

The following results are obtained in chapter III:

28. Let G be a group with identity 'e' and A_H be a Q- fuzzy subgroup of a group G, Then the relation β_q defined on G is Q-similarity relation on G. Further The fuzzy relation β_q defined on G is Q- fuzzy compatible.

29. The fuzzy relation β_q defined on G is a Q- fuzzy congruence.

30: The defined fuzzy set N is a Q- fuzzy quotient subgroup of G/H.

31: The defined fuzzy set N is a Q- fuzzy quotient normal subgroup of G/H.

32:If N is a Q- fuzzy quotient subgroupoid of finite group G/H, then N is a Q- fuzzy subgroup.

33: Let N be a Q- fuzzy quotient subgroup of a group G/H and let xH ε G/H, Then N(xHyH,q) = N(yH,q), for all yHεG/H ↔ N(xH,q) = N(H,q).

34: Let N and R be two Q- fuzzy quotient subgroups of G/H, Then N∩R is a Q- fuzzy quotient normal subgroup of G/H, and the Q-fuzzy relation μ_N is a Q- fuzzy congruence on G/H.

S-anti fuzzy right R- subgroups

Introduction: B. Schweizar and A.Sklar [1963] introduce the notions of Triangular norm (t-norm) and Triangular co-norm (S-norm) are the most general families of binary operations that satisfy the requirement of the conjunction and disjunction operators respectively. First, Abu. Osman [1987] introduced the notion of fuzzy subgroup with respect to t-norm. S. Abou. Zaid [1991] also introduced the concept of R-subgroups of a near-rings and Kyunghokim [2007] introduced the concept of fuzzy R- subgroups of a near-ring. Then J. Zhan [2005] introduced the notion of fuzzy hyper ideals in hyper near-rings with respect to t-norm. Recently , Y.U.Cho et,al [2005] introduced the notion of fuzzy sub algebras with respect to S-norm of BCK algebras and M.Akram [2006] introduced the notion of sensible fuzzy ideal.

We redefine anti-fuzzy right R- subgroups of a near-ring ‘R’ with respect to a S-norm and investigate it is related properties. Also, we review several results described in [2007] and [2007] using S-norm.

The following results are also obtained in chapter III.

35: Let ‘S’ be a s-norm. Then every sensible S-anti fuzzy right R- subgroups ‘μ’ of R is an anti-fuzzy R- subgroups of R.

36: If ‘μ’ is a S- anti fuzzy right R-subgroups of a near ring R and ‘θ’ is an endomorphism of R, then μ[θ] is a S- anti fuzzy right R- subgroups of R.

37: An onto homomorphic pre image of a S- anti fuzzy right R- subgroups of a near- ring is S-anti fuzzy right R- subgroups.

38: An onto homomorphic image of a anti fuzzy right R- subgroups with the inf property is a anti-fuzzy right R- subgroups.

39: Let f: $R \rightarrow R^1$ be a homomorphism of near-rings. If ‘μ’ is a S- anti fuzzy right R- subgroups of R^1, then μ^f is S- anti fuzzy right R- subgroup of R.

40: Let f;$R \rightarrow R^1$ be a homomorphism of near-rings. If ‘μ^f’ is a S- anti fuzzy right R- subgroups of R, then μ is S- anti fuzzy right R- subgroup R’. right R- subgroup of R^1.

41: Let ‘S’ be a continuous S- norm and let ‘f’ be a homomorphism on a near-ring R. If ‘μ’ is a S-anti fuzzy right R- subgroups of R, then μf is a S- anti fuzzy right R- subgroups of f(R).

42: A fuzzy subset ‘μ’ of R is a T- anti fuzzy right R- subgroups if and only if ‘μ^c’ is a ‘S’ anti fuzzy right R- subgroup of R.

The theorems are got about S-Product of S-anti Fuzzy right R-subgroups in Chapter III

43: Let ‘S’ be a s- norm. let ‘μ’and ‘χ’ be a S- anti fuzzy right R- subgroup of R, then μ×χ is a anti fuzzy right R- subgroup of R.

44: Let ‘ μ’ and ‘χ’ be sensible S- anti fuzzy right R- subgroups of a near- ring R. Then μ×χ is a sensible S- anti fuzzy right R- subgroup of R×R.

45: If μ×χ is a sensible S- anti fuzzy right R- subgroup of R×R , Then μ×χ need not be sensible S- anti fuzzy right R- subgroup of R.

Section 4: Some structure properties of Q-Cyclic Fuzzy group family and some algebraic properties of BCK-Algebra and Fuzzy S–Semigroup

Introduction: The original concept of fuzzy sets was firstly introduced in the pioneering work [1965] of Zadeh as an extension of crisp (usual) sets, by enlarging the truth value set of “ grade of membership” from the two sets {0,1} to the unit interval [0,1] 0f real numbers. There has been tremendous interest in the fuzzy set theory due to its many applications ranging from engineering and computer science to social behavior studies. More details and historical background of fuzzy set theory].

