Constructing a portfolio of investments is one of the most important financial decisions facing individuals and institutions. A decision-making process must be developed which identifies the appropriate weight each investment should have within the portfolio. A mathematical model is developed to optimize the portfolio selection using preemptive goal programming technique.

5.1 Introduction

Decision making regarding portfolio investments is one of the most significant financial decision in any type of institutions. The key point is identifying all sorts of different investments available and deciding the appropriate amount to be allocated within the portfolio. The best portfolio selection is one, which properly balances the risk and reward.

Essentially, the standard portfolio optimization problem is to identify the optimal allocation of the available limited resources based on the limited set of investments. In this juncture, the term optimality refers the tradeoff between the perceived risk and the expected return. For each type of investment both the values namely the perceived risk and the expected return can be computed based on the past data available related to the specific industry. Efficient portfolios are the allocations that achieve the highest possible returns for a given level of return without neglecting to reduce the risk level as much as possible.

In this chapter, Preemptive Goal Programming Model [PGPM] is discussed for the portfolio selection by considering the risk level and the expected return of various securities. This linear model taken care off two objective functions with the specified set of linear constraints. The objective functions are going to be considered based on the priority ratings. First priority is given to maximize the expected returns and the second priority is given to minimize the expected risk. To construct this model n different securities are considered. The constructed GPM model can be solved by using a special simplex method or TORA package.
5.2 Notations and Assumptions

Notations

\( n \) : \ n \) number of securities, where \( n \) is a fixed integer

\( i \) : \( i^{th} \) security \( [ i=1,2,\ldots,n] \)

\( P_i \) : \( i^{th} \) priority

\( r_i \) : expected return of the \( i^{th} \) security

\( \beta_i \) : risk level of the \( i^{th} \) security

\( a_i \) : minimum amount to be invested in the \( i^{th} \) security

\( b_i \) : maximum amount to be invested in the \( i^{th} \) security

\( A \) : the total amount available for investment in the portfolio

\( x_i \) : the proportion to be invested in the \( i^{th} \) security

\( s_i \) : the \( i^{th} \) security

\( r = [ r_1, r_2, \ldots, r_n ] \)

\( \beta = [ \beta_1, \beta_2, \ldots, \beta_n ] \)

\( x = [ x_1, x_2, \ldots, x_n ] \)

\( a = [ a_1, a_2, \ldots, a_n ] \)

\( b = [ b_1, b_2, \ldots, b_n ] \)
\[ e = [1, 1, \ldots, 1]_{1 \times n} \]

\[ \beta_0 : \text{the maximum possible total risk} \]

\[ R_0 : \text{the minimum expected total return} \]

**Assumptions**

- The first priority is to maximize the expected return.
- The second priority is to minimize the expected risk.
- It is possible to evaluate the corresponding risk level \( \beta_i \) \( [i = 1, 2, \ldots, n] \) and the expected return \( r_i \) \( [i = 1, 2, \ldots, n] \) based on the past data available.
- The summation of the total proportion should be equal to one.

**5.3 PGP Model Construction**

\( P_1 : \) maximize the expected return

\( P_2 : \) minimize the risk
Table: 10 Complete picture of the business problem

<table>
<thead>
<tr>
<th>Different Securities</th>
<th>Proportion to be Invested</th>
<th>Risk level</th>
<th>Expected return</th>
<th>Minimum amount to be invested in Rs.</th>
<th>Maximum amount to be invested in Rs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>s₁</td>
<td>x₁</td>
<td>β₁</td>
<td>r₁</td>
<td>a₁</td>
<td>b₁</td>
</tr>
<tr>
<td>s₂</td>
<td>x₂</td>
<td>β₂</td>
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</tr>
<tr>
<td>sₙ</td>
<td>xₙ</td>
<td>βₙ</td>
<td>rₙ</td>
<td>aₙ</td>
<td>bₙ</td>
</tr>
</tbody>
</table>

Total available fund limit is Rs. A
Based on the above data set, the Preemptive Goal Programming Model with the two goals can be constructed in the following way:

The two goals can be restated as

Maximize $P_1 = r x^T$

Minimize $P_2 = \beta x^T$

Subject to the restrictions

$$e x^T = 1$$

$$A x_i \geq a_i; \quad \text{for all } i = 1, 2, \ldots, n$$

$$A x_i \leq b_i; \quad \text{for all } i = 1, 2, \ldots, n$$

$$(A) e x^T = A$$

$$r x^T \geq R_0$$

$$\beta x^T \leq \beta_0$$

$$x \geq 0$$

5.4 Methodology

The constructed preemptive goal-programming model can be solved by using a special simplex method that guarantees the non-degradation of higher priority solutions. The method uses the column-dropping rule that calls for eliminating a non-basic variable $x_i$ with
\[ Z_i - C_i \neq 0 \] from the optimal tableau of goal \( G_k \) before the problem of goal \( G_{k+1} \) is optimized. This rule considers such non-basic variables, if increased above zero level in the process of optimization of succeeding goals, can degrade the quality of a higher priority goal. This process requires modifying the simplex tableau so it will carry the objective function of all the goals of the model. By changing the parameters \( R_0 \) & \( \beta_0 \) and solving multiple instances of this problem, one can generate different set of efficient portfolios. A graph can be drawn using the different sets of risk and return. The portfolio decision maker can decide and select the best point on the curve based on the acceptable balance between the risk and return.

### 5.5 Conclusion

The goal-programming model suggested in this paper is very simple and easily understandable by any financial decision makers. The model evaluates the optimum risk and optimum return of the portfolio. The computational complexity is reduced very much.