Preface

Mathematics is a central element of our current technology but few people realize that this celebrated high technology is so strongly based on Mathematics. The theory of variational inequalities is a powerful and elegant tool of the current mathematical technology and have become a rich source of inspiration for scientist and engineers. There are numerous standard textbooks and monographs dealing with various aspects of this domain. In the last four decades, this theory has been extended and generalized in various directions. Simultaneously, the theory of complementarity problems is grown up because of the applications to a wide class of mathematical models related to optimization, game theory, economics, engineering, mechanics, elasticity, fluid mechanics, stochastic optimal control, etc. For further detail on complementarity problems, we refer to a recent monograph by Isac [57] and references therein. There are three different aspects to study variational inequalities and complementarity problems (i) *Mathematical Modelling*: To convert the problems of real life or the problems from science, engineering and social sciences into a variational inequality problem/complementarity problem is called mathematical modelling. (ii) *Existence Theory*: To study the existence of solutions of variational inequalities/complementarity problems. (iii) *Numerical Methods*: To find the algorithms for computing the approximate solutions of variational inequalities/complementarity problems, which converge to the exact solution.

This thesis deals with existence theory and numerical methods of different kinds of variational inequalities, variational-like inequalities, variational inclusions and complementarity problems.

Chapter 1 deals with the brief introduction of variational inequalities, variational-like inequalities, variational inclusions and complementarity problems besides some basic definitions and results from the Functional Analysis.

In chapter 2, we consider the completely generalized nonlinear variational-like inequalities/inclusions with or without compact valued mappings. We first prove the existence of weak solutions of completely generalized nonlinear variational-like inequality problem in the setting of locally convex Hausdorff topological vector spaces. Secondly, we propose an iterative algorithm for computing the approximate solutions of completely generalized nonlinear variational-like inclusions with noncompact valued mappings in the setting of Hilbert spaces. We prove that the approximate solutions obtained by the proposed algorithm converge to the exact solution of our variational-like inclusion. We also prove that the existence of a solution of our problem. Some special cases are also discussed. In the last section of this chapter, we consider the random generalization of completely generalized nonlinear variational-like inclusions. The iterative algorithm for finding the approximate solutions, their convergence and existence of a solution are also discussed.

In chapter 3, we suggest the iterative methods for computing the approximate solutions of generalized nonlinear quasi-variational inclusion problems. The existence and convergence of solutions obtained by suggested algorithms are also studied. Several special cases are also mentioned.

In chapter 4, we consider two different classes of generalized co-quasi-variational inequalities in the setting of Banach spaces. By using the sunny nonexpensive retractions, we construct the projection iterative methods for finding the approximate solutions of our problems. Some existence and convergence results are also derived. In the last section, we consider the multivalued co-quasi-variational inequality problem for fuzzy mappings. Following the technique of the first two sections, we give an iterative algorithm and prove the convergence results for the approximate solutions obtained by proposed algorithm. The existence result for a solution of this problem is also investigated.

In chapter 5, we consider the generalized nonlinear variational inclusion problem (for short, GNVIP) in the setting of Banach spaces. Several special cases of (GNVIP) are also given. By using the resolvent operator technique for m-accretive operator defined on a Banach space, we convert our problem into a fixed point problem. This characterization is used to propose an iterative algorithm for computing the approximate solutions of (GNVIP). The convergence of approximate solutions obtained by the proposed algorithm and the existence of a solution of (GNVIP) are also studied. In the last section of this chapter, we extend the generalized nonlinear variational inclusion problem for fuzzy mappings. We also extend the iterative algorithm, and convergence and an existence result of second section of this chapter for fuzzy mappings.

Finally, in chapter 6, we study a class of generalized quasi-complementarity problems with fuzzy multivalued mappings and suggest a new algorithm for computing the approximate solutions of this class of generalized quasi-complementarity problems. We also discuss the existence of a solution of our problem without compactness assumption and the convergence of the iterative sequences generated by the algorithm. Some special cases are also given.

The section 3.2 and chapter 6 have been accepted for the publication in *Mathematical and Computational Applications* and *International Journal of Fuzzy Systems*, respectively.