PREFACE

This thesis entitled "Mathematical Programming in Sampling" is submitted to the Aligarh Muslim University, Aligarh, India, to supplicate the degree of Doctor of Philosophy in Statistics. It consists of the research work carried out by me in the department of Statistics and Operations Research, Aligarh Muslim University, Aligarh, India.

Mathematical programming is concerned with the determination of the minimum or maximum of a function of several variables which are subject to a number of constraints. Such situations exist in diverse fields of human activities e.g., engineering sciences, operations research, management sciences, computer science, numerical analysis, system analysis, economics, military operations, transportation, assignment, medical sciences, agriculture, statistical analysis etc., etc.

This thesis is an attempt to formulate some problems arising in multivariate sample survey and cluster sampling designs as a problem of mathematical programming. Attempts have also been made to develop computational procedures to solve these problems. The computational technique is used that of dynamic programming.

This thesis consists of five chapters. Chapter 1 provides an introduction to the mathematical programming, a brief history of mathematical programming techniques and its applications to various fields including sampling are presented.
In Chapter 2 the problem of obtaining a compromise allocation in multivariate stratified sampling is formulated as a nonlinear programming problem (NLPP) when the population means of various characteristics are of interest. Using the separability of the objective function and the constraint the NLPP is viewed as an L-stage decision problem which is further decomposed into L-single stage-single variable decision problems to make use of dynamic programming technique. Numerical illustrations are also presented. Few particular cases of the problem are also indicated.

This chapter is based on my joint research paper Khan, M.G.M., Ahsan, M.J. and Khan, E.A. (1995) entitled "On Compromise Allocation in Multivariate Stratified Sampling" presented in the III International Symposium on Optimization and Statistics, held during Dec.19-21, 1995, at the Aligarh Muslim University, Aligarh (INDIA) and is due to appear in its proceedings. The paper is also submitted for publication in the Naval Research Logistics (vide their manuscript # 2091).

In multivariate surveys where more than one characteristics are under study the optimum allocation of the sample sizes to various strata becomes complicated due to fact that an allocation that is optimal for one characteristic may be far from optimal for other characteristics. To deal with this situation a suitable overall optimality criterion is to be worked out. Chatterjee(1967) gave a compromise allocation by minimizing the sum of the proportional increases in the variances due to the use of non-optimum allocation assuming the
measurement cost with respect to various characteristics in a particular stratum as constant.

In Chapter 3 a generalized compromise allocation is proposed in which the cost of measurement varies with stratum as well as with various characteristics under study. The problem of obtaining the optimum compromise allocation is formulated as a mathematical programming problem which is solved by modified dynamic programming technique proposed by the author. A numerical illustration is also presented.


In Chapter 4 the problem of determining a compromise allocation is formulated as an integer NLPP with a convex objective function and single linear cost constraint with additional restrictions on the sample sizes to avoid oversampling, to made available the estimates of individual stratum variances and to get integer values of sample sizes. Using separability of the functions involved in the problem the use of dynamic programming technique is suggested to solve the resulting problem. Two numerical examples are also presented alongwith the comparison of the proposed allocation with other existing allocations. The computer programs, in C-language, for obtaining integer compromise allocation using dynamic programming are also given for both the examples.

This chapter is based on my joint research paper entitled

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"Compromise Allocation in Multivariate Stratified Sampling: An Integer Solution" submitted for publication in the Naval Research Logistics. The paper is thoroughly revised in the light of the comments of the learned referees and re-submitted in November, 1995 (vide their manuscript # 2219).

Chapter 5 deals with the problem of determining the optimum cluster size and the sample size in without replacement simple random sampling of clusters of equal sizes. The problem is formulated as an NLPP which is simplified using a double transformation. The resulting NLPP is then solved using two different approaches. The functions involved in the problem are separable with respect to cluster size and sample size this enables us to make use of dynamic programming technique. On the other hand the Lagrange multipliers technique is also used to obtain the explicit formulas for the optimum cluster and sample sizes under certain assumptions. A numerical example is also presented to illustrate both the approaches. The Kuhn-Tucker necessary conditions which are sufficient also for the transformed problem are also verified at the optimum solution.

This chapter is based on combination of two of my research papers. The first paper Khan, M.G.M., Jahan, N. and Ahsan, M.J. (1994) entitled "Determining the Optimum Cluster Size" presented in the Fourth Islamic Countries Conference on Statistical Sciences (ICCS-IV), held during Aug.27-31, 1994 at Lahore (PAKISTAN) and is appearing in its proceedings (Vol.VII). The second paper, Khan, M.G.M., Jahan, N. and Ahsan, M.J. (1995) entitled "On Optimum Choice of the Sampling Unit" was presented
in the III International Symposium on Optimization and Statistics, held during Dec. 19-21, 1995, at the Aligarh Muslim University, Aligarh (INDIA) and is due to appear in its proceedings. The paper is also submitted for publication in the Naval Research Logistics (vide their manuscript # 2104).

A comprehensive list of references is presented at the end of the thesis.