There is a quite substantial literature on fuzzy group theory. The study of fuzzy groups was started firstly by Rosenfeld [1971] . He used the *min* operating to define his fuzzy groups and showed how some basic notions of fuzzy group theory should be extended in an elementary manner to develop the theory of fuzzy groups. It was extended by Anthony and Sherwood [1979]. They used the *t- norm* operating instead of the *min* to define the t- fuzzy groups. Roventa and Spircu [2001] introduced the fuzzy group operating on fuzzy sets. Sidkey and Misherf [1991] defined t- cyclic fuzzy groups by using t- level sets in the crisp environment. Ray [1993] defined a cyclic fuzzy group of a given fuzzy group family simply by restriction. [2008],[2009] results are listed. In this paper, we give a sufficient condition for a Q- fuzzy subset to be a Q- cyclic fuzzy group. By using this Q- cyclic fuzzy group, we then define a Q- cyclic fuzzy group family and investigate its structure properties with applications.

Ray [1993] established subgroups and normal subgroups of a fuzzy group and their criteria’s. The fuzzy order of an element of a group also defined and its relationship with the order of the element is examined. The construction of the smallest fuzzy group containing a given arbitrary fuzzy set is systematized. A cyclic fuzzy group that is a restriction of a fuzzy group that is a restriction of a fuzzy group or that is generated by a fuzzy point is brought to discussion. Finally he explained the usual interactions between a cyclic and abelian fuzzy group. He also pointed out the possibility of an abelian fuzzy group might be isomorphic to a direct product of some of its cyclic subgroups is touched upon.

Kim [2006] considered the intuitionistic Q- fuzzification of the concept of sub algebra’s in BCK/BCI algebra and he explained (i) Let A be an intuitionistic Q- fuzzy sub algebra of X, Then $X_A^{(\alpha,\beta)}$ is a sub algebra of X with α+β ≤ 1. (ii) Any sub algebra of X can be realized as both a μ-level sub algebra and a γ- level algebra of some intuitionistic Q- fuzzy sub algebra of X. (iii) Let f : X →Y be a homomorphism from a BCK/BCI algebra X onto a BCK/BCI algebra. If A is an intuitionistic Q- fuzzy sub algebra of X, then the image f(A) is intuitionistic Q- fuzzy sub algebra of X.

Kim [2006] introduced the new class of algebra's related to BCK algebra's and semi groups called KS-semi group and define an ideal of a KS- semi groups and a strong KS-semi groups. He also defined a congruence relation on KS-semi groups and quotient KS-semi group and proved (i) Every p-ideal of a KS-semi group X in an ideal but the converse is not true. (ii) Let X be a strong KS-Semi group with a unity 1 and A any non- empty subset of X. If g ε A and x ≤ y imply x ε A then A is an ideal of X. (iii) Let X be a KS-semi group. Then an equivalence relation ρ on X is congruence iff is both left and right compatible. (iv) Let f : X→ Y be a homomorphism of KS-semi groups. Then kerlf is an ideal of X.

Abu osman [1987] explained the closure operator on the set of fuzzy relation on S and to show that (i) The closed hull of a fuzzy relation (s,μ) is given by Ґ́(s,μ) = (sσ,μ). (ii) The composition of two closed fuzzy relations need not be closed fuzzy relations.

Kim [2000] introduced the notion of sensible fuzzy R- subgroups in near rings, and he showed that (i) Let s be a S-norm. Every sensible fuzzy R- subgroup of R with respect to S is an anti fuzzy R- subgroup of R. (ii) An onto homomorphic image of a fuzzy right R- subgroup with respect to s is a fuzzy right R- subgroup. (iii) If μ is a fuzzy right R- subgroup of R with respect to s and θ is an endomorphism's of R, then μ[θ] is a fuzzy right R- subgroup of R with respect to s.

In Crisp environment, the notion of cyclic group on a set is well known. We study an extension of this classical notion to the Q- fuzzy sets to define the concept of Q-cyclic fuzzy groups. By using these Q- cyclic fuzzy groups, we then define a Q- cyclic fuzzy group family and investigate its structure properties with applications.

The following results are proved in chapter IV.

46: Let 'A' be a Q- fuzzy group of G Then (i)A(x,q) ≤ A(e,q) for all x εG and q ε Q. (ii) The subset G_A = { xε G / A(x,q) = A(e,q)} is a Q- fuzzy group of G.

47: Let G be a finite group and A be a Q- fuzzy group of G. Consider the subset H of G given by H = {xεG / A(x,q) = A(e,q)} .Then H is a crisp subgroup of G.

48: If 'A' is Q- fuzzy group of G, then the set U(A; t) is also Q-fuzzy group for all q ε Q, t ε Im(A).

49: If 'A' is Q- fuzzy set in G such that all non- empty level subset U(A; t) is Q- fuzzy group of G then A is Q- fuzzy group of G.

50*:* A set of necessary and sufficient conditions for a Q- fuzzy set of a group G to be a Q- fuzzy group of G is that $A(xy^{-1}, q) \geq \min (A(x,q) , A(y,q)\}$ for all x,y in G and q in Q.

51: If G is a group, then $\tilde{A}^m = \{ (a^n,q) , \mu_{\tilde{A}}(a^n,q)^m / n \in z \}$ is also a Q –cyclic fuzzy group.

