Interest in the study of ultra-relativistic heavy-ion interactions has tremendously increased owing to the possibility of creation of a new phase of nuclear matter, called the quark-gluon plasma (QGP). However, search for the evidence of QGP formation has been presenting a big challenge for the experimental high energy physicists. It is because of the idea that even if QGP is formed, it will survive for only a fraction of total evolution time and will thus, it will be difficult for discerning its existence.

During the past several years, especially with the development of heavy-ion accelerators, a thorough study of the interesting aspects of nucleus-nucleus collisions at relativistic energies has become possible. The Relativistic Heavy-Ion Collider (RHIC) at BNL and Large Hadron Collider (LHC) at CERN are two major colliders especially dedicated to the study of ultra-relativistic nuclear collisions, in general and the production of QGP in the collisions, in particular. Availability of these colliders has enabled the high energy physicists to search conclusively for the evidence of QGP formation. However, it may be noted that creating an environment for QGP formation does not necessarily mean that all heavy-ion collisions at relativistic energies would produce QGP. It is believed that fluctuations in multiplicity distributions of hadrons produced in heavy-ion collisions at high energies may be used to examine whether the nuclear matter has undergone a phase transition into QGP.

One of the possible approaches for investigating the dynamics of multiparticle production in relativistic heavy-ion collisions is to investigate fluctuations in particle density distributions of the secondary particles produced in these collisions. These
fluctuations may arise due to statistical reasons or may be due to occurrence of an uneven phenomenon during the collisions. Such uneven or anomalous fluctuations in a single event are represented as peaks, often termed as 'spikes' in narrow pseudorapidity intervals. The observed fluctuations has been found to exceed significantly the statistical noise. Several experiments have also confirmed these early observations.

The main objective of the present study is to disentangle information relating to non-statistical fluctuations in particle densities in relativistic nuclear collisions. The occurrence of non-statistical fluctuations in the interactions of 14.5A GeV/c $^{28}$Si nuclei is investigated for experimental as well as FRITIOF generated data in terms of the scaled factorial moments, $F_q$, and multifractal moments, $G_q$.

Analysis of the data on A-A collisions in terms of multifractal moments brings out clearly the existence of self-similar nature of multiparticle production following linear rise of multifractal moments in the entire rapidity space; target mass dependence is also pronounced. Moreover, the mass exponents, $t_q$, obtained from the linear fits of $\ln <G_q>$ with $-\ln(\eta)$, increase with the order of the moments, $q$, and also reveal a strong target mass dependence for both the experimental as well as FRITIOF data. Our study also confirms the dynamical contribution to the fluctuations in a definite sense as $t_q^{\text{dyn}}$ differ appreciably from $t_q$. Furthermore, the generalized dimensions, $D_q$, obtained from mass exponents $t_q$, exhibits decreasing trend with the order of the moments, $q$, for both the data sets. This result is in conformity with the predictions of the Multifractal Cascade Model. Nevertheless, there is some departure in this trend at $q = -3$ and $q = 5$. This deviation may attributed to some unknown statistical
errors in the data. It may further be noted that there exists a strong target mass
dependence in the behaviour of generalized dimensions $D_q$. The plots of $D_q$ with $q$ for
experimental, Monte Carlo and FRITIOF data sets reveal that $D_q$ differs markedly
for Monte Carlo and FRITIOF generated events for $q \leq -3$. But the behaviours of
$D_q$ for $q > -3$ are almost similar for all the three data sets considered in the present
work.

The multifractal spectra $f(\alpha_q)$, which characterize completely the dynamics of
the multiparticle production, provide no conclusive evidence for the smoothness of
rapidity distribution. In order to critically examine the relationships amongst the
spectra for different targets, $f(\alpha_q)$ spectra are rescaled for three groups of targets.
The results suggest universality in the multifractal structure for $\alpha_q < 1$ and tend
to become more broader along the $\alpha_q$ axis. The rescaled spectra for Monte Carlo,
FRITIOF and experimental data hint towards the fact the multiplicity fluctuations
are of multifractal nature and not of monofractal type.