52: Let A be a Q- cyclic group with 12 elements and generated by (a,q). Let Ã be a Q- fuzzy set of the group A defined as follows. $\mu_{\tilde{A}}(a^0,q) = 1$, $\mu_{\tilde{A}}(a^4,q) = \mu_{\tilde{A}}(a^8,q) = t_1$, $\mu_{\tilde{A}}(a^2,q) = \mu_{\tilde{A}}(a^6,q) = \mu_{\tilde{A}}(a^{16},q) = t_2$ and $\mu_{\tilde{A}}(x) = t_3$ for all other elements x in A, where $t_1,t_2,t_3 \in [0,1]$ with $t_1 > t_2 > t_3$. It is clear that Ã is a Q- fuzzy group of A, thus $\tilde{A} = \{ (a^k,q) , \mu_{\tilde{A}}(a^k,q) / k \in z \}$ is a Q- cyclic fuzzy group generated by ((a,q) , $\mu_{\tilde{A}}(a,q)$).

53*:* The Q- fuzzy group $\tilde{A}^n$ is a Q- fuzzy subgroup of $\tilde{A}^m$, if m ≤ n.

54: If $\tilde{A}^i$ and $\tilde{A}^j$ are Q- cyclic fuzzy groups, then $\tilde{A}^i \cup \tilde{A}^j$ is also Q- cyclic fuzzy group if i<j

55: If $\tilde{A}_i$ and $\tilde{A}_j$ are Q- cyclic fuzzy groups, then $\tilde{A}_i \cap \tilde{A}_j$ is also a Q- cyclic fuzzy group.

56: Since a Q- cyclic fuzzy group is an abelian group, it is clear that μ(xy,q) = μ(yx,q) for x,y ε A, q ε Q. Therefore, the Q- cyclic fuzzy groups $\tilde{A}^m$, $\tilde{A}_i \cup \tilde{A}_j$ and $\tilde{A}_i \cap \tilde{A}_j$ are also normal Q- fuzzy groups.

57: Let < A > = { A , A^1 , A^2,----, A^m, ---, E. Then $\cup\ A^P = A$ and $\cap\ A^P = E$ where m varies 1 to ∞.

58: Let Ã be a Q- cyclic fuzzy group. Then $A \supset A^2 \supset A^3 \ldots \supset A^m \ldots \supset E$.

59: Let < Ã> = { Ã , $\tilde{A}^2$, $\tilde{A}^3$ … $\tilde{A}^m$ …E }. Then $\tilde{A} \supset \tilde{A}^2 \supset \ldots \supset \tilde{A}^m \ldots \supset E$.

60: Let f be a group homomorphism's of a Q- cyclic fuzzy group Ã. Then the image of Ã under f is a Q- cyclic fuzzy group.

61: Let {$\tilde{A}^m$, $\tilde{A}^{m-1}$,…, Ã } be a finite Q- cyclic fuzzy group family. Then $\tilde{A}^m \times \tilde{A}^{m-1} \times \ldots \times \tilde{A} = \tilde{A}^m$.

Section 5: Some Algebraic properties of BCK—algebra's and fuzzy S-algebra

Y. Imai and K. Iseki [1966] introduced two classes of abstract algebras; BCK - algebras and BCI–algebras. It is known that the notion of BCI-algebras is a generalization of BCK–algebras. J. Neggers and H.S Kim [1999] introduced the class of d-algebras which is another generalization of BCK-algebras and investigated relations between d-algebras and BCK-algebras. A. Rosenfeld [1971] introduced the notion of fuzzy group. Following the idea of fuzzy groups, O.G Xi [1991] introduced the notion of fuzzy BCK-algebras. After that, Y.B Jun [1992] studied fuzzy BCK-algebras. Recently, the new class of algebraic structure introduced by Kim [2006], called S-semi group which is the combination of BCK-algebras and semi groups. In this paper, we fuzzify the new class of algebraic structure introduced by Kim [2006]. We have proved some interesting results which are very closer to the results in BCK – algebras.

The following results are showed on S-semi groups in chapter V.

62: Intersection of two BCK-algebras is BCK-algebra w.r.t* as well as Δ

63: Union of two BCK-algebras is BCK-algebra w.r.t.* if one is contained in other.

64: A subset of BCK-algebra is BCK-algebra.

65: Homomorphic image of BCK-algebra is BCK-algebra.

66 Union of fuzzy sub algebra is a fuzzy sub algebra.

67: Intersection of two fuzzy sub algebra is a fuzzy sub algebra.

68 : α - cut of fuzzy sub algebra is a fuzzy sub algebra.

69: Product of two fuzzy sub algebra is a fuzzy sub algebra.

70: Union of two intuitionistic fuzzy sub algebra is an intuitionstic fuzzy sub algebra.

71: The intersection of two intuitionistic fuzzy sub algebra's is an intuitionistic fuzzy subalgebra.

72 : α - cut of an intuitionistic fuzzy sub algebra is an intuitionistic fuzzy sub algebra.

73: Product of intuitionistic fuzzy sub algebra is also an intuitionistic fuzzy subalgebra.

74: A fuzzy set 'μ' of 'X' is a fuzzy sub S-semi group iff the upper level set μ_t is either empty or a sub S-semi group of 'X', for every $0 < t < 1$.