Similar to the phenomenon of multifractality, there is an approach known as inter­
mittency proposed by Bialas and Peschanski for studying non-statistical fluctuations
in multiparticle production. The scaled factorial moments linearly increase with de­
creasing bin size for both types of data for all the three groups of interactions. Thus,
the predicted power-law behaviour of the scaled factorial moments has been observed
in the present study, indicating thereby the presence of non-statistical fluctuations
for all the three categories of interactions.

Linear dependence of $\ln < F_q >$ on $-\ln \eta$ yields the value of intermittency indices,
\( \phi_q \), which are related with the strength of intermittency effect. The value of slopes, \( \phi_q \), show an increasing trend with q for both the data sets for all the three classes of collisions. Furthermore, the patterns of the variations for the three groups of targets, namely, CNO, emulsion and AgBr are essentially similar. The scaling behaviour of factorial moments has been related to the physics of fractal objects (particle emission sources), through the anomalous dimensions, \( d_q \), which have been computed from the slopes \( \phi_q \) using the expression \( d_q = (\frac{\phi_q}{q-1}) \). This is used to examine the fractal nature of particle emission source. It is observed that the anomalous dimensions increase with the order of the moments, q. The trends of variations of \( d_q \) exhibit almost similar pattern for both the experimental and FRITIOF data for all the three types of interactions.

The presence of non-thermal phase transition in relativistic nuclear collisions may be due to intermittent behaviour in the final state of multiparticle production. In order to investigate this aspect \( \lambda_q \) are plotted against q. The \( \lambda_q \) versus q plots show distinct minima at \( q_c = 4 \) for both data sets and for all the three types of interactions. This may be a positive signal for the occurrence of the non-thermal phase transition.

Multiparticle final state in high energy collisions under self-similar multifractal system is characterized by a parameter known as Levy index, \( \mu \). It gives a measure of the degree of multifractality. The Levy index, \( \mu \), for the hadronic system in 14.5A GeV/c \( ^{28}\text{Si}-\text{nucleus} \) collisions is found to be 1.509 \( \pm 0.059 \) and for the FRITIOF data the value of \( \mu \) turns out to be 1.489 \( \pm 0.040 \). These results support the Levy stability theory.
The Renyi dimensions, \( D_q \), and multifractal spectra \( f(\alpha) \) are calculated for \( q \) ranging between 1.0 to 9.0, in steps of 0.2: using the relations i) \( D_q = \frac{F_q}{q-1} \) and ii) \( f(\alpha) = q\alpha - T_q \). Renyi dimensions, \( D_q \) are observed to decrease with \( q \), indicating thereby the occurrence of multifractal behaviour in multiparticle production in nuclear collisions at high energies. Furthermore, multifractal spectra \( f(\alpha) \) attain their maxima at around \( \alpha = 5 \) for both experimental and FRITIOF data and tend to decrease thereafter up to \( \alpha = 7 \) for the FRITIOF data and \( \alpha = 9 \) for the experimental data.

The methods of studying non-statistical fluctuations also give better interpretation of thermodynamic behaviour of multiparticle production in high energy nuclear collisions. Multifractal specific heat, \( c \), is computed using \( F_q \) moments, modified \( G_q \) moments and Takagi moments for the experimental and FRITIOF data for the three categories of interactions. The common feature of the multifractal specific heat, \( c \), obtained from all the three approaches is that there is no systematic variation in the value of multifractal specific heat, \( c \), for different order of the moments. The differences in the value of multifractal specific heat obtained from the three methods are mainly due to different values of \( D_q \) determined in terms of the parameter used in the three approaches. Further data are yet required to formulate any general conclusion in respect of multifractal specific heats in high energy interactions.