Structure properties of M-Fuzzy groups

Several researches were conducted on the generalizations of the notion of fuzzy sets. The study of fuzzy group was started by Rosenfeld [1971] and it was extended by Roventa [2001] who have introduced the concept of fuzzy groups operating on fuzzy sets. Wu [1981] studied the fuzzy normal subgroups. Gu [1994] put forward the notion of fuzzy groups with operators. In this paper, we introduce the concept of M- fuzzy groups with operators and obtain some related results. For the sake of convenience, we set out the former concepts.

The following results are found in chapter V.

75: Let A be M- fuzzy group and S be a fuzzy subset of A then S is a M- fuzzy subgroup of A iff $A_S(m(xy)) \geq \min(\{A_S(mx), A_S(my)\}$ for all x,y in S.

78: for all a,b ε [0,1], m ε M and p is any positive integer , verify that (i) If ma ≤ mb then $(ma)^p \leq (mb)^p$ and (ii) $\min\{ma,mb\}^p = \min\{(ma)^p, (mb)^p\}$

79: If A is a M- fuzzy group, then $A^p= ((mx, (A(mx)^p) / mx \, ε \, A \}$ is M- fuzzy group.

80: The M- fuzzy group A^q is a M- fuzzy subgroup A^p , if q ≤ p.

81: If A^i and A^j are M- fuzzy groups, then $A^i \cup A^j$ is also M- fuzzy group for positive integers i & j.

82: If A^i and A^j are M- fuzzy groups, then $A^i \cap A^j$ is also M- fuzzy groups, where i and j are natural numbers.

83: Prove that $A^p \subset A$ for all p..

84: Let G and G^1 be M – groups and 'f 'an M- homomorphism from G onto G^1, (i) if A is M – fuzzy group of G^1 then A^f is M- fuzzy group of G. (ii) if A^f is M- fuzzy group of G then A is M- fuzzy group of G^1.

85: Let $< A > = \{ A , A^1 , A^2,----, A^P,---, E\}$. Then $\cup A^P = A$ and $\cap A^P = E$ where p varies 1 to ∞.

86: Let G and G^1 be M- groups and f is homomorphism from G onto G^1. If A^f is M- fuzzy group of G , then A is M- fuzzy group of G^1.

87: Let A be a M- fuzzy group, then $A \supset A^2 \supset A^3 \ldots\supset A^p \ldots E$.

The following results are found for w-fuzzy groups in chapter V.

88: Let 'A' be a fuzzy subset in a group X such that A(e) =1, where 'e' is the identity of X then 'A' is a w- fuzzy group iff $A(xy^{-1}) \geq A(x) A(y)$ for all x,y ε X.

89{ If the w- fuzzy groupoid 'A' on X has left identity e_λ and a right identity e_μ then $e_\lambda = e_\mu$.

90: Let G and G^1 be groups and f a homomorphism from G onto G^1, (i) if A is w- fuzzy group of G^1 then A^f is w- fuzzy group of G. (ii) if A^f is w- fuzzy group of G then A is w- fuzzy group of G^1.

91: If A and B be w-fuzzy groups of G_1 & G_2 respectively then A×B is w-fuzzy group of $G_1 \times G_2$.

92: If $A_1, A_2, \ldots, A_n$ are w- fuzzy groups of $G_1, G_2, \ldots, G_n$ respectively then $A_1 \times A_2 \times \ldots \times A_n$ is w-fuzzy groups of $G_1 \times G_2 \times \ldots \times G_n$.

93: Let A and B be fuzzy subsets of G_1 and G_2 respectively such that A×B is a w- fuzzy group of $G_1 \times G_2$ then A and B is w- fuzzy group of G_1 and G_2 respectively.

94:Let f: $G \rightarrow G^1$ be a group homomorphism and let 'A' be a w- fuzzy group of G^1 then $f^{-1}(A)$ is w- fuzzy group of G.

95: Let A be a w- fuzzy group of group G and A* be a fuzzy set in G defined by $A^*(x) = A(x) + 1 - A(e)$ for all x ε G. Then A* is w- fuzzy group of G containing A.

Section 6: Generalized product of fuzzy groups and P-level subgroups

Since its inception, there has been a tremendous interest in the fuzzy set theory due to its many applications ranging from engineering and computer sciences to social behavior studies. There is quite substantial literature on fuzzy group theory. The study of fuzzy groups was started firstly by Rosenfeld [1971] . He used the min operating to define his fuzzy groups and showed how some basic notions of group theory should be extended in an elementary manner to develop the theory of fuzzy groups. It was extended by Antony and Sherwood [1982]. They used the-t-norm operating instead of the min to define the t-fuzzy groups. Roventa and Spircu [2001] introduced the fuzzy group operating on fuzzy sets.

We first generalized the results of the product of fuzzy groups which were done by Ray [1999]. We also define p-level subset and P-level subgroups, and then we study some of their properties.

The following results are found in Product of fuzzy groups & t-fuzzy groups in chapter VI

96: If $\hat{G}_1, \hat{G}_2, \ldots, \hat{G}_n$ be a fuzzy groups of the groups $X_1, X_2, \ldots, X_n$ respectively, then $\hat{G}_1 \times \hat{G}_2 \times \ldots \times \hat{G}_n$ is fuzzy groups of $X_1 \times X_2 \times \ldots \times X_n$.

97: Let $\hat{G}_1 \times \hat{G}_2 \times \ldots \times \hat{G}_n$ be fuzzy groups of the groups $X_1 \times X_2 \times \ldots \times X_n$, respectively then $\hat{G}_1 \times \hat{G}_2 \times \ldots \times \hat{G}_n$ is a fuzzy normal subgroup of $X_1 \times X_2 \times \ldots \ldots X_n$.

98: Let the fuzzy groups $\hat{G}_1 \times \hat{G}_2 \times \ldots \times \hat{G}_n$ of $X_1 \times X_2 \times \ldots \times X_n$ conjugate to fuzzy subgroups $H_1, H_2, \ldots, H_n$. Then $\hat{G}_1 \times \hat{G}_2 \times \ldots \times \hat{G}_n$of the group $X_1 \times X_2 \times \ldots \times X_n$ is conjugate to the $H_1, H_2, \ldots, H_n$ of $X_1 \times X_2 \times \ldots \times X_n$

The following results are found in p-level subgroups In chapter VI

99: Let 'X' a group and 'A' a t-fuzzy of X, then the P-level subset A_cT for c ε [0,1] , c < T(A(e) , r), is a subgroup of X, where of X, where 'e' is the identity of 'X'.

100: Let ‘A’ and ‘B’ be P-level subsets of the sets X and Y respectively and let c ε [0,1], then A×B is also a P-level subset of X×Y.

101: Let ‘X’ and ‘Y’ be two groups , A and B a t-fuzzy group of X and Y respectively. Then the P-level subset $(A\times B)_c t$, for c ε [0,1] is a fuzzy group of X×Y, where e_x and e_y are identies of X and Y respectively.

102: Let ‘X’ be a group and A_cT a P-level subgroup of X. If A is a normal t-fuzzy subgroup of X, then A_cT is a normal subgroup of X.

103 Let A and B be fuzzy subsets of X and Y respectively, T be a t-norm and c ε [0,1]. Then $A_cT \times B_cT = (AB)_cT$.

Characterization of interval valued anti fuzzy lefy h-ideals over hemi ring in interval valued anti fuzzy characteristic function

Introduction: Fuzzy set theory has been developed in many directions by many researchers and has evoked great interest among mathematicians working in different fields of mathematics, such as topological spaces , functional analysis , loop , group , ring , near ring, vector spaces , automation. There have been wide - ranging applications if the theory of fuzzy sets , from the design of robots and computer simulations to engineering and water resource planning. Since then many researchers have been involved in extending the concepts and results of abstract algebra to the broader frame work of the fuzzy setting. The notion of fuzzy left h- ideals in hemi ring was introduced in Y.V.Jun et.al.[2004] . The notion of (i-v) fuzzy set , a kind of well-known generalization of ordinary fuzzy set, was introduced by Zadeh [1965], Biswas [1994] investigate (i-v) fuzzy subgroup, wang and Li [2000] investigated TH- interval valued fuzzy subgroup and SH – interval valued fuzzy subgroup Zeng [2006] proposed concepts of cut set of (i-v) fuzzy set and investigated decompositions of theorems and representation theorems of (i-v) fuzzy set and so on. These works show the importance of (i-v) fuzzy set. In this paper, we apply the notion of (i-v) fuzzy sets to anti fuzzy left h- ideals of hemi ring. We introduce the notion of (i-v) anti fuzzy left h- ideals of R with respect to max norm and investigate some of their properties. Using lower level set, we give a characterization of $\max^{i}$ – anti fuzzy left h- ideal. Finally we establish the theorems of the homomorphic image and the inverse image.

Jun. Y.B [2004] showed that the fuzzy setting of a left h- ideal in a hemi ring is constructed and basic properties are investigated. Using a collection of left h- ideals of a hemi ring S are established. He also explained the notion of a finite valued fuzzy left h- ideals and its characterization. fuzzy relations on a hemi ring S are also discussed. X.P.Li, [2000] explained the idempotent interval co norm SH induced by a T- co norm on the space on the interval valued fuzzy sets on fuzzy groups and SH interval valued fuzzy groups. In the mean time some of its basic properties and structural characterizations are discussed. Also he showed that the theorems of the homomorphic image and the inverse image are given. D. M. Olson [1978] showed that the fundamental homomophisms theorems for rings is not generally applicable in hemi ring theory. He explained also the class of N- homomorphisms of hemi rings the fundamental theorem is valid. In addition, the concept of N- homomorphism is used to prove that every hereditarily semi subtractive hemi ring is of type (k). W. J. Liu [1987] proved that some basic concepts of fuzzy algebra as a fuzzy invariant subgroups, fuzzy ideals and some fundamental properties. He also showed that characteristic of a field by fuzzy ideals.

The following theorems are showed in chapter VI:

104: Let 'R' be a hemi ring and A be a fuzzy set in R then A is max ^{i} - anti fuzzy h- ideal in R if and only if A ^{c} is a fuzzy left h- ideal in R.

105: Let A and B are max ^{i} - anti fuzzy left h- ideal of R , then A $\cup$ B also max ^{i}- anti fuzzy left h- ideal in R.

106 Let f : $R_1 \rightarrow R_2$ be an onto homomorphism of hemi ring. If 'A' is max ^{i} - anti fuzzy left h- ideal of R_2, then f^{-1}(A) is a max ^{i} - anti fuzzy left h- ideal of R_1.

107: Let f : $R_1 \rightarrow R_2$ be an epimorphisms of hemi ring. Let 'A' be an f- invariant max ^{i} - anti fuzzy left h- ideal of R_1, then max ^{i} - anti fuzzy left h- ideal of R_2.

108: Let 'A' be max^i – anti fuzzy left h – ideal in a hemiring R such that L(A;α) is a left h – ideal of 'R' , for each α ε Im(A), α ε[0,1] then 'A' is max^i- anti fuzzy left h – ideal of 'R'.

109: Let 'A' be maxi – anti fuzzy left h – ideal in a hemiring R. Let A^+ be a fuzzy lower cut set in 'R' defined by $A^+(x)$ = A(x) +1 –A(0) for all x in 'R' then A^+ is lower cut of max^i – anti fuzzy left h – ideal in 'R' which contains A.

Section 7: Fuzzy α-cuts on fuzzy groups and L-fuzzy number

We give some properties on α-cuts fuzzy groups. The concept of fuzzy subset of a non-empty set first was introduced by Zadeh [1965] . In Rosenfeld formulated the concept of fuzzy subgroup of a group. This work was the first fuzzification of any algebraic structures and thus opened a new direction, new exploration, new path of thinking to mathematicians, engineers, computer scientists any many others in various ways of various tests. The fuzzy concept is taking the vital role in all engineering fields especially for the design of analysis part.

The following theorems are showed in chapter VII:

110: Let Ĝ be a fuzzy group and $\hat{G}_\alpha$ be its α -cut. Then $\hat{G}_\alpha$ forms a fuzzy group.
111: If $\hat{G}_\alpha^i$ and $\hat{G}_\alpha^j$ are α-cut fuzzy groups, then $\hat{G}_\alpha^i \cup \hat{G}_\alpha^j$ is also a α-cut fuzzy group if i < j
112: $\hat{G}_\alpha^i$ and $\hat{G}_\alpha^j$ are fuzzy groups then $\hat{G}_\alpha^i \cap \hat{G}_\alpha^j$ also a fuzzy group if i < j.
113: 'λ' is fuzzy number with convex function. Then λ is constant function.
114: Let a fuzzy number 'λ' be a convex function. Then (R, +, O) is a fuzzy group.

Some Structure Properties of Intuitionistic Fuzzy Group

We consider the intuitionistic fuzzification of the concept of groups and investigate some properties of such groups. In this paper defined intuitionistic fuzzy groups (IFG) intuitionistic normal groups (ING) and takes some characterizations of them.

The concept of Intuitionistic fuzzy set was introduced by Atanassov [1986] as a generalization of the notion of fuzzy sets. In this section, we present now some preliminaries on the theory of intuitionistic fuzzy sets. Since then it has been applied in wide varieties of field like computer science, management science, Medical sciences, Engineering problems etc., to list a few only. Consequently, there is a genuine necessity of a different kind of fuzzy sets. We consider the intuitionistic fuzzification of the concept of groups and investigate some properties of such groups.

The following theorems are showed in Intuitionistic fuzzy group in chapter VII:
115: If A is an intuitionistic Fuzzy group of 'X' Then I_A $(x^{-1}) = I_A$ (x) for all x ε X.

116: Unit Intuitionistic fuzzy set, Zero Intuitionistic fuzzy set and α -Intuitionistic fuzzy set of a group 'X' are intuitionistic fuzzy group of X.

117: A set of necessary and sufficient conditions for al intuitionistic fuzzy set 'A' to be an intuitionistic fuzzy group of X is that I_A $(xy^{-1}) \geq$ Min $\{I_A(x), I(y)\}$

118: If A and 'B' an two intuitionistic fuzzy groups of a group 'X' then A∩B is also intuitionistic Fuzzy Group of 'X'.

119: If 'A' and 'B' are two intuitionistic Fuzzy Group of a Group x then AUB is also intuitionistic Fuzzy group of 'X'.

120: If A = <,x, t_A, f_A > is an Intuitionistic fuzzy group of a group X then t_A is a fuzzy group of X and 1- f_A is a fuzzy group of 'X'

121: Let 'A' be an intuitionisitc fuzzy group of a group X then (α,β) – cut forms a subgroup of X for all α, β.

The following theorems are showed in Intuitionistic Fuzzy Semi group and subsemigroup in chapter VII:

122: **A**n intuitionistic fuzzy Sub semi group A = (t_A, f_A) of 'S' is called an intuitionistic fuzzy group of 'S' if $t_A(x^{-1}) \geq t_A(x)$; $f_A(x^{-1}) \leq f_A(x)$, and every intuitionistic fuzzy sub semi group of 'S' is constant.

123: Let 'A' be a intuitionistic normal group of a group X then I_A (x, y) = I_A(e) for all x, y $\in$ X.

Section 8: Generalized product and some algebraic properties of fuzzy group

In this section, the theory of fuzzy sets has developed in many directions and is finding applications in a wide variety of fields. Rosenfeld in 1971 used this concept to develop the theory of fuzzy groups in this paper we have given independent proof of several theorems on fuzzy groups. We discuss about Fuzzy groups and investigate some of their structures on the concept of Fuzzy Group family.

The following results are obtained in chapter VIII:

124: If G is a group, then prove that $G^m = \{ (x , \mu_G(x)^m / x \varepsilon G \}$ is a fuzzy group, and the fuzzy group G^n is a fuzzy subgroup of G^m , if m ≤ n.

125: If $\hat{G}^i$ and $\hat{G}^j$ are fuzzy groups, then $\hat{G}^i \cup \hat{G}^j$ is also a fuzzy group if i<j

126: $\hat{G}^i$ and $\hat{G}^j$ are fuzzy groups then $\hat{G}^i \cap \hat{G}^j$ also a fuzzy group if i < j. Thus if $< A >$ = $\{ A , A^1 , A^2,----, A^P,----, E\}$ then $\cup$ $A^P = A$ & $\cap$ A^P = E where p varies 1 to ∞.

Pseudo fuzzy cossets of fuzzy normal subgroups

We have given Independent proof of Several theorems on pseudo fuzzy cossets of fuzzy normal subgroups We introduce the notion of pseudo fuzzy double cossets, pseudo fuzzy middle cossets of a group and consider its fundamental properties

We are formulated the fuzzification of any algebraic structures and thus opened a new directions new exploration, new path of thinking to mathematicians engineers, computer scientists and many others in various ways of various tests. The fuzzy concept is taking the vital role in all Engineering fields especially for the design and Analysis port. Mukherjee and Bhattacharya [1986] introduced the fuzzy right cossets and fuzzy left cossets of a group. Here we introduce the notion of pseudo fuzzy cossets of a group and how they are related with fuzzy normal subgroups.

The following results are obtained in Pseudo fuzzy cosets In chapter VIII:

127: A fuzzy subset μ of a group 'G' is a fuzzy subgroup of $\hat{G}$ if and only if

$\mu (x y^{-1}) \geq$ Min { μ (x), μ (y) for every x, y in G

128: Let 'μ' be a fuzzy subgroup of a group 'G' then the pseudo fuzzy coset $(a \mu)^p$ is a fuzzy sub group of 'G' for every $a \in G$.

129: Every pseudo fuzzy double coset is a fuzzy subgroup of a group 'G'

130 Every fuzzy middle cosset of a group ' G' is a fuzzy subgroup of G.

131: Every pseudo fuzzy cosset is a fuzzy normal subgroup of a group 'G'

132 the intersection of two pseudo fuzzy cosset normal subgroup is also fuzzy normal subgroup of a group.

133: Pseudo fuzzy double coset is a fuzzy normal subgroup of a group 'G'

134: Fuzzy middle cosets forms a fuzzy normal subgroup of G.

135: Let μ_A be a strongest Fuzzy relation on 'S' and 'A^+_α' be the strong α-cut the μ_A forms a strong α- cut Fuzzy group on S.

136: Let λ and μ be strong Fuzzy α- cuts on S then $\lambda \times \mu$ is a strong Fuzzy group α- cut.

137: Let $\mu_A{}^i$ and $\mu_A{}^j$ be two strong Fuzzy relations and $A_\alpha{}^+$ be strong Fuzzy α- cut then $\mu_A{}^i{}_{UA}{}^j$ forms a strong Fuzzy α-cut on S.

138 Let μ_A be the strongest Fuzzy Relation on S then $A_\alpha{}^+$ is a strong α-cut then μ_A forms a Fuzzy compatible..

139: Let μ_A be a fuzzy compatible then it forms a strong fuzzy α-cut

Section 9: Multi-stage decision making approach to opti8mize the product mix in assignment level under fuzzy group parameters

Aryanezhad, M.B [2004] showed that one of the most important decisions made in production systems is determining the product mix in such a way that maximum throughput would be obtained. Several algorithms to determine the product mix under theory of constraints (TOC) have been developed. He also explained the inefficiency of the traditional TOC algorithm in handling the multiple bottleneck problem is discussed through an example, then the latest algorithm and its advantages will be discussed and an improved algorithm, which is much more efficient and can reach the optimum solution with considerable speed will be presented. Finally showed that the improved algorithm and the integer linear programme (ILP) method will be compared with each other through the examples.

Chen. J [1992] explained a method of fuzzy multi attribute decision making based on rough sets, the uniqueness, as well as the crucial part is that we define rough sets under priority relationship. He also establish priority class using priority relationship R as the categorization criteria, then we study the mathematical properties of the concepts, define the priority degree between two objects and define the summary priority degree as well, by using inclusion degree priority degree measurement. Using summarization method, we can get the evaluation value for a given object and thus study the multi-attribute decision making. Finally he showed the decision making example given of rough sets, which helps give as insight into the method of fuzzy multi decision making based as rough sets.

Inventory models in which the demand rate on the inventory level are based on the common real life observation that greater product availability tends to stimulate more sales. Theory of constraints (TOC) is a production planning philosophy that tries to improve the throughput of the system management of inventory levels. Due to the existing of inventory levels in a production system the demands of all products can not be fully met. So one of the most important decisions made in production systems is product mix problem. Although many algorithms have been developed in the fields using the concept of theory of constraints. This paper benefits from a variety of advantages. In order to consider the importance of all inventory levels, group decision making approach is applied and the optimal product mix is reached. In the algorithm presented in this paper, each inventory level is considered as a decision maker. The new algorithm benefits from the concept of fuzzy group decision making and optimizes the product mix problem in inventory environment where all parameters are fuzzy values.

Theory of constraints (TOC) is a production planning philosophy that aims to improve the system through put by efficient use of inventory levels. In this paper product mix optimization is considered as a decision making problem. Regarding this analogy decision making criteria should be first defined (5).Two important criteria are throughput and the later delivery cost. Later delivery cost is the most of mission one unit of each product. Assuming each inventory level as a decision maker, product mix optimization is a group decision making problem. In all previous researchers all parameters (such as processing time, demand etc) are assumed as crisp values.

A new algorithm is developed to optimize the product mix problem with all inputs are fuzzy values and Borda methods is used in group decision making process as ordinal techniques are preferred to cardinal ones.

Contributions for application of fuzzy group in decision making in chapter IX:

140: The improved algorithm benefits from the advantage of reaching optimal solution. In the previous researchers all inputs were considered as crisp values. The assumption is not in real cases. This paper considers product mix problem as a group decision making problem in which all inputs are fuzzy. In this paper, a new algorithm for optimizing product mix under fuzzy parameters is developed. For this method, ordering methods are used in order to make decision in a fuzzy group decision making environment.

PUBLICATIONS

1. " Generalized product of fuzzy groups and P- level subgroups" Journal of Applied Mathematical Analysis and Applications, vol.3 , No.2, PP 137-145 , (July- December} 2007.

2. "Q- fuzzy left R- subgroup of near rings w.r.t T- norms" Antarctica Journal of Mathematics , Vol.5 , No. 2, PP 59-63 , 2008.

3. "Characterizations of Weaker fuzzy groups in terms of Rosenfeld's fuzzy groups" Journal of Applied Mathematical Analysis and Applications,Vol.4, No.1-2 PP 37-43 , (January – December) 2008.

4. "A New Structure and Construction of Q-fuzzy groups" Advances in Fuzzy Mathematics,Vol.4,No.1,PP 23-29, 2009.

5. "Lattice Valued Q- fuzzy Sub modules of Near rings with respect to T- norms" Advances in Fuzzy Mathematics, Vol.4, No.2,(2009),PP 137-145.

6. "Characterization of interval-valued anti fuzzy left-h ideals over hemi rings" Advances in Fuzzy Mathematics, Vol.4, No.2(2009), PP 129-136.

ACCEPTED FOR PUBLICATIONS

1.S-Product of S-anti fuzzy right R-Sub groups of Near rings w.r.t. T-Norm" , Accepted For Publications Advances in Fuzzy sets & Systems (AFSS),2009.

"2.Multi- Stage Decision Making Approach To Optimize the Product Mix in Assignment Problem Under Fuzzy Parameters" Accepted For Publications in International Journal of Applied Mathematical Sciences (IJAMS)" 2009.

*3.*Enumeration of Maximal Chains and Flags interms of Finite Abelian groups" Accepted For Publications in Antarctica Journal of Mathematics, Vol.7, 2010.

*4.*Some Algebraic Properties of BCK algebra's and Fuzzy S- semi groups" Accepted For Publications in Antarctica Journal of Mathematics, Vol.7, 2010.

*5.*Some Structure Properties of M- fuzzy groups" Accepted For Publications in Applied Mathematical Sciences (AMS), 2010.

*6.*Some Structure Properties of Q-cyclic Fuzzy group Family" Accepted for Publications in Antarctica Journal of Mathematics, Vol.7, 2010.

7. Q-Fuzzy subgroups of beta fuzzy congruence relations on a groups" Accepted For Publications in International Journal of computer science, Network and (IJCNS), 2010.
8.Structure Properties of Q- Anti fuzzy left h- ideal in a hemi rings" International Journal of Computer Applications (IJCA), 2010.

SEMINAR'S / CONFERENCES ATTENDED & PRESENTED

1.Some Structure properties of Intuitionistic Fuzzy group" presented in National Seminar on Intellectual perspective of Mathematics, held on 5th April 2007 in Cauvery College for women, Trichy.

2."On d-algebra's" presented in State level seminar on Applied mathematics, held on 14th February 2007 in Bon Secure College for women, Thanjavur.

3."work shop on applications of Mathematics in Engineering and Technology" held on 6 sept 2002 at the PSNA College of Engineering and Technology, Dindigul.

4.UGC National seminar on Recent advances in pure and applied mathematics" held on 23rd September 2003 in kandaswami kandor's college, velur on 23.08.2003.

5."Third annual convention of I.S.T.E Tamilnadu and pondichery section" Modern Trends in Engineering Education" held on 18th November 2000 in Moderator Gnanadasan polytechnic, Nagarcoil.

6. "Workshop on Research Methodology" held on 30th January 2010, in JJ College of Engineerig & Technology, Trichy.

7.One day workshop on "Fuzzy logic and its Applications" held on 22nd March 2010 in Periyar Maniammai University, Vallam, Tanjore.

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ATTENDENCE

<table>
<tr><td colspan="3">The following staff members and research scholars were presented during the presentation of SYNOPSIS for the Ph. D. work of Mr. R. Nagarajan in PG & Research Department of Mathematics, Jamal Mohamed College (Autonomous), Tiruchirappalli – 620 020 at 03.00 pm on 28. 06. 2010.</td></tr>
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